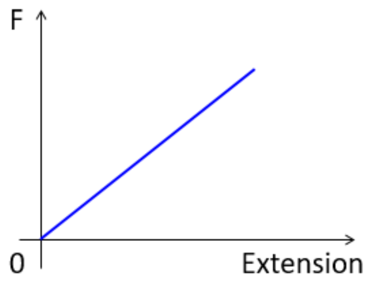
Answers to Examples, Quizzes & Mini-test [Forces]

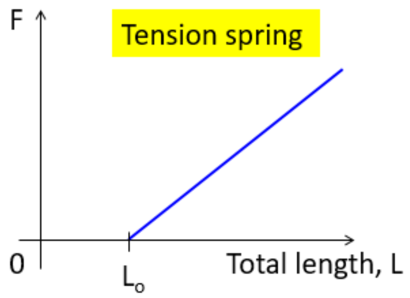
Quiz

The natural length of a spring is L_o. Sketch the graph of elastic force (F) against TOTAL length of the spring (L).

Is F proportional to L?

Solution:





- F-L graph does not pass through origin.
- For the same spring, the gradient should remain the same as its Force-extension graph (i.e. parallel)
- F is proportional to extension (x); but F is not proportional to L!
  - "proportional' relationship means that the graph is not only linear, but also passes through the origin

MINI-TEST 1

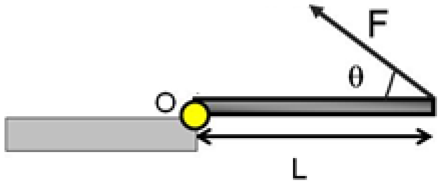
A force of 16 N is required to stretch a spring a distance of 40 cm from its rest position. A second identical spring is fixed parallel and adjacent to this spring. How much force is required to stretch this pair of springs by 20 cm?

My solution:

The applied force needs to overcome the total restoring force of the spring.
By Hooke's Law, restoring force is directly proportional to spring's extension (for the same spring constant); and directly proportional to the spring constant (for the same extension).
Thus stretching ONE spring to HALF the distance requires HALF the force (i.e. 8 N).
With TWO springs in parallel & identical, the effective spring constant DOUBLES. This DOUBLES the force required (i.e. 16 N).
Final answer: 16 N

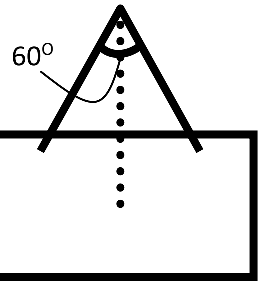
Quiz

A drawbridge of length L being is raised by a chain attached as shown.
What is the perpendicular distance from O? What is the moment or torque about O?



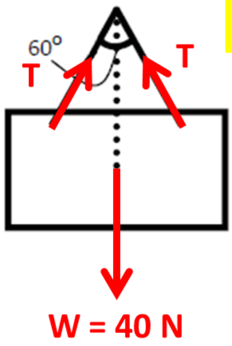
Perpendicular distance between O & the line of action of the force =
Moment or torque about O, $\tau =$ **F L sin θ**

Example 6
A picture weighing 40 N is hung by a wire which passes over a small hook.
The two parts of the wire are inclined at 60^O to each other.
Find the tensile force in the wire.

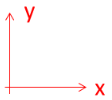


Solution:

Step 1: Draw the forces acting on the picture.



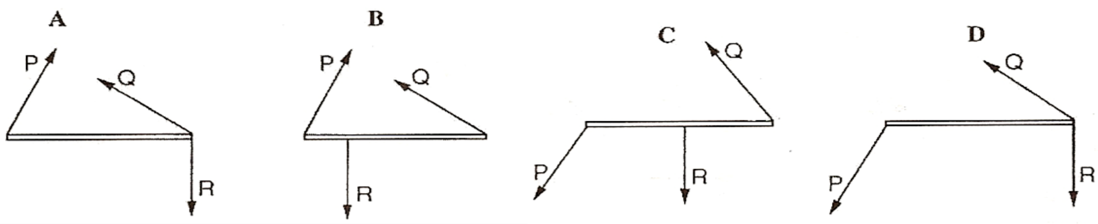
Step 2: Choose 2 perpendicular directions to resolve the forces in.



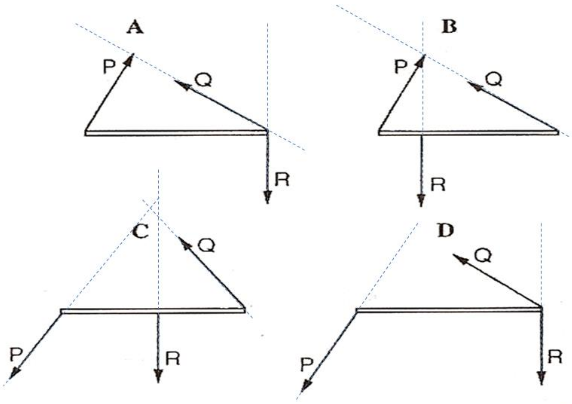
Since the picture is in equilibrium, the vector sum of the vertical components of the forces must add up to zero, i.e. $\sum F_y = 0$

$2 \times T \cos 30^O + (-40) = 0$
T = 23 N (2 s.f.)

Example 7 (N01/1/5)
A light rod is acted upon by three forces P, Q and R. Which diagram could show the position and direction of each of the forces when the rod is in equilibrium?



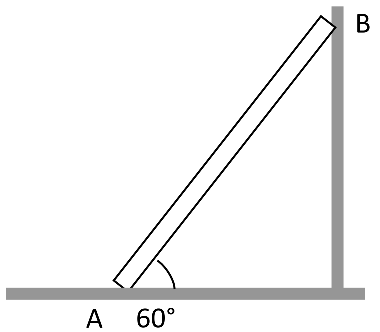
Solution:



Answer: B

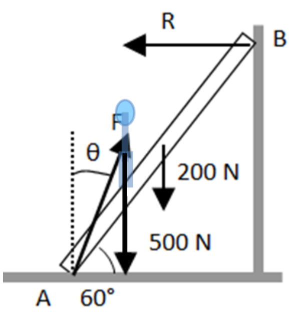
Example 9

A uniform ladder of length 10 m and weight 200 N leans against a smooth wall so that it is inclined 60° to the horizontal ground. A boy of weight 500 N stands on the ladder ¼ of the way from its lower end. Calculate the normal reaction at the smooth wall and the magnitude and direction of the force acting on the lower end of the ladder.



Solution:

There are currently two unknown forces (F & R). It is therefore convenient to take moments about point A to obtain an equation with only one unknown, in this case, R.

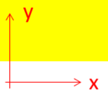


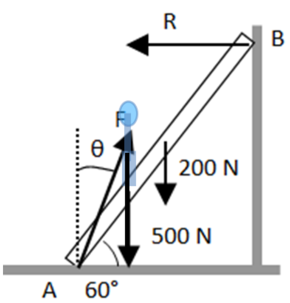
Taking moments about point A,
Sum of clockwise moment
= Sum of anticlockwise moment

$500\,(2.5\cos 60^\circ) + 200\,(5\cos 60^\circ)$
$= R\,(10\sin 60^\circ)$

$\underline{R = 130\text{ N (2 s.f.)}}$

Now to find F…
We apply the other condition: $\sum F_x = 0$ and $\sum F_y = 0$





Balancing horizontal forces,
$F\sin\theta = R = 130$ (1)

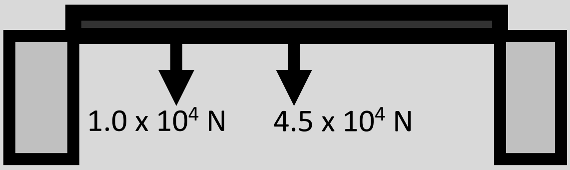
Balancing vertical forces,
$F\cos\theta = 500 + 200 = 700$ (2)

(1) ÷ (2): $\tan\theta = 129.9 \div 700$
$\underline{\theta = 11^\circ\text{ (2 s.f.)}}$

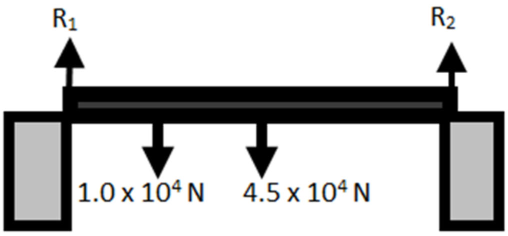
Sub $\theta = 10.5^\circ$ in (1), $\underline{F = 710\text{ N (2 s.f.)}}$

MINI-TEST 2

A uniform bridge, resting horizontally on supports at each end, weighs 4.5×10^4 N and is 8.0 m long. Find the force acting downward on each support when the bridge carries a load of 1.0×10^4 N placed 2.0 m from one end.



My solution:



Sum of upward forces = Sum of downward forces

$R_1 + R_2 = (1.0 + 4.5) \times 10^4 = 5.5 \times 10^4$ N

Take moments about the left end.

Sum of Anticlockwise moment = Sum of Clockwise moment

(R_2) (8.0 m) = (1.0×10^4) (2) + (4.5×10^4) (4)

$R_2 = 2.5 \times 10^4$ N

$R_1 = 3.0 \times 10^4$ N

$R_1 = 3.0 \times 10^4$ N, upward $R_2 = 2.5 \times 10^4$ N, upward

R1 & R2: forces exerted by each support on the bridge.

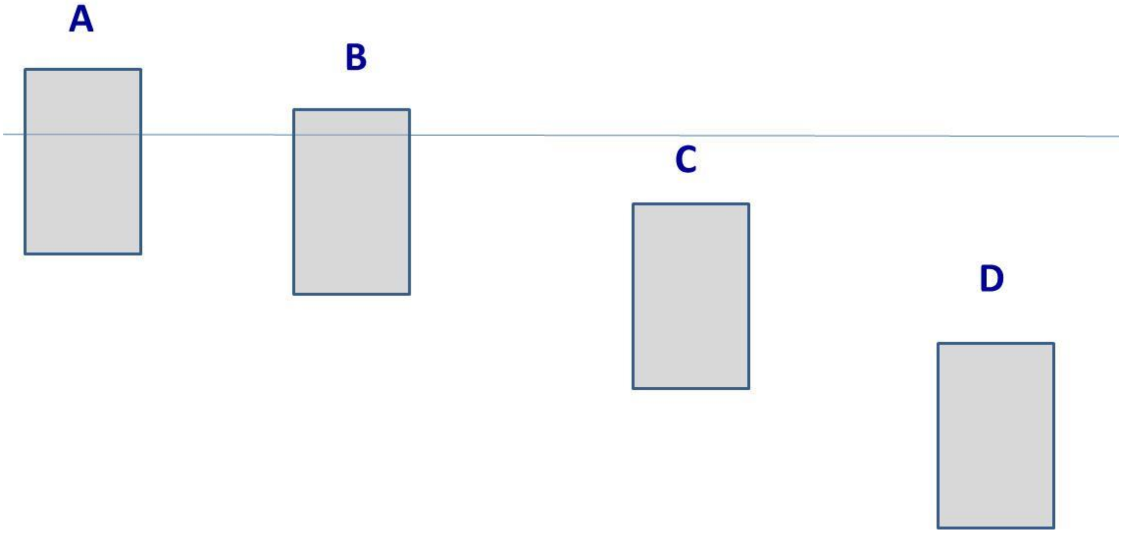
To determine the force acting on each support, apply Newton's 3rd Law of motion:

Force on Left support by bridge = 3.0×10^4 N, downward

Force on Right support by bridge = 2.5×10^4 N, downward

Quiz

Which object experiences the greatest value of upthrust?



Solution:

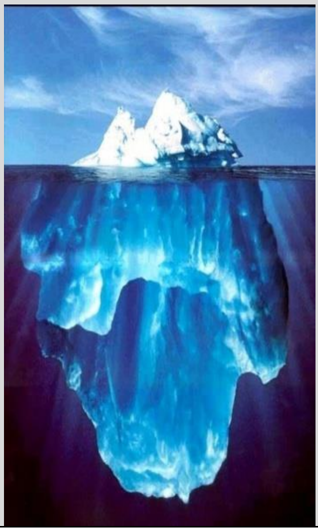
Upthrust = Weight of fluid displaced

→ B > A

→ C = D

MINI-TESTS 3

What proportion of an iceberg (floating in water) is submerged?
[density of ice = 0.92 g / cm^3; density of water = 1 g / cm^3]



My solution:

Vertical forces are balanced since the iceberg floats and thus in equilibrium (Newton's 1st law of motion). Thus,

Weight of ice berg = Upthrust

Weight of ice berg = Weight of water displaced (by Archimedes' Principle)

$m_{ice}\, g = m_{water\ displ}\, g$

$V_{ice}\, \rho_{ice}\, g = V_{water\ displ}\, \rho_{water}\, g$

$$\frac{V_{water\ displ}}{V_{ice}} = \frac{\rho_{ice}}{\rho_{water}} = 0.92$$

92 % of iceberg is underwater!