TEMASEK JUNIOR COLLEGE, SINGAPORE
Preliminary Examination 2015
Higher 2 Mathematics Paper 2

Section A: Pure Mathematics [40 marks]

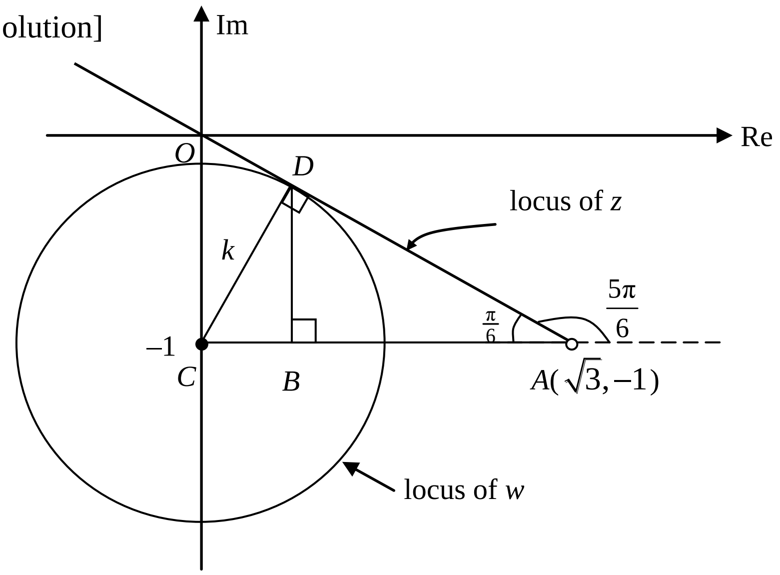
1 Sketch, on an Argand diagram, the locus of the point representing the complex number z such that $\arg(z-\sqrt{3}+\mathrm{i})=\frac{5\pi}{6}$. [2]

Give a geometrical description of the locus of the point representing the complex number w such that $|w+\mathrm{i}|=k$, where k is real. [1]

(i) Given that the two loci intersect at exactly one point, show that $k = a$ or $k \geq b$ where a and b are real constants to be determined. [3]

(ii) In the case when k takes the value of a, find the complex number representing the point of intersection, in the form $x + \mathrm{i}y$, where x and y are exact. [3]

[Solution]



Locus – <u>half line with correct argument and excluded point, passes through origin</u>

The locus of w is a <u>circle centered at (0, -1) and radius k units</u>.

(i) Two loci intersect exactly at one point: $k = CD$ or $k \geq AC$

In triangle ACD: $\sin\frac{\pi}{6}=\frac{CD}{\sqrt{3}} \Rightarrow CD=\frac{\sqrt{3}}{2}$

and $AC=\sqrt{3}$, $\therefore k=\frac{\sqrt{3}}{2}$ or $k\geq\sqrt{3}$

i.e. $a=\frac{\sqrt{3}}{2}$, $b=\sqrt{3}$

(ii) Consider triangle BCD: $\angle BCD = \frac{\pi}{2}-\frac{\pi}{6}=\frac{\pi}{3}$

$$BC = k\cos\frac{\pi}{3}=\frac{\sqrt{3}}{2}\left(\frac{1}{2}\right)=\frac{\sqrt{3}}{4},$$

$$BD = k\sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}\left(\frac{\sqrt{3}}{2}\right)=\frac{3}{4}$$

At point D: $x=\frac{\sqrt{3}}{4}, y=-(1-\frac{3}{4})=-\frac{1}{4}$, $\therefore$ complex number representing point D = $\frac{\sqrt{3}}{4}-\frac{1}{4}\mathrm{i}$

Alternatively,

$OD = 1\cos\frac{\pi}{3} = \frac{1}{2}$

$\therefore$ complex number representing point D = $\frac{1}{2}\left(\cos\left(-\frac{\pi}{6}\right)+\mathrm{i}\sin\left(-\frac{\pi}{6}\right)\right) = \frac{1}{2}\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)$

$= \frac{\sqrt{3}}{4}-\frac{1}{4}\mathrm{i}$

2 **(a)** Given that the sequence $5,\ 11,\ 17,\ \cdots,\ x$ is arithmetic, solve the equation

$$5+11+17+\cdots+x = 2760.$$ [4]

(b) Mr Tan set aside \$80,000 for his two sons. On the first day of the year that his sons turned 7 and 17 years old, he deposited \$$x$ into the younger son's bank account and the remaining sum of money into the elder son's bank account. Mr Tan adds a further \$1000 into the younger son's account on the first day of each subsequent year. The bank pays a compound interest at a rate of 2% per annum on the last day of each year. Each son will withdraw the full sum of money from his account (after interest had been added) on the last day of the year that he turns 21 years old.

(i) Find the amount of money the elder son will withdraw in terms of x. [1]

(ii) Show that the younger son will withdraw $\$\left(1.02^{15}x+51000\left(1.02^{14}-1\right)\right)$. [3]

Find the value of x if Mr Tan wanted both sons to receive the same withdrawal amount, giving your answer to the nearest integer. [2]

[Solution]

2(a) Let x be the nth term of the given AP.

$x = 5+6(n-1)$

$\quad = 6n-1$ --------- (1)

$\frac{n}{2}(5+x) = 2760$ ---------- (2)

Sub (1) into (2): $3n^2+2n-2760=0$

$\Rightarrow\ n=30$ or $n=\frac{-92}{3}$ (reject as $n\in\mathbb{Z}^+$)

$\Rightarrow\ x = 179$

2(b) **(i)** At the time of withdrawal, the amount in the elder son's account

$=1.02^5(80000-x)$

(ii)

Year	Balance at the end of the year in younger son's account
1(age 7)	1.02(x)
2(age 8)	1.02(1.02(x) + 1000) = 1.02^2 (x) + 1.02 (1000)
3(age 9)	1.02 (1.02^2 (x) + 1.02 (1000)+1000)
	=1.02^3 (x) + 1.02^2 (1000)+1.02(1000)
…	
15(age 21)	1.02^{15}(x) + 1.02^{14} (1000)+ 1.02^{13} (1000)+…+ 1.02(1000) [B1] AEF

At the time of withdrawal, the amount in the younger son's account

$$=1.02^{15}(x)+1.02(1000)\left(\frac{1.02^{14}-1}{1.02-1}\right)$$

$$=1.02^{15}(x)+51000\left(1.02^{14}-1\right)$$

If they should receive the same amount of money at the time of withdrawal,

$$1.02^{5}(80000-x)=1.02^{15}(x)+51000\left(1.02^{14}-1\right)$$

$$x=\frac{1.02^{5}(80000)-51000\left(1.02^{14}-1\right)}{1.02^{15}+1.02^{5}}=29401.85$$

Therefore, \$$x$ = \$29402 (correct to nearest integer)

3 **(a)** By considering a standard series expansion, find the general solution of the differential equation $x=1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)+\frac{1}{2!}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2+\frac{1}{3!}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^3+\cdots+\frac{1}{r!}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^r+\cdots$. [4]

(b) An empty rectangular tank has vertical sides of depth H metres and a horizontal base of unit area. Water is pumped into the tank at a constant rate such that if no water flows out, the tank can be filled up in time T seconds. Water flows out at a rate which is proportional to the depth of water in the tank. At time t seconds, the depth of water in the tank is x metres.

When the depth of water is 1 metre, it remains at this constant value. Show that

$\frac{\mathrm{d}x}{\mathrm{d}t}=k(1-x)$, where k is a constant in terms of H and T. [2]

Find x in terms of t, H and T. [4]

[Solution]

3(a) $x=1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)+\frac{1}{2!}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2+\frac{1}{3!}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^3+\cdots+\frac{1}{r!}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^r+\cdots$

$$=\mathrm{e}^{\frac{\mathrm{d}y}{\mathrm{d}x}}$$

i.e. $x=\mathrm{e}^{\frac{\mathrm{d}y}{\mathrm{d}x}} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\ln x$

Integrate wrt x:

$$y=\int \ln x\,\mathrm{d}x = x\ln x-\int x\cdot\frac{1}{x}\,\mathrm{d}x$$

$$=x\ln x - x + C$$

3(b) "In" rate: $\dfrac{H\cdot 1}{T}=\dfrac{H}{T}$

"Out" rate: px

$$\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{H}{T}-px$$

(Note $V=1\times x$ as horizontal base is of unit area) $\therefore \dfrac{\mathrm{d}V}{\mathrm{d}t}=\dfrac{\mathrm{d}x}{\mathrm{d}t}$

When $x=1, \dfrac{\mathrm{d}x}{\mathrm{d}t}=0$

$$\Rightarrow 0=\frac{H}{T}-p$$

$$\therefore p=\frac{H}{T}$$

$$\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{H}{T}-\frac{H}{T}x=\frac{H}{T}(1-x)$$

i.e. $k=\dfrac{H}{T}$

$$\frac{1}{1-x}\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{H}{T}$$

Integrate wrt t :

$$\int\frac{1}{1-x}\,\mathrm{d}x=\frac{H}{T}\int 1\,\mathrm{d}t$$

$$\Rightarrow -\ln|1-x|=\frac{H}{T}t+C$$

$$1-x=\pm\mathrm{e}^{-C}\mathrm{e}^{-\frac{H}{T}t}$$

$$\therefore x=1-A\mathrm{e}^{-\frac{H}{T}t}$$

When $t=0, x=0$

$$\Rightarrow 0=1-A \quad \therefore A=1$$

$$\therefore x=1-\mathrm{e}^{-\frac{H}{T}t}$$

4 The curve C has equation $y=\dfrac{a+bx^2}{b+ax^2}$ where $x\in\mathbb{R}$ and, a and b are constants such that $0<a<b$.

(i) **(a)** Using an algebraic method, find the range of values of y in terms of a and b. [3]

(b) Find the equation of the asymptote and the coordinates of the stationary point of C. [4]

(c) Sketch C, indicating the intercept(s) on the axes. [1]

(ii) Given that $b=2a$, find $\displaystyle\int \frac{a+bx^2}{b+ax^2}\,\mathrm{d}x$. [3]

[Solution]

(i) **(a)**

$$y=\frac{a+bx^2}{b+ax^2} \Rightarrow (ay-b)x^2+(by-a)=0$$

Since $x\in\mathbb{R}$,

discriminant = $0-4\,(ay-b)\,(by-a)\ge 0$

$$(ay-b)\,(by-a)\le 0$$

$\because y\neq\dfrac{b}{a}$ since coefficient of x^2 cannot be zero

$$\therefore \frac{a}{b}\le y<\frac{b}{a}$$

Alternative

$$y=\frac{a+bx^2}{b+ax^2} \Rightarrow by+ayx^2=a+bx^2$$

$$x^2=\frac{a-by}{ay-b}$$

Since $x^2\ge 0$, $\dfrac{a-by}{ay-b}\ge 0 \Rightarrow (a-by)(ay-b)\ge 0$

$$\therefore \frac{a}{b}\le y<\frac{b}{a}$$

(b)

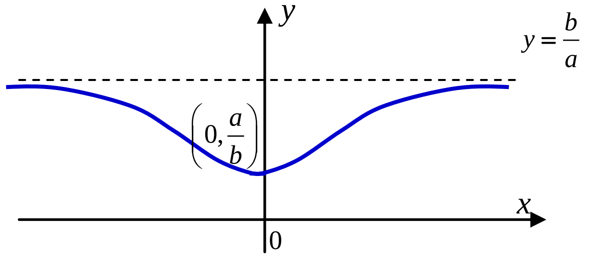
$$y=\frac{a+bx^2}{b+ax^2}=\frac{b}{a}+\frac{-\frac{b^2}{a}+a}{b+ax^2}=\frac{b}{a}+\frac{a^2-b^2}{a\left(b+ax^2\right)}$$

Equation of asymptote: $y=\dfrac{b}{a}$

$$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-\left(a^2-b^2\right)}{a\left(b+ax^2\right)^2}2ax=\frac{2x\left(b^2-a^2\right)}{\left(b+ax^2\right)^2}=0$$

$\Rightarrow x=0,\ y=\dfrac{a}{b}$ Coordinates of stationary point is $\left(0,\ \dfrac{a}{b}\right)$.

(c)



(ii) Given that $b=2a$, $\int \frac{a+bx^2}{b+ax^2}\,dx = \int \frac{a+2ax^2}{2a+ax^2}\,dx = \int \frac{1+2x^2}{2+x^2}\,dx$

$$= \int 2 + \frac{-3}{2+x^2}\,dx$$

$$= 2x - 3\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + C$$

Section B: Statistics [60 marks]

5 A popular online retail store sells and provides home delivery of clothes and electrical appliances. The store manager wants to find out from the store customers their opinions on the quality of the products they have purchased.

(i) Describe how systematic sampling can be carried out with 5% of the customers. [2]

(ii) Explain briefly why the sample obtained in part **(i)** may not be a representative sample. [1]

[Solution]

5(i) Arrange all customers in an ordered list, using customer's identification number or other unique identification code/email address (Note: Not home address of customer as there may be more than 1 customer, like another family member, also using the same online retail store for purchases).

Assign each customer a number according to the list.

Randomly select a number and select the first customer with this number.

Select every 20th customer thereafter, going back to the beginning of the list when it goes beyond the last number until the desired sample size is obtained.

(ii) The sample may consist of customers who bought clothes only from the online retail store. Hence the customers who bought electrical appliances may not be represented in the sample.

6 Find the number of ways to arrange the twelve letters of the word PERSEVERANCE

(i) in a row, [1]

(ii) in a circular way such that the vowels are placed adjacent to one another. [2]

Four letters are selected at random from the twelve letters of the word PERSEVERANCE to form a code word. The code word can contain at most two identical letters. Find the number of possible code words. [3]

[Solution]

4E , 2R, P, S, V, A, N, C

6 (i) Number of ways = $\frac{12!}{4!2!} = 9979200$

(ii) Number of ways = $\frac{(8-1)!}{2!} \times \frac{5!}{4!}$ = 12600

Number of possible code words

$$= {}_8P_4 + \binom{7}{2}\binom{2}{1}\times\frac{4!}{2!} = 1680 + 504 = 2184$$

No repeat   2E or 2R

Alternative Method

Case 1: 0R0E ${}_6P_4 = 360$

Case 2: 1R1E $\binom{6}{2}\times 4! = 360$

Case 3: 1R0E or 0R1E $\binom{6}{3}\times 4!\times 2 = 960$

Case 4: 2R0E or 0R2E $\binom{6}{2}\times\frac{4!}{2!}\times 2 = 360$

Case 5: 2R1E or 1R2E $\binom{6}{1}\times\frac{4!}{2!}\times 2 = 144$

Total number of possible code words = 2184.

7 The manufacturer of Mola snack biscuits claims that the mean biscuit content in a packet is 15 g. A random sample of 80 packets is taken and the weight, x g, of the content in each packet is measured. The results are summarised by

$$\sum(x-15) = 53.6 \text{ and } \sum(x-15)^2 = 795.7.$$

(i) Calculate unbiased estimates of the population mean and variance. [2]

(ii) Test whether the mean weight differs from 15 g at 4% level of significance. [4]

(iii) Another large random sample of n packets is taken. Assume now that the population standard deviation is 3 g. Find the range of values of the sample mean, in terms of n, for which the claim that the mean weight differs from 15 g is supported at 4% level of significance. [3]

[Solution]

7(i) $\bar{x} = \dfrac{\sum(x-15)}{80} + 15 = 15.67$ (exact)

$$s^2 = \frac{1}{79}\left\{\sum(x-15)^2 - \frac{\left(\sum(x-15)\right)^2}{80}\right\} = \frac{1}{79}\left(795.7 - \frac{53.6^2}{80}\right) = 9.62 \text{ (3 s.f)}$$

(ii) $H_0: \mu = 15$

$H_1: \mu \neq 15$

Level of significance: 4%

Since $n = 80$ is large, by Central Limit Theorem, $\bar{X}$ follows a normal distribution approximately,

Test statistic: $\dfrac{\bar{X}-\mu}{s/\sqrt{n}} \sim N(0, 1)$ approximately

If H_0 is true, p-value = 0.0533 > 0.04

Since p-value > level of significance, we do not reject H_0.

There is insufficient evidence at 4% level of significance that the mean weight of a packet differs from 15 g.

(iii) If H_0 is true, $z_{cal} = \dfrac{\bar{x}-15}{\sqrt{3/n}}$

To reject H_0, $\dfrac{\bar{x}-15}{3/\sqrt{n}} < -2.05375$ or $\dfrac{\bar{x}-15}{3/\sqrt{n}} > 2.05375$

$\Rightarrow \bar{x} < 15 - \dfrac{6.16}{\sqrt{n}}$ or $\bar{x} > 15 + \dfrac{6.16}{\sqrt{n}}$

8 In a school, there are 28 and 18 teachers in the Science and Mathematics Departments respectively. Of the 28 Science teachers, there are 10 single men, 4 single women, 2 married couples, 8 married men and 2 married women. Of the 18 Mathematics teachers, there are 3 single men, 3 single women, 1 married couple, 5 married men and 5 married women. No Science teacher is married to a Mathematics teacher. Two teachers are chosen at random from the two departments to attend a focus group discussion.
Find the probability that

(i) they are married to each other, [2]

(ii) they are a man and a woman in the same department, [2]

(iii) they are married to each other given that a man and woman are chosen, [2]

(iv) they are from different departments given that a married man and a single woman are chosen. [3]

[Solution]

	single men	**single women**	**couples**	**Married Men**	**Married Women**	**Sub total**
Science	10	4	1M1W 1M1W	8	2	28
Mathematics	3	3	1M1W	5	5	18

(i) P(they are married to each other) $\dfrac{{}^{3}C_1}{{}^{46}C_2} = \dfrac{3}{1035} = \dfrac{1}{345}$.

or $\dfrac{6}{46} \times \dfrac{1}{45} = \dfrac{1}{345}$

(ii) P(they are a man and a woman in the same department)

$$= \frac{\left({}^{20}C_1 \times {}^{8}C_1\right) + \left({}^{9}C_1 \times {}^{9}C_1\right)}{{}^{46}C_2} = \frac{241}{1035}$$

or $\frac{20}{46} \times \frac{8}{45} \times 2 + \frac{9}{46} \times \frac{9}{45} \times 2 = \frac{241}{1035}$

(iii) P(they are married to each other | a man and woman are chosen)

$$= \frac{{}^{3}C_1}{{}^{29}C_1 \times {}^{17}C_1} = \frac{3}{493}$$

or $\frac{\frac{1}{345}}{\frac{29}{46} \cdot \frac{17}{45} \cdot 2} = \frac{3}{493}$

(iv) P(they are from different departments | a married man and a single woman are chosen)

$$= \frac{\text{P(A married man and a single woman are from different dept)}}{\text{P(A married man and a single woman are chosen)}}$$

$$= \frac{\left({}^{10}C_1 \times {}^{3}C_1\right) + \left({}^{4}C_1 \times {}^{6}C_1\right)}{{}^{16}C_1 \times {}^{7}C_1} = \frac{54}{112} = \frac{27}{56}$$

or $\frac{\frac{10}{46} \cdot \frac{3}{45} \cdot 2 + \frac{4}{46} \cdot \frac{6}{45} \cdot 2}{\frac{16}{46} \cdot \frac{7}{45} \cdot 2} = \frac{54}{112} = \frac{27}{56}$

9 When a large number of mangoes are harvested, 37% of them are unripe. The mangoes are packed randomly in boxes of six.

(i) Show that the probability that there is no unripe mango in a box is 0.062524, correct to 5 significant figures. [1]

(ii) Find the probability that there are exactly 2 unripe mangoes in 2 boxes. [2]

A supermarket orders 50 boxes of mangoes daily.

(iii) By using a suitable approximation, find the probability that there are at least 9 boxes with no unripe mango in a particular day. [4]

(iv) When the supermarket places a daily order for 10 weeks, estimate the probability that the mean number of boxes with no unripe mango in a day is between 3 and 9. [3]

[Solution]

9 Let X be number of unripe mangoes in a box (out of 6).

$X \sim \text{B}(6, 0.37)$

(i) P(no unripe mango) = $\text{P}(X = 0) = 0.062524$ (correct to 5 significant figures)

(ii) $X_1 + X_2 \sim \text{B}(12, 0.37)$

P(exactly 2 unripe mangoes) = $\text{P}(X_1 + X_2 = 2) = 0.0890$ (correct to 3 sig fig)

<u>Alternative Method</u>

P(exactly 2 unripe mangoes)

$= \text{P}\left((X_1 = 2, X_1 = 0) \text{ or } (X_1 = 0, X_1 = 2) \text{or } (X_1 = 1, X_1 = 1)\right)$

$= 2 \times \text{P}(X = 2)\text{P}(X = 0) + \text{P}(X = 1)^2$

$= 0.0890$ (correct to 3 sig fig)

(iii) Let Y be number of boxes of mangoes (out of 50) with no unripe mango in a day.

$Y \sim \mathrm{B}(50, 0.062524)$

Since n = 50 is large and $np = (50)(0.62524) = 3.1262 < 5$

$Y \sim \mathrm{P_o}(3.1262)$ approximately

P(at least 9 boxes with no unripe mango) = $\mathrm{P}(Y \geq 9) = 1 - \mathrm{P}(Y \leq 8)$

= 0.00494 (correct to 3 sig fig)

(iv) In 10 weeks, there are 70 days altogether.

Mean number of boxes with no unripe mango = $\bar{Y} = \dfrac{Y_1 + Y_2 + Y_3 + \cdots + Y_{70}}{70}$

Since sample size = 70 is large,

$\bar{Y} \sim \mathrm{N}\left(50 \times 0.062524, \dfrac{50 \times 0.062524 \times (0.93748)}{70}\right)$ by Central Limit Theorem.

i.e. $\bar{Y} \sim \mathrm{N}(3.1262, 0.041868)$ approximately

$\mathrm{P}\left(3 < \bar{Y} < 9\right)$ = 0.731 (correct to 3 sig fig)

<u>Alternative method [1] (Using Possion approximation)</u>

$Y \sim \mathrm{P_o}(3.1262)$ approximately

Let T = $Y_1 + Y_2 + Y_3 + \cdots + Y_{70} \sim \mathrm{P_0}(3.1262 \times 70)$ i.e. $T \sim \mathrm{P_0}(218.834)$ approximately

$\mathrm{P}\left(3 < \bar{Y} < 9\right)$ = $\mathrm{P}(3 \times 70 < T < 9 \times 70)$

$= \mathrm{P}(210 < T < 630)$

$= \mathrm{P}(T \leq 629) - \mathrm{P}(T \leq 210)$

= 0.711

<u>Alternative method [2] (Approximating Binomial to Normal)</u>

Let T = $Y_1 + Y_2 + Y_3 + \cdots + Y_{70} \sim \mathrm{B}(70 \times 50, 0.062524)$ i.e. $T \sim \mathrm{B}(3500, 0.062524)$

Since $n = 3500$ is large, $np = 218.834 > 5$, $n(1 - p) = 3281.166 > 5$

$T \sim \mathrm{N}(218.834, 205.151623)$ approximately

P(mean number of boxes with no unripe mango is between 3 and 9)

= $\mathrm{P}\left(3 < \bar{Y} < 9\right)$ = $\mathrm{P}(3 \times 70 < T < 9 \times 70) = \mathrm{P}(210 < T < 630)$

= $\mathrm{P}(210.5 < T < 629.5)$ by continuity correction

= $\mathrm{P}(210.5 < T < 629.5)$

= 0.720 (correct to 3 sig fig)

Alternative method [3] (Approximating Poisson to Normal)

Let $T = Y_1 + Y_2 + Y_3 + \cdots + Y_{70} \sim \mathrm{P_0}(3.1262 \times 70)$ i.e. $T \sim \mathrm{P_0}(218.834)$

Since $\lambda = 218.834 > 10$ $T \sim \mathrm{N}(218.834, 218.834)$ approximately

P(mean number of boxes with no unripe mango is between 3 and 9)

$= \mathrm{P}(3 < \overline{Y} < 9) = \mathrm{P}(3 \times 70 < T < 9 \times 70) = \mathrm{P}(210 < T < 630)$

$= \mathrm{P}(210.5 < T < 629.5)$ by continuity correction

$= \mathrm{P}(210.5 < T < 629.5)$

= 0.713 (correct to 3 sig fig)

10 The table gives the research and development (R&D) expenditures, *x*, and earnings, *y*, in suitable units for 11 pharmaceutical companies in a particular year.

R&D expenditure, *x*	8.5	12	6.5	4.5	2	0.5	1.5	6	9	7.5	2.5
Earnings, *y*	83	147	69	50	43	35	40	64	97	53	45

(i) Draw the scatter diagram for these values, labelling the axes clearly. [2]

(ii) Give a possible reason why one of the data points does not seem to follow the trend. [1]

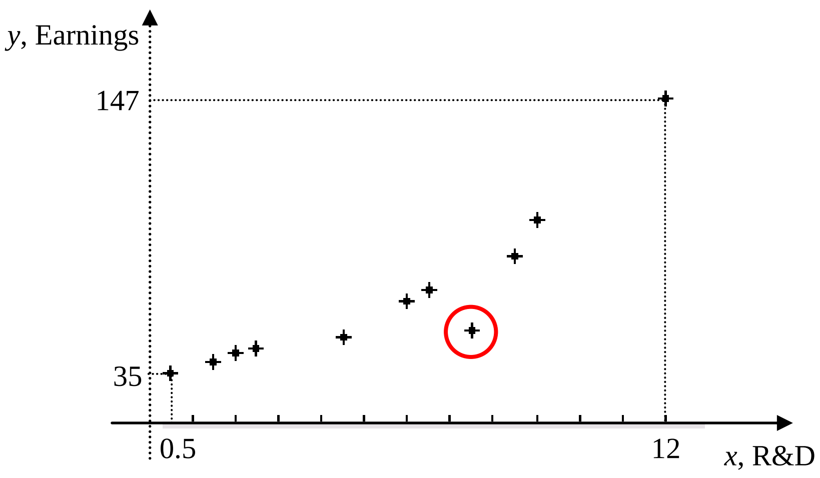
(iii) Using your answer in part **(i)**, explain whether $y = a + bx$ or $y = c + dx^2$ is a better model for the data. [1]

(iv) Explain how you would verify your choice of model in part **(iii)** by calculating the product moment correlation coefficients. [2]
Hence estimate the earnings of a company with a R&D expenditure of 10 units, showing your calculations clearly. [3]

(v) For the model $y = c + dx^2$, give an interpretation, in context, of the value of *d*. [1]

[Solution]

10(i)



(ii) The point (7.5, 53) does not follow the trend. This could be because

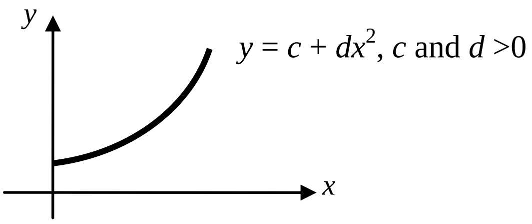
- company could have just started its research and development and need time for R&D to reap benefits and better earnings
- company's R&D did not produce any new drugs or did not improve the quality of drugs for the particular year
- new direct competitor selling the same drugs as company could have entered the market

Not accepted:

- company experiencing economic downturn (all companies equally affected)
- company experience lower sales (did not really explain why despite R&D experience lower sales)
- R&D failed without any elaboration

(iii) $y = c + dx^2$ is a better model for the data as the scatter diagram shows that as x increases, y increases at an increasing rate / concave upwards with a positive gradient.

or sketch a quadratic curve that fits the data points well.



(iv) Compute the product moment correlation coefficients for the two models:

Model $y = a + bx$: r-value = 0.903

Model $y = c + dx^2$: r-value = 0.968

This verifies that $y = c + dx^2$ is a better model as its r-value of 0.968 is closer to 1 than r-value of 0.903 for the $y = a + bx$ model.

Equation of least-squares regression line of y on x^2 is

$y = 35.57226 + 0.72018\,x^2$

When $x = 10$, $y = 35.57226 + 0.72018\,(100) = 107.591$

$= 108$

(v) The value of d is the increase in the value of earnings (y) for every unit increase in the squared value of the R&D expenditure (x^2).

OR When the square of the R&D expenditure (x^2) increases by 1 unit, the earnings (y) increases by d units.

11 A power station has one generator. It has been observed that the generator has on average 3 breakdowns in 2 years. Assume that the number of breakdowns can be modelled using a Poisson distribution.

(i) Given that the probability that no breakdown occurs in a period of n months is 0.8, find n, correct to 1 decimal place. [3]

(ii) Using a suitable approximation, find the probability that the number of breakdowns in 10 years is less than 10. [4]

For each breakdown, the time taken for the repair work is normally distributed with mean 2 hours and standard deviation 0.7 hour.

(iii) Find the least integer m such that the probability that the total time taken to repair 3 randomly chosen breakdowns is less than m hours is at least 0.85. [3]

The government imposes a fine of $10,000 for the first breakdown of the generator. The fine is increased to $20,000 for each subsequent breakdown within a year of the first breakdown. The power station faces an additional fine of $50,000 if the generator is not repaired within 3 hours of a breakdown.

(iv) Find the probability that the power station pays less than $50,000 in fines in a particular year. [3]

[Solution]

(i) Let X be the number of breakdowns in ***n* months**. $\quad E(X) = n\,(3/24) = n/8$

$$X \sim \text{Po}\left(\frac{n}{8}\right)$$

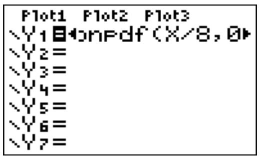
$$P(X = 0) = 0.8 \quad \Rightarrow \quad \frac{e^{-\frac{n}{8}}\left(\frac{n}{8}\right)^0}{0!} = 0.8 \quad \Rightarrow \quad e^{-\frac{n}{8}} = 0.8$$

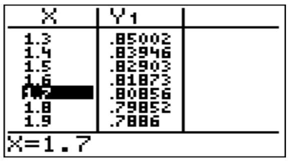
$\Rightarrow \quad n = -8\ln 0.8 \approx 1.7851 \approx$ **1.8** months (correct to 1 d.p.)

<u>Alternative method (Using GC)</u>

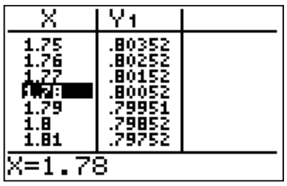
$X \sim \text{Po}\left(\frac{n}{8}\right)$, $P(X = 0) = 0.8$

From GC, we have





n	$P(X = 0)$
1.78	0.80052
1.79	0.79951
1.80	0.79852



$n = 1.8$ months

(ii) Using a suitable approximation, find the probability that the number of breakdowns in 10 years is less than 10. [4]

Let Y be the number of breakdowns in 10 years. $Y \sim \text{Po}(15)$

Since $\lambda = 15 > 10$, $Y \sim \text{N}(15, 15)$ approximately

$\text{P}(Y < 10) = \text{P}(Y < 9.5)$ by continuity correction

$= 0.0778$

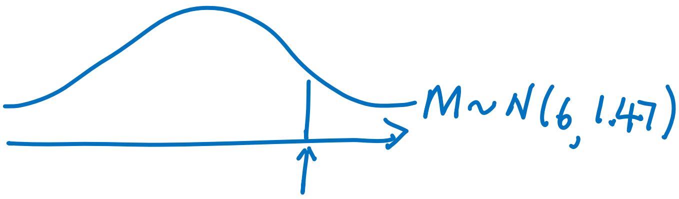
(iii) Find the least integer m such that the probability that the total time taken to repair 3 randomly chosen breakdowns is less than m hours is at least 0.85. [3]

Let T be the time taken to repair a randomly chosen breakdown. $T \sim \text{N}(2, 0.7^2)$

Then, $M = T_1 + T_2 + T_3 \sim \text{N}(6, 1.47)$

$\text{P}(M < m) \geq 0.85 \quad \Rightarrow \quad m \geq \mathbf{7.2566}$

Least integer $m = 8$



(OR) $\text{P}(M < m) \geq 0.85 \quad \Rightarrow \text{P}\left(Z < \dfrac{m-6}{\sqrt{1.47}}\right) \geq 0.85$

$\Rightarrow \dfrac{m-6}{\sqrt{1.47}} \geq \mathbf{1.0364} \quad \Rightarrow \quad m \geq \mathbf{7.2566}$

(OR) Use GC table.

The government imposes a fine of $10,000 for the first breakdown of the generator. The fine is increased to $20,000 for each subsequent breakdown within a year of the first breakdown. The power station faces an additional fine of $50,000 if the generator is not repaired within 3 hours of a breakdown.

(iv) Find the probability that the power station pays less than $50,000 in fines in a particular year. [3]

Let W be the number of breakdowns in a year. $W \sim \text{Po}(1.5)$

P(pay less than $50,000 fine in a year)

$$= \mathbf{P(W = 0) + P(W = 1)\,P(T < 3) + P(W = 2)\,[P(T < 3)]^2}$$

$$= 0.22313 + 0.3347(0.92344) + 0.25102(0.92344)^2$$

$= 0.746$ (to 3 sf)