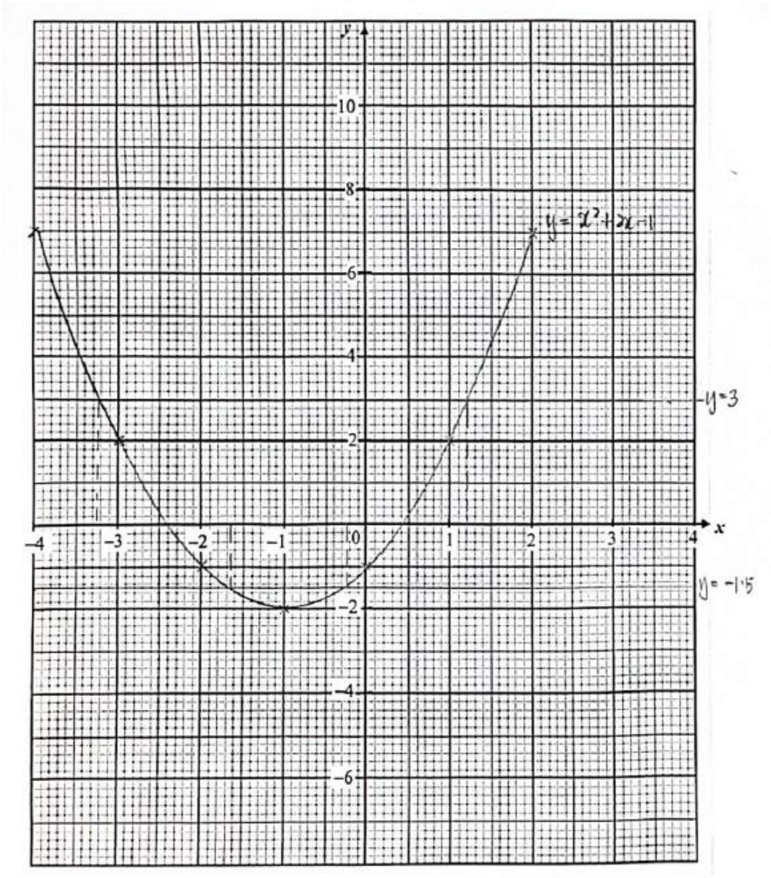
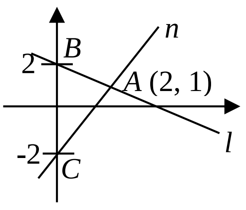
2024 4E/5N EM Prelim Paper 2 Marking Scheme (90 marks)

1a	$\frac{3x+1}{3} < \frac{2-5x}{4}$ $4(3x+1) < 3(2-5x)$ $12x+4 < 6-15x$ $27x < 2$ $x < \frac{2}{27}$	M1 A1
1bi	$a = \frac{2(0.2)-3(-1.5)}{0.2+2(-1.5)}$ $a = -\frac{7}{4}$ / -1.75	B1
1bii	$a = \frac{2b-3c}{b+2c}$ $a(b+2c) = 2b-3c$ $ab+2ac = 2b-3c$ $ab-2b = -3c-2ac$ $b(a-2) = -3c-2ac$ $b = \frac{-3c-2ac}{a-2}$ $b = \frac{3c+2ac}{2-a}$	M1 M1 Either A1
1c	$\frac{5x}{x-3} - \frac{2}{2x-3} = 1$ $\frac{5x(2x-3)-2(x-3)}{(x-3)(2x-3)} = 1$ $10x^2-15x-2x+6 = 2x^2-3x-6x+9$ $10x^2-17x+6 = 2x^2-9x+9$ $8x^2-8x-3=0$ $x = \frac{-(-8) \pm \sqrt{(-8)^2-4(8)(-3)}}{2(8)}$ $x = 1.2906, -0.29057$ $x = 1.291, -0.291\,(3dp)$	M1 – common denominator M1 – either expansion M1 A1A1

2a	$y = 7$	B1
2b		7 points – B2 4 points – B1 Curve – B1
2ci	$-3.25 < x < 1.2$	B1
2cii	$x^2 + 2x + \frac{1}{2} = 0$ $x^2 + 2x + \frac{1}{2} - 1.5 = 0 - 1.5$ $x^2 + 2x - 1 = -1.5$ $y = -1.5$ $x = -1.65 \quad or \quad x = -0.25$	M1 A1A1
2ciii	$x^2 + 2x - 1$ $= \left(x + \frac{2}{2}\right)^2 - \left(\frac{2}{2}\right)^2 - 1$ $= (x+1)^2 - 2$ Therefore, the minimum point is $(-1, -2)$	M1 - either x or y value seen A1 – in the form $(x+p)^2 + q$

3a	Vol of hemisphere $=\frac{2}{3}\times\pi\times(3x)^3$ $=18\pi x^3$ Vol of cone $=\frac{1}{3}\times\pi\times\left(\frac{3}{2}x\right)^2\times y$ $=\frac{1}{3}\times\pi\times\frac{9}{4}x^2\times y$ $=\frac{3\pi}{4}x^2y$ $18\pi x^3=3\times\frac{3\pi}{4}x^2y$ $18x^3=\frac{9}{4}x^2y$ $y=\frac{18x^3\times 4}{9x^2}$ $y=8x\ (shown)$	M1 M1 A1
3b	$l=\sqrt{y^2+\left(\frac{3}{2}x\right)^2}$ $=\sqrt{(8x)^2+\left(\frac{3}{2}x\right)^2}$ $=\sqrt{64x^2+\frac{9}{4}x^2}$ $=\sqrt{\frac{265x^2}{4}}$ $=8.1394x\ /\ \frac{\sqrt{265}}{2}x$ CSA of cone $=\pi\left(\frac{3}{2}x\right)\times l$ $=\pi\left(\frac{3}{2}x\right)\times 8.1394x$ $=38.356x^2$ CSA of cylinder $=2\pi\times\left(\frac{3}{2}x\right)\times 3x$ $=28.274x^2$ CSA of hemisphere $=2\pi\times(3x)^2$ $=56.549x^2$ rim area $=\pi(3x)^2-\pi\left(\frac{3}{2}x\right)^2$ $=21.206x^2$	M1 4 correct – M3 3 correct – M2 2 correct – M1

3b	Total SA = 450 $38.356x^2+28.274x^2+56.549x^2+21.206x^2=450$ $144.385x^2=450$ $x^2=3.1167$ $x=1.7654$ Total ht, $h=y+3x+3x$ $h=8x+3x+3x$ $h=14x$ $h=14(1.7654)$ $h=24.7156$ $h=24.7\ (3sf)$	M1 M1 A1
4a	$\frac{1}{25}$	B1
4b	$T_n=\frac{16-3n}{5n}$	M1 - $-3n$ M1 – $5n$ A1 - T_n
4c	$T_{n+1}-T_n=\frac{16-3(n+1)}{5(n+1)}-\frac{16-3n}{5n}$ $=\frac{16-3n-3}{5(n+1)}-\frac{16-3n}{5n}$ $=\frac{n(13-3n)-(16-3n)(n+1)}{5n(n+1)}$ $=\frac{13n-3n^2-(16n+16-3n^2-3n)}{5n(n+1)}$ $=\frac{13n-3n^2-16n-16+3n^2+3n}{5n(n+1)}$ $=\frac{-16}{5n(n+1)}$ (*shown*)	M1 M1 A1
4d	For $n>0$ $5n(n+1)>0$ $\frac{-16}{5n(n+1)}<0$ $T_{n+1}-T_n<0$	B1

5a	grad of line $l = \frac{-2-4}{8-(-4)}$ $= -\frac{1}{2}$ $y = -\frac{1}{2}x + c$ *subst* $(-4, 4)$ into $y = -\frac{1}{2}x + c$ $4 = -\frac{1}{2}(-4) + c$ $c = 2$ $y = -\frac{1}{2}x + 2$	M1 A1
5b	Line m $6y = 18 - 3x$ $y = -\frac{1}{2}x + 3$ The gradient of line l and line m are equal (gradient = $-\frac{1}{2}$) and the y-intercept not equal. Line l and line m are parallel. Therefore, line m does not intersect the line l.	M1 – gradient value A1 – parallel
5c	$y = -\frac{1}{2}x + 2$ -------(1) $\quad 2y = 3x - 4$ --------(2) *Subst* (1) *into* (2) $2\left(-\frac{1}{2}x + 2\right) = 3x - 4$ $-x + 4 = 3x - 4$ $4x = 8$ $x = 2$ *Subst* $x = 2$ *into* (1) $y = -\frac{1}{2}x + 2$ $y = -\frac{1}{2}(2) + 2$ $y = 1$ $A(2,1)$	M1 M1 – either x or y coordinate A1
5d	*Area of* Δ *ABC* $= \frac{1}{2} \times 4 \times 2$ $= 4\ units^2$  	M1 – BC = 4 units M1 – height of 2 units A1

6a	$\angle OPV = \angle ORV = 90^\circ$ (tan ⊥ rad) $\angle POR = 360 - 90 - 90 - 40$ (sum of quad) $= 140$ $reflex\ \angle POR = 360 - 140$ ($\angle$ at a pt.) $= 220$ $\angle PQR = \dfrac{220}{2}$ ($\angle$ at ctr = 2× $\angle$ at circum) $=110$	M1 M1 A1
6b	$\angle OPR = \dfrac{180-140}{2}$ (sum of isos Δ) $=20$ $\angle SPR = \angle STR$ ($\angle$ in same seg) $=43$ $\angle OPU = 43 - 20$ $= 23$ $\angle POU = 180 - 140$ ($\angle$ on str. line) $=40$ $\angle TUP = \angle OPU + \angle POU$ (ext. $\angle$ of Δ) $=23+40$ $= 63^\circ$	M1 M1 A1
6c	$\angle SOR = 2 \times \angle STR$ ($\angle$ at ctr = 2× $\angle$ at circum) $=2 \times 43$ $= 86$ reflex $\angle SOR = 360 - 86$ $= 274$ area of minor sector $= \dfrac{86}{274} \times$ area of major sector $=0.31387 \times$ area of major sector No because the area of the minor sector is 0.314 of the area of major sector, which is less than $\dfrac{1}{3}$ (0.333)	M1 M1 A1

7ai	$\begin{pmatrix}8\\-4\end{pmatrix}+\begin{pmatrix}u\\v\end{pmatrix}=\begin{pmatrix}6\\4\end{pmatrix}$ $\begin{pmatrix}u\\v\end{pmatrix}=\begin{pmatrix}6\\4\end{pmatrix}-\begin{pmatrix}8\\-4\end{pmatrix}$ $=\begin{pmatrix}-2\\8\end{pmatrix}$	B1
7aii	$\overrightarrow{XY}=\begin{pmatrix}-2\\8\end{pmatrix}$ $\lvert\overrightarrow{XY}\rvert=\sqrt{(-2)^2+(8)^2}$ $=8.2462$ $=8.25\ \text{units}^2$	M1 A1
7aiii	Grad of ZY = Grad of XY $\frac{k-4}{-1-6}=\frac{8}{-2}$ $\frac{k-4}{-7}=-4$ $k=32$ $\overrightarrow{OZ}=\begin{pmatrix}-1\\32\end{pmatrix}$	M1 A1
7bi	$\Delta ABD,\ \overrightarrow{AB}+\overrightarrow{BD}=\overrightarrow{AD}$ $\overrightarrow{BD}=8\mathbf{b}-4\mathbf{a}$ $4\ units,\ \overrightarrow{BD}-8\mathbf{b}-4\mathbf{a}$ $3\ units,\ \overrightarrow{BE}-\frac{3}{4}(8\mathbf{b}-4\mathbf{a})$ $=6\mathbf{b}-3\mathbf{a}$	M1 A1
7bii	$\Delta BEF,\ \overrightarrow{BE}+\overrightarrow{EF}=\overrightarrow{BF}$ $\overrightarrow{BF}=6\mathbf{b}-3\mathbf{a}+2\mathbf{a}-2\mathbf{b}$ $=4\mathbf{b}-\mathbf{a}$ $\overrightarrow{BC}=2\times\overrightarrow{BF}$ $=8\mathbf{b}-2\mathbf{a}$	M1 A1
7biii	$\Delta BCD,\ \overrightarrow{BD}+\overrightarrow{DC}=\overrightarrow{BC}$ $\overrightarrow{DC}=\overrightarrow{BC}-\overrightarrow{BD}$ $=8\mathbf{b}-2\mathbf{a}-(8\mathbf{b}-4\mathbf{a})$ $=2\mathbf{a}$ $\overrightarrow{AB}=2(2\mathbf{a})$ $\overrightarrow{AB}=2\overrightarrow{DC}$ $ABCD$ is a trapezium because $\overrightarrow{AB}=2\overrightarrow{DC}$, AB is parallel to DC 1 pair of opposite side parallel.	M1 M1 - $\overrightarrow{AB}=2\overrightarrow{DC}$, AB is parallel to DC A1 – awarded for 'trapezium' only if M1 awarded

8ai	$\frac{50}{100}\times 80=40$ Median = \$550	B1
8aii	$\frac{25}{100}\times 80=20 ==> LQ=\$420$ $\frac{75}{100}\times 80=60 ==> UQ=\$680$ $IQR \quad =680-420$ $\quad =\$260$	M1 A1
8b	No of workers ($\le \$540) = 38$ No of workers ($> \$540) = 80-38$ $=42$ % of workers ($> \$540) \quad = \frac{42}{80}\times 100\%$ $=52.5\%$	M1 A1
8c	No of workers ($\le \$460) = 26$ No of workers ($\le \$800) = 73$ No of workers btw \$460 and \$800 $=73-26$ $=47$	M1 - either A1
8d	Let the additional no of workers be *x*. No of workers now ($\le \$650) = 56$ $\frac{56+x}{80+x}=\frac{11}{15}$ $840+15x=880+11x$ $4x=40$ $x=10$	M1 M1 A1
9a	Total amt $=90+3\times 150$ $=\$540$	B1
9b	Size of hall required $=(300\times 15)\times 1.65$ $=7425 \text{ m}^2$ Hall C and Hall E (total area = 7600 m^2) daily rental $=\$8000+\$5400$ $=\$13400$	M1 A1

9c	Cost for 1. hall rental $= \$13400 \times 4$ $= \$53600$ 2. long tables $= 300 \times \$2 \times 4$ $= \$2400$ 3. square tables $= \frac{600}{100} \times \$30 \times 4$ $= \$720$ 4. round tables $= \frac{75}{5} \times \$20 \times 4$ $= \$1200$ 5. chairs $= \frac{1500}{50} \times \$10 \times 4$ no of chairs needed $= 300 \times 4 + 300$ $= \$1200$ $= 1500$ 6. security guard $= 2 \times \$10 \times 12 \times 3$ $= \$720$ 7. part-timers $= 4 \times \$8 \times 12 \times 3$ $= \$1152$ Total cost $= 53600 + 2400 + 720 + 1200 + 1200 + 720 + 1152$ $= \$60992$ Total amt collected for 3 days $= 300 \times \$540$ $= \$162000$ Total entrance fee collected $= 35000 \times 6$ $= 210000$ Profit $= 162000 + 210000 - 60992$ $= \$311008$ The profit of \$311008 is more than the minimum target of \$300000. $\therefore$ The event organiser is correct.	5 items – M3 4 items – M2 2 items – M1 M1 – either item 6 or 7 M1 M1 A1- comparison made