Candidate Name: ___________________________ (   ) Class: ________

4N(A)
Session 2

KRANJI SECONDARY SCHOOL
Preliminary Examination
Secondary 4 Normal (Academic)

ADDITIONAL MATHEMATICS **4051/02**
Paper 2

Wednesday **2 Aug 2023** 1 hr 45 min

READ THESE INSTRUCTIONS FIRST:
Do not open this question paper until you are told to do so.

Write your name and register number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 70.

Question	Marks
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Setter: Mdm J Yap

This question paper consists of 16 printed pages including the cover page

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 **(a)** Solve the equation $4^{x-3} = \dfrac{2^{x^2}}{8^{3x+2}}$. [5]

(b) The coordinates of the points P and Q are $(-6, 5)$ and $(2, 10)$.
Find the equation of the perpendicular bisector of PQ. [4]

2 A rectangular block has a square base of side $\left(\sqrt{10}+\sqrt{2}\right)$ cm and a height of h cm.

The volume of the rectangular block is $\left(52+28\sqrt{5}\right)$ cm^3.

Without using a calculator, find the height of the rectangular block, in cm.
Give your answer in the form $\left(a+b\sqrt{5}\right)$ cm where a and b are integers. [5]

3 **(a)** Given that A is an acute angle and that $\sin A = \frac{5}{7}$, find the exact value of $\sin 2A$. [3]

(b) Solve the following equations

(i) $\sec(2x - 15°) = 2$ for $0° < x < 180°$, [4]

(ii) $7\sec^2 A + 6\tan A - 23 = 0$ for $0 < A < \pi$. [4]

4 **(a)** A biologist modelled the population, N, of bacteria in a sample by the equation

$$N = -t^3 + \frac{41}{2}t^2 - 60t + 500, \qquad \text{for } 5 \le t \le 25,$$

where t is the temperature of the sample in degree Celsius.

(i) At what temperature does the number of bacteria reached the maximum? [3]

(ii) Find the maximum bacteria population. [2]

(b) Given that $y = \frac{3}{x^2 + 4}$ for $x > 0$, find $\frac{dy}{dx}$ and explain why y is a decreasing function.

[3]

5 **(i)** Differentiate $\left(1-\frac{1}{x}\right)^5$ with respect to x. [2]

(ii) Hence find the exact value of $\int_1^2 x^{-6}(x-1)^4 dx$. [4]

6 The equation of a circle is $x^2 + y^2 - 4x + 6y - 12 = 0$.

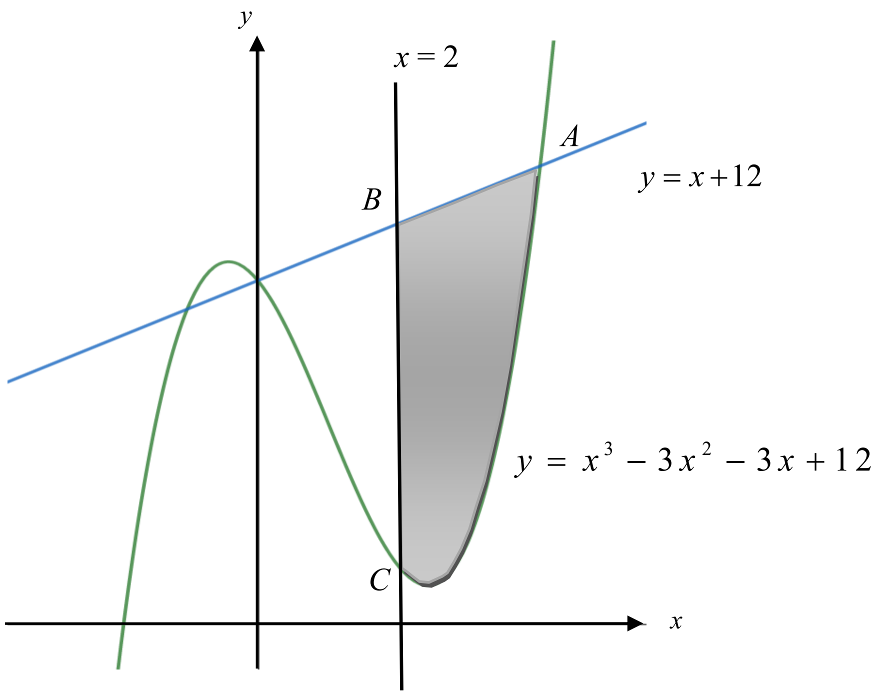
(i) Find the radius and the coordinates of the centre, *C*, of the circle. [3]

The point $P(5,1)$ lies on the circle.

(ii) Find the equation of the tangent to the circle at *P*. [4]

(iii) The tangent at P intersects the axes at A and B. Find the area of the triangle ABC. [4]

7

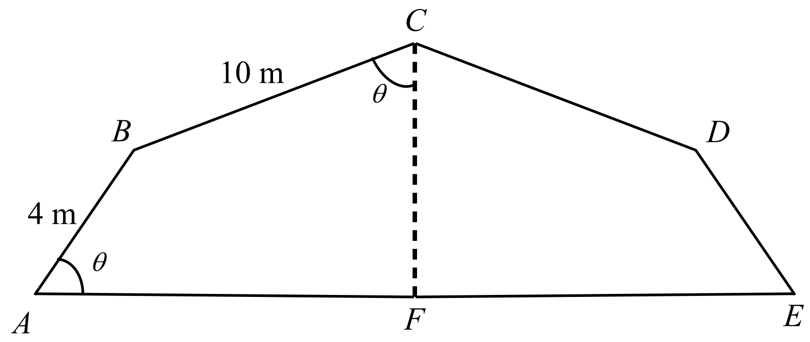


The diagram shows the line $y = x + 12$ intersecting the curve $y = x^3 - 3x^2 - 3x + 12$ at the point A and intersecting the line $x = 2$ at the point B. The line $x = 2$ also intersects the curve at the point C. Find

(i) the coordinates of A and of B. [5]

(ii) the area of the shaded region ABC. [6]

8 The diagram shows a cross-sectional area of a tunnel *ABCDEF*.
$AB = 4$ m, $BC = 10$ m and $\angle BAF = \angle BCF = \theta$ where θ is an angle in degrees.
The tunnel is symmetrical about the vertical line *CF*.



(i) Show that $AF = 10\sin\theta + 4\cos\theta$. [2]

(ii) Express *AF* in the form $R\sin(\theta + \alpha)$ where $R > 0$ and $0° < \alpha < 90°$. [3]

(iii) State the maximum length of AF and the corresponding value of θ. [2]

(iv) The tunnel was built such that the length of AF is the maximum. Is it possible for a train of width 12 m and height 3.6 m to pass through the tunnel? You can assume that the train is a rectangular cuboid and that the train will pass through the center of the tunnel. [2]

End of Paper