SPRINGFIELD SECONDARY SCHOOL

End-Of-Year Examination 2023

Sec 3 Express

STUDENT NAME	

CLASS		REGISTER NUMBER		

MATHEMATICS 4052/01

Paper 1 **5 October 2023**

2 hours 15 minutes

Candidates answer on the question paper
Additional Materials: NIL

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams, graphs or rough working.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

For Examiner's Use	
Total	**/90**

Do not turn over this question paper until you are told to do so.

This question paper consists of 21 printed pages.

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Answer **all** the questions

1 **(a)** By prime factorization, explain why 484 is a perfect square.

Answer [2]

……

……

……

……

(b) Find the highest common factor (HCF) of 66 and 150.

Answer ..... [2]

2 The expression $x^2 - 6x - 2$ can be written in the form $(x-3)^2 + b$.

(a) Find the value of *b*.

Answer b = ... [2]

(b) Explain whether the *y* value at $x = 3$ is a maximum or minimum value.

Answer [1]

……..

……..

……..

……..

3 Solve $\frac{2}{x+1} - \frac{3}{x-6} = 2$

Answer $x =$ ..... [4]

4 Find the equation of the straight line passing through (1 , -3) and is parallel to the line $3y + 5x = 15$.

Answer ..... [3]

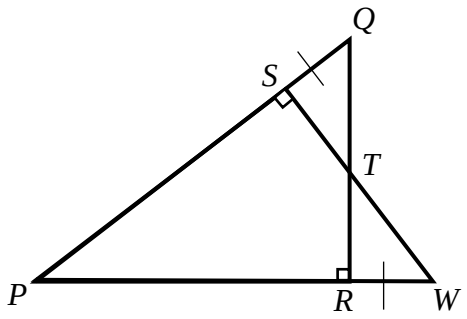
5 **(a)** Simplify $\left(\frac{2}{3a}\right)^{-3}$, leaving your answer in positive index notation.

Answer ..... [2]

(b) Simplify $25x^2y \times \frac{1}{10x^{-3}y^{-2}}$

Answer ..... [2]

6 In the diagram, SW and RQ intersect at T and QSP, WRP, QTR and WTS are straight lines. Given that $\angle PSW = \angle PRQ = 90°$ and $SQ = RW$.



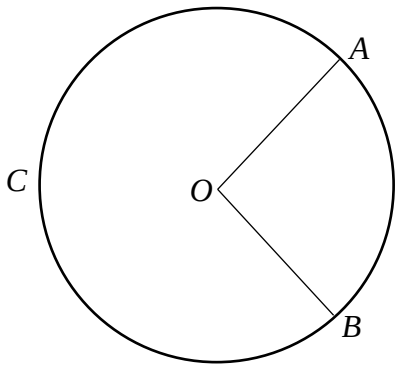
(a) Prove that triangle STQ is congruent to triangle RTW.

Answer [3]

(b) Find $\angle RTW$ if $\angle RPS = 62°$

Answer .....° [2]

7 In the circle, the ratio of the arc length of the minor arc AB to the circumference of the whole circle is 1:6.



(a) Calculate the acute angle AOB in radians.

Answer ..... rad [2]

(b) Given that the area of the minor sector AOB is $8\frac{1}{3}$ cm^2. Calculate the radius of the circle.

Answer .....cm [2]

(c) Find the perimeter of the minor sector *OBA*.

Answer .....cm [2]

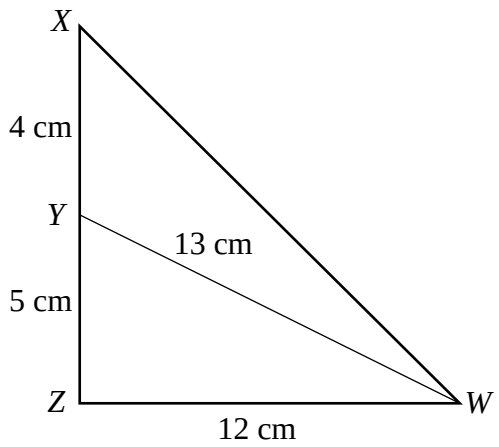
8 **(a)** Expand and simplify $5x - 3(2x - 7) + 9$.

Answer ..... [2]

(b) Simplify $\frac{3}{2x-1} - \frac{7}{x+1}$.

Answer ..... [3]

9 In the diagram, XYZ is a straight line, $XY = 4$ cm, $YZ = 5$ cm, $ZW = 12$ cm and $WY = 13$ cm.



(a) Explain why angle WZY is a right angle,

Answer [2]

…..

…..

…..

…..

(b) Find the length of WX,

Answer .....cm [2]

(c) Find the value of $\cos \angle XYW$.

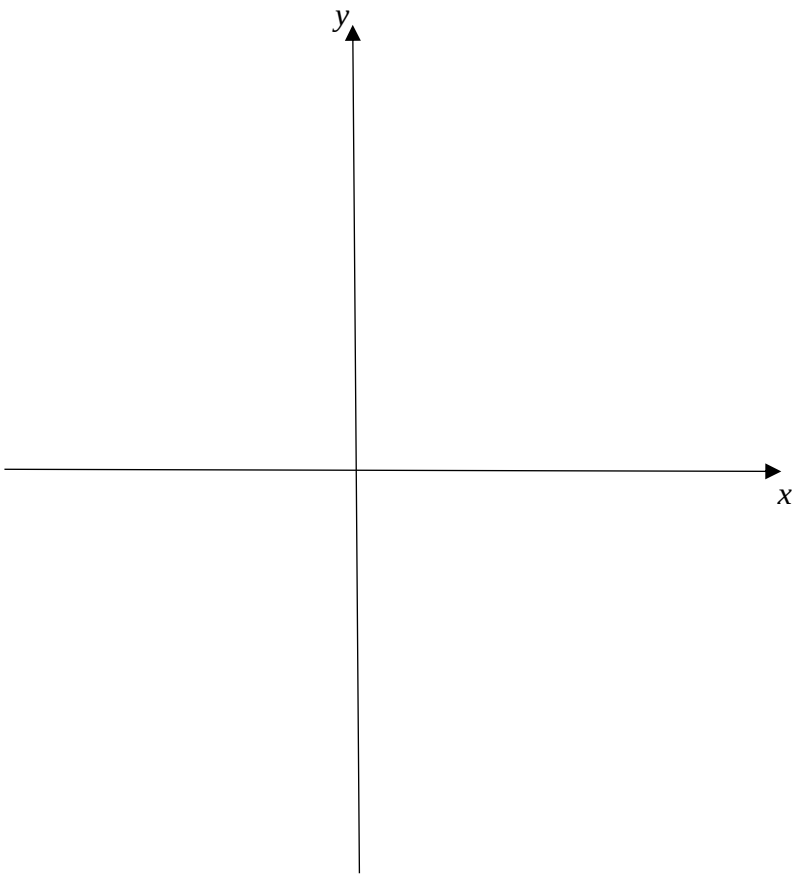
Answer ..... [1]

(d) the area of triangle WXY.

Answer .....cm^2 [1]

10 **(a)** Sketch the graph of $y = -(x+1)^2 + 2$. Indicating clearly the values where the curve crosses the x and y axes and the turning point of the curve.

Answer [3]



(b) State the equation of the line of symmetry for $y = -(x+1)^2 + 2$.

Answer ..... [1]

11 Given that $-4 \le x \le 2$ and $-1 \le y \le 3$ where x and y are integers.

Find

(a) the largest possible value of $y - x$,

Answer ..... [1]

(b) the smallest possible value of $\frac{y}{x}$,

Answer ..... [1]

(c) the largest possible value of $x^2 + y^2$.

Answer ..... [1]

12 The school drink stall sells 500 cups of milo drink daily at $0.80 per cup.
Each cup of milo need 22g of milo powder.

(a) How much powder is needed for 5 days?

Answer .....g [1]

The drink stall owner wanted to get 5 days worth of milo powder supply. The owner can choose between two supermarkets for the supply.

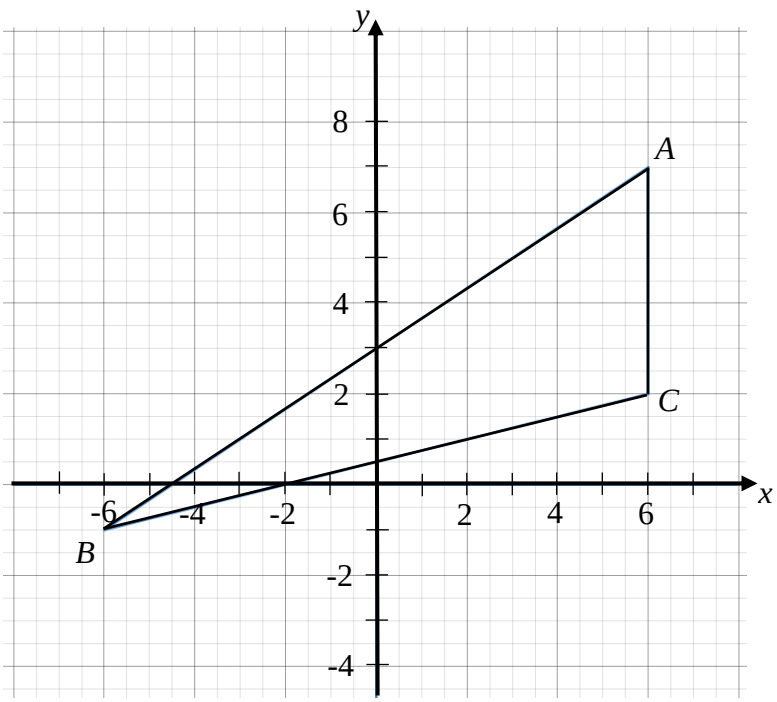
Supermarket A
$8.45 for one pack 900g
Buy 2 get 1 free

Supermarket B
1 tin of 1.8kg cost $16
$91 for 6 tins of 1.8kg

(b) Based on the advertisement above, which supermarket should the owner choose? Justify the decision you make and show your calculations clearly.

Answer [5]

13 The diagram shows triangle ABC with vertices $A(6 , 7)$, $B(-6 , -1)$ and $C(6 , 2)$



Find

(a) the length of AB,

Answer ..... [2]

(b) the equation of BC,

Answer ..... [3]

14 Factorise completely

(a) $2ab^2 + 8ab^3$

Answer ..... [1]

(b) $2x^2 - 5x + 3$

Answer ..... [1]

(c) $6ab - 2ad + 3bc - cd$

Answer ..... [3]

15 **(a)** John borrowed a sum of money at 24% per annum compound interest. After 2 years, he owed a total of $768.80. Calculate the sum of money he borrowed.

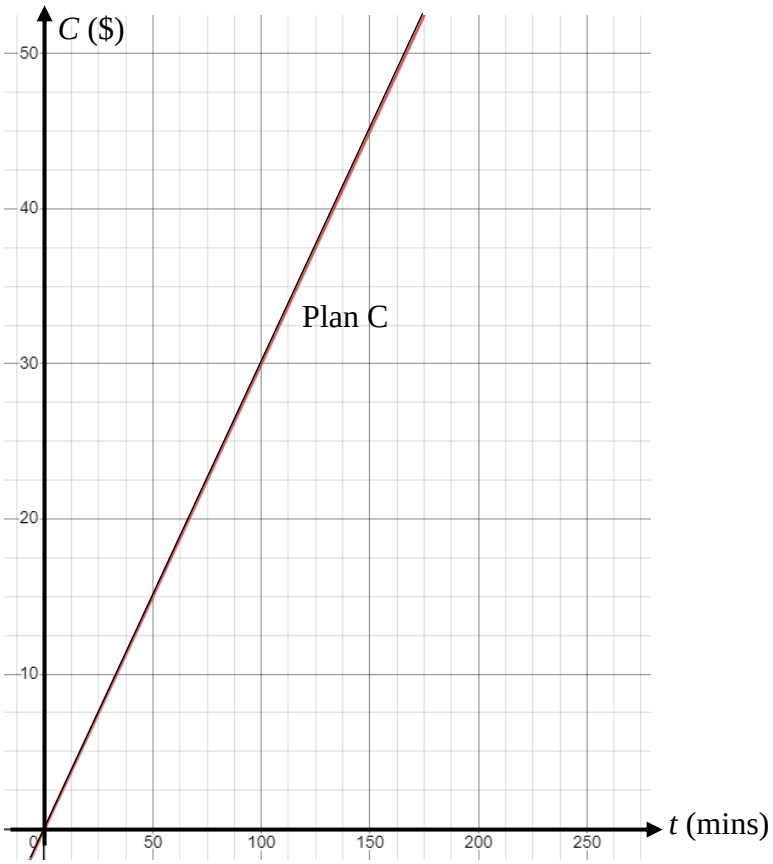
Answer $..... [2]

(b) Using your answer in part **(a)**, if the compound interest rate is now 24% per annum compounded half-yearly. What is the total amount John must pay after 2 years?

Answer $..... [3]

16 Mr Tham went overseas to study and wanted to purchase a mobile plan there.
The table below shows three plans that were available.

Plan A	Plan B	Plan C
Monthly fee = \$15	Monthly fee = \$40	Monthly fee = \$0
100 minutes free talk time	Unlimited free talk time	0 minute free talk time
\$0.20 for each additional minute of talk time		\$0.30 for each minute of talk time



(a) On the same axis, draw the graphs of Cost C (\$) against the talk time t (mins) for Plan A and B. [2]

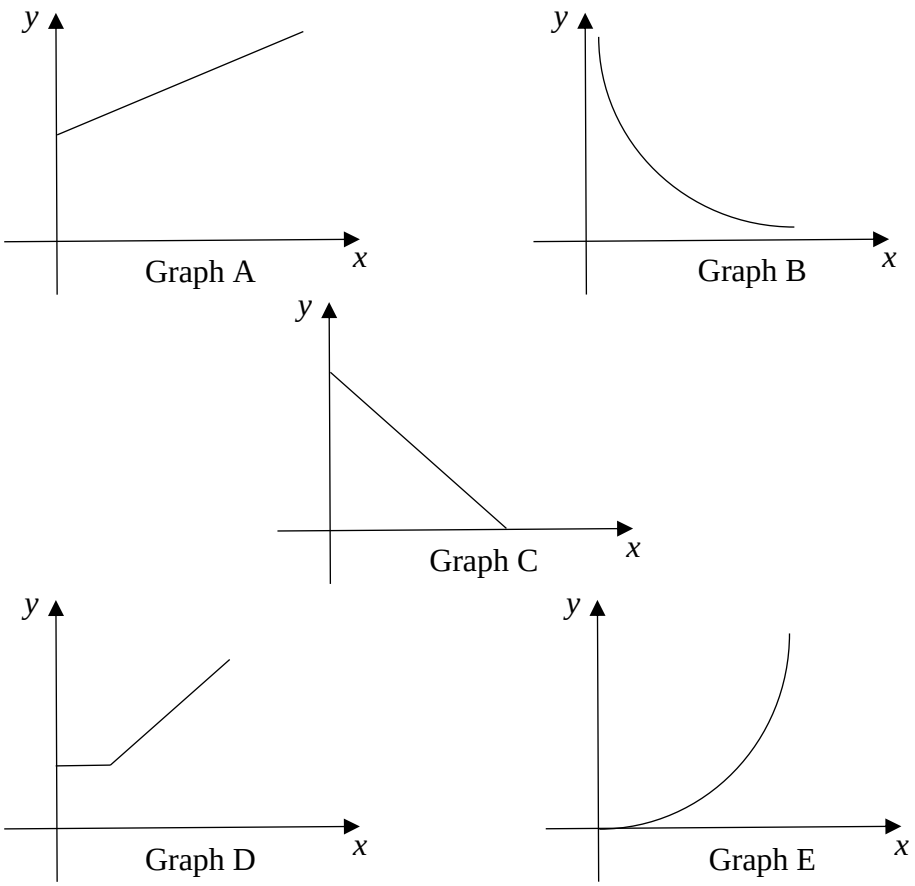
(b) Using your graphs, state the plan that Mr Tham should choose if he uses 150 minutes of talk time monthly.

Answer ..... [1]

(c) After how many minutes of talk time, will Plan B be more worth than Plan A?

Answer .....mins [1]

17 Choose one of the graphs shown below to match each of the following statements.



(a) The population of microorganism, *y*, increased exponentially within a short period of time *x*.

Answer ..... [1]

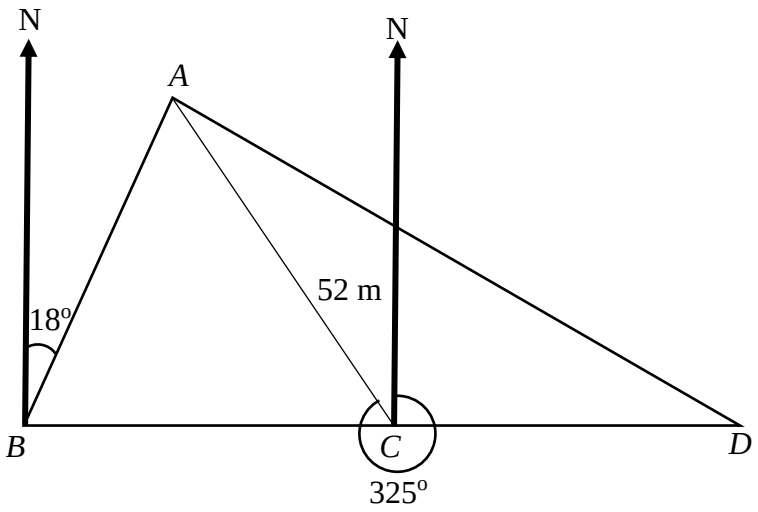
(b) The water in a cylindrical container drips from a hole. The height of the water level in the cylinder, *y*, decreases at a constant rate over a period of time, *x*.

Answer ..... [1]

(c) Taxi fare, *y*, with a starting fare of $4.10 for the first kilometre and $0.35 per km over a distance, *x*.

Answer ..... [1]

18



In the diagram, *A*, *B*, *C* and *D* represent four points on a piece of horizontal land. *AC* = 52 m and point *B* is due west of point *D*. Point *A* is on a bearing of 018° from point *B* and 325° from point *C*.

(a) Calculate the distance *AB*.

Answer .....m [3]

(b) Given that the area of triangle *ACD* = 1256 m^2, calculate

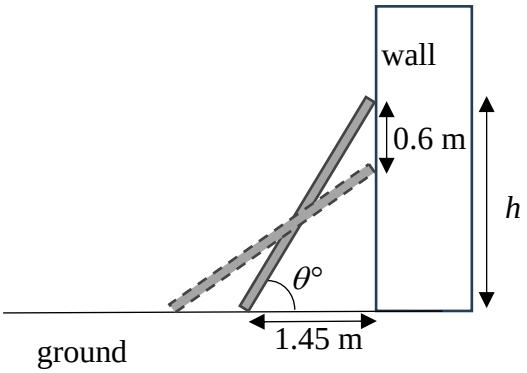
(i) the distance *CD*,

Answer .....m [3]

(ii) the distance AD.

Answer .....m [2]

19 A ladder of length 6 m is leaning against a wall and forming an angle $\theta°$ with the ground. The base of the ladder is 1.45 m from the wall.



(a) Find the initial height, h m of the top of the ladder from the ground.

Answer .....m [2]

(b) Find the value θ.

Answer ..... [2]

(c) There is a 4 to 1 safety rule governing the use of a ladder. The base of the ladder should be placed so that it is one metre away from the wall for every four metres of height to where the ladder rests against the wall.

If the ladder slides down 0.6 m, is the ladder safe to be use? Justify your answer with workings.

Answer [3]

End of Paper