Anglo - Chinese School
(Independent)

FINAL EXAMINATIONS 2014

YEAR 3 INTEGRATED PROGRAMME

CORE MATHEMATICS
PAPER 2

WEDNESDAY **8th October 2014** **1 hour 30 minutes**

INSTRUCTIONS TO STUDENTS

Do not open this examination paper until instructed to do so.
A calculator is required for this paper.
Answer all the questions on the answer sheets provided.
At the end of the examination, fasten the answer sheets together.
Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures. Answers in degrees are to be given to one decimal place.

INFORMATION FOR STUDENTS

The maximum mark for this paper is 80.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for correct method, provided this is shown by written working. You are therefore advised to show all working.

Answer all the questions on the answer sheets provided. ***Please start each question on a new page.***

1 *[Maximum mark: 5]*

(a) Given that the equation $\frac{7}{n-5} = \frac{6}{n} + 4$ can be expressed as $pn^2 + qn + r = 0$ find the values of *p*, *q* and *r*. [3]

(b) Solve for *n*. [2]

2 *[Maximum mark: 7]*

(a) Solve the inequality $-2 < \frac{1}{2}x - 1 < 5$. [3]

(b) Find the range of values of *x* for which $(2-x)(2x+3) \leq 5$. [4]

3 *[Maximum mark: 9]*

(a) Solve the equation $e^{-2x+1} = 15e + 4e^{-x+1}$, giving your answer correct to two decimal places where appropriate. [5]

(b) **(i)** Sketch the graph of $y = 3^{-x}$, indicating the *y*-intercept clearly. [2]

(ii) Insert on your sketch the additional graph required to obtain a graphical solution of the equation $x^3(3^x) = 2$. [2]

4 *[Maximum mark: 8]*

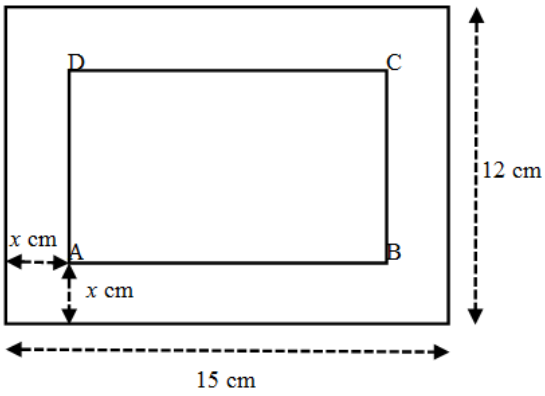
The straight line $y = 2x + 2$ cuts the curve $x^2 + 3y = 1$ at the points A and B. Find

(i) the coordinates of A and B, [4]

(ii) the equation of the perpendicular bisector of AB. [4]

5 *[Maximum mark: 8]*

The diagram shows a rectangular sheet of metal measuring 15 cm by 12 cm. A square of side x cm is cut out from each corner and the metal sheet is used to make an open box with base $ABCD$ and height x cm.



(i) Express the length of the side AB and BC in terms of x. [2]

(ii) If 60 cm^3 of water is added into the box, the height of the water level will be $1\frac{1}{2}$ cm. Form an equation in x and show that it reduces to $2x^2 - 27x + 70 = 0$. [2]

(iii) Solve the equation $2x^2 - 27x + 70 = 0$. Hence, find the capacity of the box. [4]

6 *[Maximum mark: 12]*

(a) Express $\frac{2}{2x+1} - \frac{1}{(2x+1)^2} - 1$ as a single fraction. [3]

(b) Solve the equations

(i) $3\log_x 2 = 4 - \log_2 x$, [5]

(ii) $\log_2\left(3x^2 - 32\right) + \log_2 \frac{1}{x} = 2 + \log_2 3$. [4]

7 *[Maximum mark: 9]*

(a) Given that the line $y = 3x - m$ meets the curve $y = 2x^2 - 6x + 5$, find the range of values of *m*. [4]

(b) P is a point on the curve $y = \frac{1}{x}$ and Q is a fixed point with coordinates $(2,\ 6)$. The midpoint of PQ is (*h*, *k*).

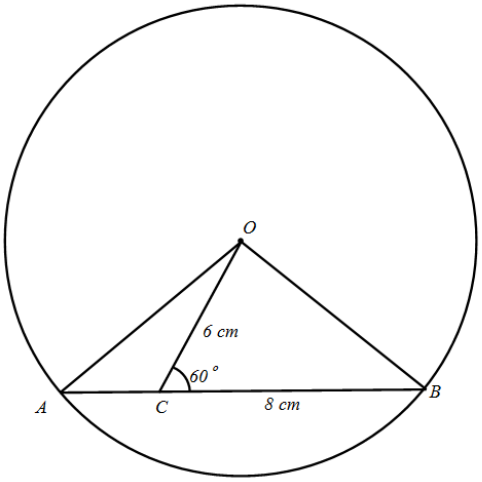
(i) If the *x*-coordinate of P is *n*, write down the coordinates of the midpoint of PQ in terms of *n*. [2]

(ii) Hence, express *k* in terms of *h*. [3]

8 *[Maximum mark: 12]*

(a) A triangle has sides 15, 41, and 52. What is its area? [5]

(b) *AB* is a chord on the circle with centre *O* and *C* is a point on *AB* such that $\angle OCB = 60°$.



Given that *OC* = 6 cm and *BC* = 8 cm, find

(i) OB, [3]

(ii) $\angle AOC$. [4]

9 *[Maximum mark: 10]*

Answer the whole of Question 9 on a sheet of graph paper.

The table shows experimental values of 2 variables, *x* and *y*.

x	1	2	2.5	3
y	2.12	2.86	3.44	4.18

(i) It is known that *x* and *y* are connected by the equation $ay^2 - bx^3 = 1$. By plotting y^2 against x^3, obtain a straight line to represent the above data. [4]

(ii) Use your graph to estimate the value of *a* and of *b*. [2]

(iii) By drawing a suitable straight line, find the value of *x* and of *y* which satisfy the simultaneous equations

$$ay^2 - bx^3 = 1$$
$$y^2 - x^3 = 1$$

[4]

End of Paper

Answer Key

1 a) $p = 4,\ q = -21,\ r = -30$ b) n = 6.42 or -1.17

2 a) $-2 < x < 12$ b) $x \le -\frac{1}{2}$ or $x \ge 1$

3 a) $x = -1.85$

4 i) A (-5, -8) and B (-1, 0). ii) $2y + x - 11 = 0$

5 i) AB = $(15 - 2x)$ cm BC = $(12 - 2x)$ cm iii) 140 cm³

6 a) $-\left(\frac{2x}{2x+1}\right)^2$ b) i) $x = 2$ *or* 8 ii) 5.83

7 a) $m \le 5\frac{1}{8}$ b) i) $\left(\frac{m+2}{2},\ \frac{1+6m}{2m}\right)$ ii) $k = \frac{12h-11}{4(h-1)}$

8 a) 234 cm² b i) 7.21 cm ii) $\angle AOC = 13.9°$

9 ii) $a = \frac{1}{4}$ $b = \frac{1}{8}$ iii) $x = 1.82$ $y = 2.56$