

RAFFLES INSTITUTION 2019 Year 6 Preliminary Exam Higher 2

# **Knowledge and Inquiry**

Paper 2

9759/02

24 September 2019

2 hours

Additional Materials: Answer Paper

## READ THESE INSTRUCTIONS FIRST

Do not turn this sheet over until you are told to do so. Write your name and CT group on all the work you hand in. Write in dark blue or black ink. Do not use staples, paper clips, highlighters, glue or correction fluid/tape.

Section A

Answer Question 1.

Section B

Answer any **two** questions.

At the end of the examination, fasten your answers to each section separately. You will be asked to submit your answer to each section separately.

The number of marks is given in brackets [] at the end of each question or part question.

### You must answer question one.

#### 1 Why on earth is mathematics beautiful? Does it matter?

Looking at two contrasting solutions to a mathematical problem, I am struck by how different they are, but also by how I am instantly drawn to one over another. Not because one is explained better than the other (both are described well, in fact), but because one is shorter. The longer one doesn't quite get to the heart of the matter; it is a bit cluttered with unnecessary distractions, although it does eventually answer the question. The other uses a different approach, which captures the essence of the ideas – it helps the reader to understand *why* this piece of mathematics works this way, not just that it does. For a mathematician, the "why" is critical, and the best mathematical solutions are the ones that answer the question simply and elegantly. It is no different from Science adopting the law of parsimony – that we choose the simpler solution of two competing hypotheses that make the same predictions. After all, all assumptions introduce possibilities for error; if an assumption does not improve the accuracy of a theory, its only effect is to increase the probability that the overall theory is wrong. If it matters to Science, then, it should, all the more, matter to Mathematics.

Besides, there is something beautiful about a beautiful proof. Euler's equation  $e^{i\pi} + 1 = 0$ , for example, demonstrates this very well. In addition to containing three of the basic arithmetic operations once each (addition, multiplication, exponentiation), it also knits together five fundamental mathematical constants: i) the number 0, ii) the number 1, iii) the number  $\pi$  (pi), which is used everywhere in Euclidean geometry, iv) the number *e*, the base of natural algorithms and a figure that is widely used in mathematical analysis, and v) the number *i*, the imaginary unit of complex numbers, which is crucial in providing insight into algebra and calculus. All that information, in such an elegant and powerful package.

Some might argue that we should not concern ourselves with beauty in mathematics – computer proofs are long, tedious, and inelegant, but they unmistakably prove theorems that humans have been unable to prove for centuries. The formal proof of the Four Colour Theorem is just but one case in point. Others argue that demanding beauty in mathematics may be inconsequential or just part of someone's lofty ideal, but that would be going against centuries of mathematicians who valued beauty. Henri Poincare once quipped, "The feeling of mathematical beauty, of the harmony of numbers and forms, and of geometric elegance, is a true aesthetic feeling that all real mathematicians know", and the mathematical community is almost unanimous in agreeing with Poincare. Lest you think that this demand for beauty is really inconsequential, consider the fact that it has resulted in significant breakthroughs over the years. Mathematical Physicist Hermann Weyl chose beautiful proofs which eventually resulted in him reconciling electromagnetism with relativity; Nobel Physicist Paul Dirac made his most impressive discoveries and predictions, such as that of the positron, by demanding elegant, simple mathematical descriptions.

I think that mathematical principles are experienced as beautiful because they point directly to the fundamental structure of the universe. After all, the reason why mathematics works so well, and so elegantly, is because the universe is ultimately just mathematics – mathematical structures and the relations that connect them constitute the ultimate irreducible "stuff" of which our world is made. There is an aesthetic nature to mathematics, and it matters because it is only when we learn to appreciate the beauty in mathematics and begin to demand it in mathematical inquiry can we truly continually experience breakthrough. We may never really know the source of mathematical beauty, but we do know that this is an ancient affinity that has never left humankind. As G. H. Hardy said, "Beauty is the first test; there is no permanent place for ugly mathematics."

Adapted from "Mathematics is Beautiful" by Vicky Neale

Critically evaluate the above argument with reference to the nature and construction of knowledge in mathematics. Respond with your own critical comments to support or challenge the author's position. [30]

### Answer any two questions.

2 The process of human unification has taken two distinct forms: establishing links between distinct groups, and homogenising practices across groups. Links may be formed even between groups that continue to behave very differently, even between sworn enemies. War itself can generate some of the strongest of all human bonds. It spreads ideas, technologies and people far more quickly than commerce. In 1918, the United States was more closely linked to Europe than in 1913. The two then drifted apart in the interwar years, only to have their fates meshed together inextricably by the Second World War and the Cold War. War also makes people far more interested in one another. Never had the US been more closely in touch with Russia than during the Cold War, when every cough in a Moscow corridor sent people scrambling up and down Washington staircases. People care far more about their enemies than about their trade partners. And look where we are today: the sworn enemies of yesterday have become closer friends today despite them continuing to behave very differently. So, war is not necessarily not a means of achieving unification across very different people groups.

Homogenising practices across groups also contributes to human unification, but in my opinion, may not be the best way to do so. Uniformity is not the same as unity, and emphasising uniformity often does not cause people to unite. There is no one fixed way of doing things. Also, getting everyone to adopt the same practice just isn't feasible, and might even result in rebellion and insurgency.

Finding the best way to unification requires a lot of understanding, sensitivity and wisdom, which conflict might just be better at helping us achieve.

Critically assess the reasoning in this argument, explaining why you do or do not accept its conclusion(s).

[15]

3 Globally, a staggering 40% of men smoke compared to only 9% of women. In developing countries, the percentage is higher, at 50% of men smoking compared to 35% in developed countries. The reasons for the disproportionately higher percentage in men could be because men smoke to control their emotions, since they are known to not be able to appropriately manage their emotions compared to women. Research has shown that men were found more likely than women to grab a cigarette if they felt stressed, angry, anxious, or sad. Another reason could be peer pressure: choosing not to join your friends for a smoke break on the patio would mean that you would end up being alone. Social pressure to exhibit a certain ruggedness, masculinity, independence or mystery could also be another reason.

But smoking is a very unhealthy way to control one's emotions, especially given the known negative consequences associated with habitual smoking. While smoking provides a temporary relief to the negative emotions one feels, it does not and cannot address the root causes of those emotions, whether they be relational conflict, workplace problems, or stressful situations. In fact, it might even cause people to not want to address the root causes of problems, since it provides a convenient escape route, and lulls people into believing that things are okay now that they feel better after a puff. We need to raise awareness of these facts and actively work to discourage smoking. Maybe we should start by making cigarette packs pink in colour to discourage men from smoking.

Critically assess the reasoning in this argument, explaining why you do or do not accept its conclusion(s).

[15]

4 Everyone talks about environmental and wildlife conservation in the name of protecting whatever we have left, but has anyone actually asked whether the assumptions of this approach are justified?

Western-style conservation projects and ecotourism are, in fact, harming wildlife, damaging the environment, and displacing and criminalising local people. The pristine beaches and wildlife tours demanded by overseas tourists has led to developments that do not benefit wildlife, such as beaches being built, mangroves stripped out, waterholes drilled and forests cleared. And when wildlife reserves are established, local communities can suddenly find that their everyday subsistence activities, such as hunting and collecting wood, have been outlawed. Some attempts to conserve wildlife have ended up pitting local communities against conservationists because they are regarded as unjust impositions, despite their presumably good intentions. The international conservation movement and their supporters assume that they are making ethical and environmentally sound decisions to save wildlife such that the ends justify the means, but is this really true? They are, in fact, supporting practices that have counterproductive, unethical and highly unjust outcomes.

And what does environmental conservation even mean anyway? Retaining the "natural look and feel" of a rainforest even though bulldozers have ploughed through a big chunk of the area to create so-called eco-resorts replete with man-made swimming pools, treehouses and concrete paths does not constitute conservation. Even a 10% alteration to a natural environment can be detrimental. The assumption that the picture-perfect landscape of an eco-friendly Carribean beach or tropical resort is a natural feature couldn't be further from the truth; tourist playgrounds are manufactured environments, and the more naïve people are about this fact, the worse it will be for environmental and wildlife conservation.

Critically assess the reasoning in this argument, explaining why you do or do not accept its conclusion(s).

[15]

End of Paper