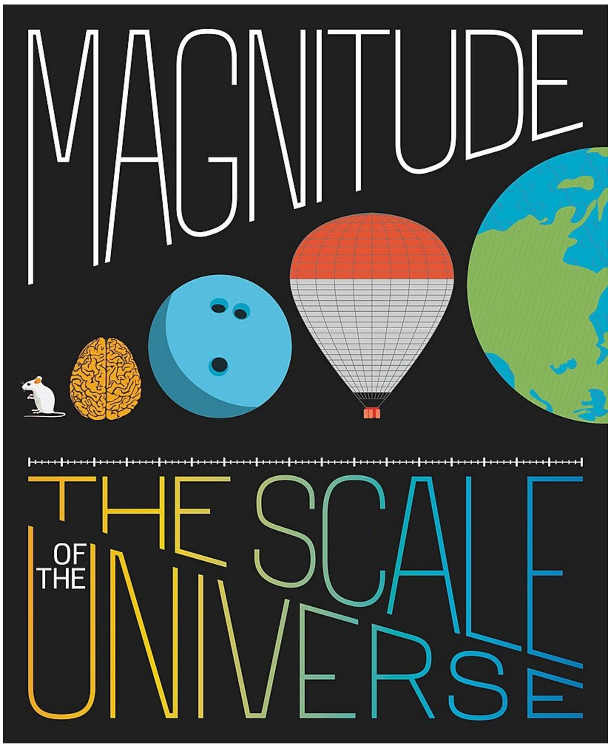
H2 Topic 01 – Measurement



Content

- Physical quantities and SI units
- Scalars and vectors
- Errors and uncertainties

Learning Outcomes

Candidates should be able to:

(a) recall the following base quantities and their SI units: mass (kg), length (m), time (s), current (A), temperature (K), amount of substance (mol)
(b) express derived units as products or quotients of the base units and use the named units listed in 'Summary of Key Quantities, Symbols and Units' as appropriate
(c) use SI base units to check the homogeneity of physical equations
(d) show an understanding of and use the conventions for labelling graph axes and table columns as set out in the ASE publication *Signs, Symbols and Systematics (The ASE Companion to 16-19 Science, 2000)*
(e) use the following prefixes and their symbols to indicate decimal sub-multiples or multiples of both base and derived units: pico (p), nano (n), micro (μ), milli (m), centi (c). deci (d), kilo (k), mega (M), giga (G), tera (T)
(f) make reasonable estimates of physical quantities included within the syllabus
(g) distinguish between scalar and vector quantities, and give examples of each
(h) add and subtract coplanar vectors
(i) represent a vector as two perpendicular components
(j) show an understanding of the distinction between systematic errors (including zero error) and random errors
(k) show an understanding of the distinction between precision and accuracy
(l) assess the uncertainty in a derived quantity by addition of actual, fractional, percentage uncertainties or by numerical substitution (a rigorous statistical treatment is not required).

** Additional requirement for 8867 H1 Physics (2020) subsumed under H208 Temperature and Ideal Gases*
(e) state that 1 mole of any substance contains 6.02×10^{23} particles & use the Avogadro number $N_A = 6.02 \times 10^{23}$ mol^{-1}

1.0 Introduction: The Nature of Science

Metrology is the scientific study of measurement. *A common understanding of units is crucial in collaborative human activities. It has roots in the French Revolution's political motivation to standardise units in France, when a length standard taken from a natural source was proposed. It has evolved into the Système international (SI) units that we are used to seeing.*

Science is a *durable* body of knowledge derived from empirical and repeatable observations. Despite this, science knowledge is also *tentative*: it is neither set in concrete nor perfect. Rather, it is subject to change.

A *scientific law* describes an observed phenomenon while a *scientific theory* explains it. The tentative nature of science also means that laws and theories may be updated in the light of new evidence or new interpretation of existing evidence.

Since science demands and relies on empirical evidence, a system of measurements is important to allow meaningful and objective comparisons between observations.

1.1 Physical Quantities and Units

A *physical quantity* is a property of a material or system that can be quantified by measurement. Each quantity consists of a numerical *magnitude* and a *unit.*

Systems of measurements are generally inter-convertible. *However, conversion from SI to "English" units (pounds/feet/inches used in the US) can result in mistakes. In 1999, NASA lost the $125 million Mars Climate Orbiter spacecraft after a 286-day journey to Mars. Thrusters used to nudge spacecraft onto a correct path were fired incorrectly. Lockheed Martin, sent thruster data in pounds to NASA, but NASA's navigation team was expecting metric units (newtons).*

We adopt SI units. The SI was founded on 7 *base quantities*, by which all other physical quantities in the SI are defined. The 7 SI base units are related to the base quantities where each is defined without referring to other units.

base quantity	SI base unit	symbol
mass	kilogram	kg
length	metre	m
time	second	s
electric current	ampere	A
temperature	kelvin	K
amount of substance	mole	mol
luminous intensity*	candela*	cd*

* Not in syllabus.

Other quantities such as force (in newtons) and energy (in joules) are referred to as *derived quantities.* These can be expressed in terms of the 7 base quantities.

When expressing a derived unit in terms of base units, consider the equations that relate to its associated quantity.

Example 1

Express the following derived SI units in terms of base SI units.

(**a**) the newton (**b**) the joule

(a)

$$F = ma$$

$$\text{unit of } F = (\text{unit of } m)(\text{unit of } a)$$
$$= \text{kg m s}^{-2}$$

(b)

$$E_K = \frac{1}{2}mv^2$$

$$\text{unit of } E = (\text{unit of } m)(\text{unit of } v)^2$$
$$= \text{kg m}^2\ \text{s}^{-2}$$

Notes: Present properly the working for such questions asking for units.

1.1.1 Homogeneous Equations and Dimensional Analysis

All correct physical equations are *homogenous*: all the terms on both sides of the equation have the same base units. If a quantity does not have an associated physical unit (dimension) and is purely a number, it is a *dimensionless* quantity.

Dimensional analysis may be used to determine the

- unit of a constant in an equation
- unit of an unknown quantity (which can be used to deduce possibilities for the unknown quantity)
- plausibility of a proposed physical equation

When performing dimensional analysis:

- addition or subtraction must involve only terms of the same units
- any exponent (the "power" by which a quantity is raised to) must be dimensionless

Example 2

The Bernoulli's equation used for fluid flow in a pipe may be expressed in the form

$$p + \frac{1}{2}\rho x^2 + A\rho h = B$$

where p is the pressure of the fluid, ρ is the density of the fluid, h is the height of the pipe, A and B are constants, and x is a physical quantity.

(a) Determine, in terms of SI base units, the units of A and x.

$$\text{unit of } P = (\text{unit of A})(\text{unit of } \rho)(\text{unit of } h)$$
$$\frac{(\text{kg})(\text{m s}^{-2})}{(\text{m}^2)} = (\text{unit of A})(\text{kg m}^{-3})(\text{m})$$
$$\text{unit of A} = \text{m s}^{-2}$$

$$\text{unit of } P = (\text{unit of } \rho)(\text{unit of } x)^2$$
$$\frac{(\text{kg})(\text{m s}^{-2})}{(\text{m}^2)} = (\text{kg m}^{-3})(\text{unit of } x)^2$$
$$\text{unit of } x = \text{m s}^{-1}$$

(b) Suggest a possible physical quantity that is represented by x.

speed (of the fluid)

1.1.2 Prefixes

Prefixes tell us the multiple or sub-multiple to cater to larger or smaller values respectively.

prefix	symbol	multiplier
tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}

prefix	symbol	multiplier
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}

Example 3

(a) Singapore's total land area is about 730 km^2. Express this value in m^2.
(b) Mercury has a density of 1.36×10^{4} kg m^{-3}. Express this value in g cm^{-3}.

$$
\begin{aligned}
730\text{ km}^2 &= 730\ (\text{km})^2 \\
&= 730\ (10^3\text{m})^2 \\
&= (730\times10^6)\ \text{m}^2 \\
&= (7.30\times10^8)\ \text{m}^2
\end{aligned}
$$

$$
\begin{aligned}
1.36\times10^4\text{ kg m}^{-3} &= \frac{1.36\times10^4\text{ kg}}{(1\text{ m})^3} \\
&= \frac{(1.36\times10^4)(10^3\text{g})}{(100\text{ cm})^3} \\
&= \frac{(1.36\times10^4)(10^3)\text{g}}{(100^3)\text{ cm}^3} \\
&= 13.6\text{ g cm}^{-3}
\end{aligned}
$$

Notes: A tip is to use brackets (*parentheses*) to keep track of prefixes before applying the power. When the numbers are very large, it is also a good practice to leave the number in standard form.

1.1.3 Estimation of Physical Quantities

The *order of magnitude* of a given quantity is the power of ten exponent of the numerical value when given in *scientific notation*. A number is written in scientific notation when a number between 1 and 10 is multiplied by a power of 10.

The order of magnitude is a useful estimate of values.

numerical value	scientific notation	order of magnitude
125	1.25×10^{2}	2
12 500	1.25×10^{4}	4

Example 4

Which estimate is realistic?

A The kinetic energy of a bus travelling on an expressway is 30 000 J.
B The power of a domestic light is 300 W.
C The temperature of a hot oven is 300 K.
D The volume of air in a car tyre is 0.03 m^3.

Solution

A) Estimate using "5 tonner" and 50 kmph

$$E_K = \frac{1}{2}mv^2$$

$$\approx \frac{1}{2}(5000)\left(\frac{50\times10^3}{60^2}\right)^2$$

$$\approx 500\ 000\ \text{J}$$

B) From Lazada/Shopee: typically <100 W

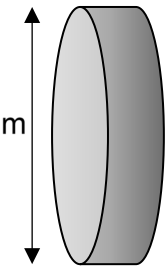
C) 300 K is 27°C

D) Estimate using a cylinder:

volume $=\left(\pi r^2\right)h$

$$\approx \pi(0.25)^2(0.2)$$

$$\approx 0.04\ \text{m}^2$$



$\approx$ 0.5 m

$\approx$ 0.2 m

ANSWER: D

Notes: SG 2008 A-Level Physics MCQ. Estimates can be made from typical values of other commonly encountered quantities.

We should be more aware of our everyday environment and develop a good sense of the quantities. We can, for e.g., read product specifications for power ratings, mass etc.

Product labels and specifications. *This is a typical charging adapter for mobile devices. Its output power is 10 W (use P = iV). Look for the battery capacity of your mobile phone– how long will it take for this charger to fully charge your mobile phone from a flat-out state?*

Heavy vehicles. *The truck above is commonly referred to as a "5-tonner" because it has a mass of about 5000 kg.*

Room dimensions. *A typical HDB apartment has a floor-to-ceiling height of about 2.6 m*

1.2 Errors

The experimental error in measuring a physical quantity can be understood as the difference between the measured value and the true value of a physical quantity.

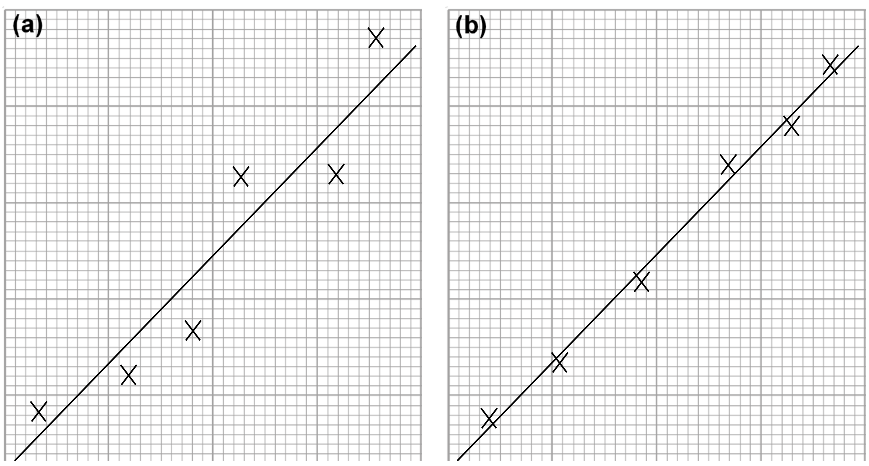
Experimental error = measured value – true value

1.2.1 Random Errors and Systematic Errors

A **random error** occurs when the measured values are scattered about the true value with no fixed pattern.	A **systematic error** occurs when the measured values are consistently larger or consistently smaller than the true value.

	random errors	**systematic errors**
eliminate	cannot be eliminated	can be eliminated
reduce	can reduce by averaging	can reduce by correct laboratory practice
examples	• fluctuations in the readings of a sensitive measuring instrument (e.g. an electronic balance) • fluctuations in the conditions of the environment (e.g. temperature or vibrations) • Irregularities in the quantity measured (e.g. diameter of a long wire) • human judgement	• zero errors • mis-calibration of measuring instrument • imperfect experimental procedures

Experimental data around a line of known equation:
Consider the scattering of data points about a best fit line.
The larger the scatter, the larger the extent of random errors in the experiment.



A <u>best fit line</u> compensates for values that are higher than the true values with the ones that are lower.

1.2.2 Accuracy and Precision

Accuracy is

how close the measured reading is to the true value

Take note of 2 different meanings of precision here. The precision of a measuring instrument will influence the *uncertainty* of a measurement which we will discuss later.

Precision of a set of readings is

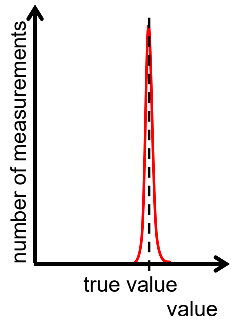
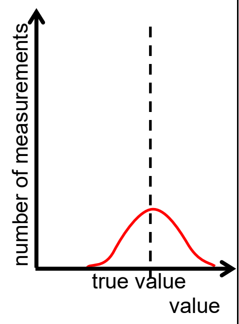
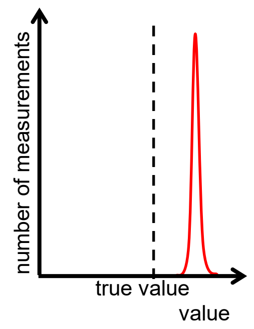
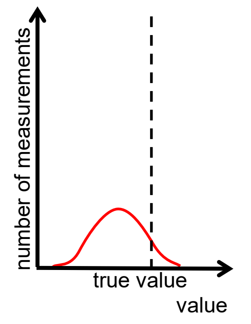
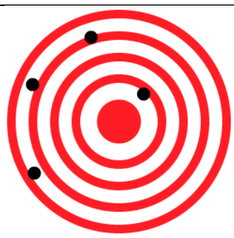
determined by
the range in the values.

In other words, the higher the precision of a set of values, the closer the values are to each other.

Precision of an instrument is determined by

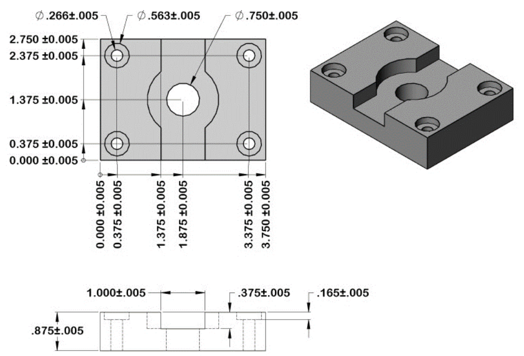
the size of the smallest division.

Example 5
Fill in the blanks with “high” or “low”.

				
accuracy	high	high	low	low
precision (of readings)	high	low	high	low
systematic error	low	low	high	high
random error	low	high	low	high
				

1.3 Uncertainty

An uncertainty quantifies the extent of errors. It gives the total range of values on both sides of a measurement in which the actual value of the measurement is expected to lie.



Different lingo, same meaning. *For carpenters and mechanical engineers, they speak of "tolerances" which convey the same concept i.e the max permissible error a certain feature can have.*

1.3.1 Absolute Uncertainty

A measurement x and its associated uncertainty Δx can be expressed in the form

$$x \pm \Delta x$$

where Δx is the *absolute uncertainty* of x.

Notes:

1. When a **calculation for uncertainty Δx** is required in an intermediate step or asked for in a standalone calculation, follow the general significant figures / decimal place (s.f. / d.p.) rule:
   - For **addition/subtraction** of values, the **d.p. of the answer** should follow the **lowest d.p**. used in the calculation. [See Example 8]
   - For **multiplication**/**division** of values, the **s.f. of the answer** should follow the **least s.f.** used in the calculation, subject to a minimum of 2 s.f. [See Example 9-11]
   -
2. When required to provide a **value with its associated uncertainty in the form of (x ± Δx),**
   - Δx must be rounded off to **1 s.f**;
   - *x* takes the same d.p. as the absolute uncertainty e.g.

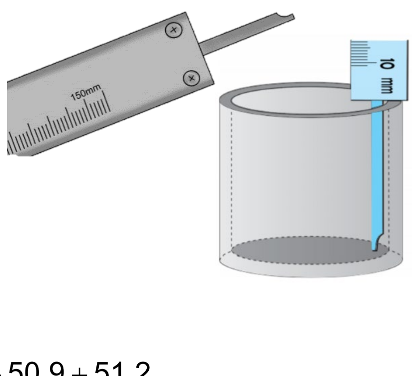
l = 52.<u>1</u> ± 0.<u>1</u> cm   or m = 21.0<u>3</u> ± 0.0<u>5</u> kg   or V = 8<u>30</u> ± <u>10</u> cm^3

Example 6

Salt is poured into a wide beaker. The height of the salt *h* is desired in order to determine the volume occupied. Since it is not a liquid, the height is not even. Several measurements at peaks and troughs of the salt surface is made using the depth rod of a vernier calliper. The readings are shown below.

(i) Estimate the uncertainty of the height *h*, and

(ii) express the measurement *h* with its uncertainty.



h_1 / mm	h_2 / mm	h_3 / mm	h_4 / mm
51.1	50.9	50.9	51.2

Solution

(i)

$$\Delta h \approx \frac{1}{2}(h_{max} - h_{min})$$
$$= \frac{1}{2}(51.2 - 50.9)$$
$$= 0.15 \text{ mm } (2 \text{ s.f.})$$

(ii)

$$\langle h \rangle \approx \frac{51.1 + 50.9 + 50.9 + 51.2}{4} = 51.025 \text{ mm}$$

$$h = 51.0 \pm 0.2 \text{ mm}$$

1.3.2 Fractional and Percentage Uncertainty

$$\text{fractional uncertainty} = \frac{\Delta x}{x}$$

$$\text{percentage uncertainty} = \frac{\Delta x}{x} \times 100\%$$

By A-Level convention, give fractional or percentage uncertainty to 2 s.f.

Fractional uncertainty is the ratio of absolute uncertainty to the measured value of a quantity. Percentage uncertainty is the fractional uncertainty in percentage. Both provide the relative size of its absolute uncertainty to its value.
To reduce the fractional uncertainty of a measurement, we can
- reduce the absolute uncertainty
- increase the value of the measurement

1.3.3 Calculating Uncertainties in Derived Quantities

In many experiments, measurements made are for calculating the value of another quantity. In doing so, the uncertainties of the measurements will *propagate* through the calculations.

1.3.4 The "Max - Min" Method

We can estimate the uncertainty of a calculated quantity as half the difference between the maximum and the minimum possible values of the quantity. i.e.

$$\Delta z \approx \frac{1}{2}\left(z_{\text{max}} - z_{\text{min}}\right)$$

Δz : uncertainty of quantity z
z_{max} : maximum value of z
z_{min} : minimum value of z

Example 7

The equation connecting object distance u, image distance v and focal length f is $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$.

A student measures values of u and v with their associated uncertainties. These are

$$u = 50 \text{ mm} \pm 3 \text{ mm}; \qquad v = 200 \text{ mm} \pm 5 \text{ mm}$$

He calculates the value of f = 40 mm. What is the uncertainty in this value?

Solution

$$f = \frac{1}{\left(\frac{1}{u} + \frac{1}{v}\right)}$$

$$f_{\text{max}} = \frac{1}{\left(\frac{1}{u} + \frac{1}{v}\right)_{\text{min}}} = \frac{1}{\frac{1}{u_{\text{max}}} + \frac{1}{v_{\text{max}}}} = \frac{1}{\frac{1}{53} + \frac{1}{205}}$$

$$f_{\text{min}} = \frac{1}{\left(\frac{1}{u} + \frac{1}{v}\right)_{\text{max}}} = \frac{1}{\frac{1}{u_{\text{min}}} + \frac{1}{v_{\text{min}}}} = \frac{1}{\frac{1}{47} + \frac{1}{195}}$$

$$\Delta f \approx \frac{1}{2}\left(f_{\text{max}} - f_{\text{min}}\right) = \frac{1}{2}\left(\frac{1}{\frac{1}{53} + \frac{1}{205}} - \frac{1}{\frac{1}{47} + \frac{1}{195}}\right) = 2.1 \text{ mm (2 s.f.)}$$

Notes: Adapted from SG 2011 A-Level Physics MCQ. Give calculated uncertainties to 2 s.f. as intermediate answers or when asked as a standalone quantity in a structured question.

While powerful, the Max-Min method can be slow. The next table below shows a set of formulae that allows us to quickly calculate the uncertainty of the quantity that is the subject of **simple** equations.

For more complicated equations like Example 7, stick to the Max-Min method. (See Appendix 1 to see how the Max-Min method can show the validity of the formulae.)

1.3.5 Error Propagation

Operation	Calculating Uncertainty for Simple Equations
Add or Subtract	Sum up contributing **absolute** uncertainties $z = m(x_1) + x_2 - n(x_3) + ...$ $\Delta z = m(\Delta x_1) + \Delta x_2 + n(\Delta x_3) + ...$
Multiply, Divide or Power	Sum up contributing **fractional** uncertainties $z = \frac{kA^nB}{qC^m}$ $\frac{\Delta z}{z} = \frac{\Delta k}{k} + n\frac{\Delta A}{A} + \frac{\Delta B}{B} + \frac{\Delta q}{q} + m\frac{\Delta C}{C}$ $= n\frac{\Delta A}{A} + \frac{\Delta B}{B} + m\frac{\Delta C}{C}$ **Note:** Constants *k* and *q* have zero uncertainty.

Uncertainties are always *added*. So the uncertainty of a quantity can never be less than any of its contributing uncertainties. This is a good reminder that uncertainty estimation errs on side of being *conservative*. **To determine the uncertainty for any quantity, make it the "subject of formula" before calculating its uncertainty** to avoid accidentally subtracting contributing uncertainties.

Example 8

Given *x, y* and *z,* find the quantities with their associated uncertainties:

$$x = (3.3 \pm 0.1) \text{ cm}; \qquad y = (5.4 \pm 0.2) \text{ cm}; \qquad z = (10.0 \pm 0.7) \text{ cm}$$

(a) $Q = x + y$

(b) $T = 2z - x - y$

Solution

(a)

$$\begin{aligned} Q &= x + y \\ &= 3.3 + 5.4 \\ &= 8.7 \text{ cm} \end{aligned}$$

$$\begin{aligned} \Delta Q &= \Delta x + \Delta y \\ &= 0.1 + 0.2 \\ &= 0.3 \text{ cm} \end{aligned}$$

$$Q = (8.7 \pm 0.3) \text{ cm}$$

(b)

$$\begin{aligned} T &= 2z - x - y \\ &= 2(10.0) - 3.3 - 5.4 \\ &= 11.3 \text{ cm} \end{aligned}$$

$$\begin{aligned} \Delta T &= 2\Delta z + \Delta x + \Delta y \\ &= 2(0.7) + 0.1 + 0.2 \\ &= 1.7 \text{ cm} \end{aligned}$$

$$T = (11 \pm 2) \text{ cm}$$

Example 9

Given *x*, *y* and *z*, find the quantities with their associated uncertainties.

$x=(0.33\pm0.01)$ cm $\quad$ $y=(5.4\pm0.2)$ cm $\quad$ $z=(10.0\pm0.7)$ cm

(a) $Q=xy$

(b) $T=\dfrac{yz}{x}$

Solution

(**a**)

$$Q=xy$$
$$=(0.33)(5.4)$$
$$=1.782\text{ cm}^2$$

$$\frac{\Delta Q}{Q}=\frac{\Delta x}{x}+\frac{\Delta y}{y}$$
$$\frac{\Delta Q}{1.782}=\left(\frac{0.01}{0.33}+\frac{0.2}{5.4}\right)$$
$$\Delta Q=0.12\text{ cm}^2$$

$$Q=(1.8\pm0.1)\text{ cm}^2$$

(**b**)

$$T=\frac{yz}{x}$$
$$=\frac{(5.4)(10.0)}{(0.33)}$$
$$=163.64\text{ cm}$$

$$\frac{\Delta T}{T}=\frac{\Delta y}{y}+\frac{\Delta z}{z}+\frac{\Delta x}{x}$$
$$\frac{\Delta T}{163.64}=\left(\frac{0.2}{5.4}+\frac{0.7}{10.0}+\frac{0.01}{0.33}\right)$$
$$\Delta T=22\text{ cm}$$

$$T=(160\pm20)\text{ cm}$$

Example 10

Given *x*, *y* and *z*, find the quantities with their associated uncertainties

$x=(0.33\pm0.01)$ cm $\quad$ $y=(5.4\pm0.2)$ cm $\quad$ $z=(10.0\pm0.7)$ cm

(a) $Q=x^2y$

(b) $W=\dfrac{x^2}{y^3}$

Solution

(**a**)

$$Q=x^2y$$
$$=(0.33)^2(5.4)$$
$$=0.588\text{ cm}^3$$

$$\frac{\Delta Q}{Q}=2\left(\frac{\Delta x}{x}\right)+\frac{\Delta y}{y}$$
$$\frac{\Delta Q}{0.588}=\left(\frac{(2)(0.01)}{0.33}+\frac{0.2}{5.4}\right)$$
$$\Delta Q=0.057\text{ cm}^3$$

$$Q=(0.59\pm0.06)\text{ cm}^3$$

(**b**)

$$W=\frac{x^2}{y^3}=\frac{0.33^2}{5.4^3}$$
$$=0.0006916\text{ cm}^{-1}$$

$$\frac{\Delta W}{W}=2\left(\frac{\Delta x}{x}\right)+3\left(\frac{\Delta y}{y}\right)$$
$$\frac{\Delta W}{0.0006916}=\frac{(2)(0.01)}{0.33}+\frac{(3)(0.2)}{5.4}$$
$$\Delta W=0.00012\text{ cm}^{-1}$$

$$W=0.0007\pm0.0001\text{ cm}^{-1}$$
$$\text{or }=(7\pm1)\times10^{-4}\text{ cm}^{-1}$$

Example 11

Given the following measurements, find the percentage uncertainty of the cylinder's density.

diameter of cylinder $D = 5.08 \pm 0.01$ cm
height of cylinder $h = 20.3 \pm 0.1$ cm
mass of cylinder $m = 252.5 \pm 0.1$ g

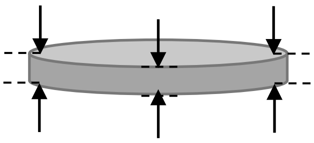
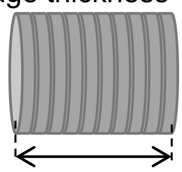
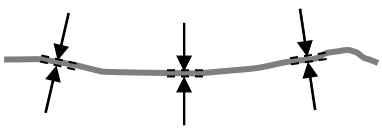
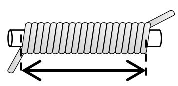
Solution

$$\rho = \frac{\text{mass}}{\text{volume}} = \frac{m}{\pi r^2 h} = \frac{m}{\pi\left(\frac{D}{2}\right)^2 h} = \frac{4m}{\pi D^2 h}$$

$$\frac{\Delta\rho}{\rho}\times 100\% = \left(\frac{\Delta m}{m} + 2\frac{\Delta D}{D} + \frac{\Delta h}{h}\right)\times 100\% = \left(\frac{0.1}{252.2} + 2\frac{0.01}{5.08} + \frac{0.1}{20.3}\right)\times 100\% = 0.93\% \ (2 \text{ s.f.})$$

Note: "4" and "*π*" are constants with no uncertainty. For terms with "power", multiply the power as a factor. Give standalone fractional/percentage uncertainty to 2 s.f.

1.3.6 Average of Repeated Measurements vs Averaging from Single Measurement

	Average of Repeated Measurements	**Average from Single Measurement**
random error	minimised	present in that single measurement
uncertainty	follow instrument precision at best	can be reduced to be better than instrument precision
concept	make multiple measurements of one item	make one measurement of multiple items
comparative examples	measuring the thickness of a single coin at 3 points using a vernier calliper average across the 3 separate thicknesses recorded 	measure the total thickness of 10 coins stacked together using a vernier calliper divide the total thickness by 10 to obtain average thickness 
	measure the diameter of a metal wire at different parts of the wire using a micrometer screw gauge average across the separate diameters recorded 	wind the wire with closed coils tightly around a rod for 20 rounds measure the total length for 20 diameters using a vernier calliper divide the total length by 20 to obtain average diameter 
	measure time taken for a ball to drop a fixed height using a stop watch repeat the measurement for 3 tries average the timings across the 3 tries	measure the total time taken for 15 complete oscillations using a stop watch divide the total time taken by 15 to obtain average period

Example 12

We demonstrate a lower uncertainty in measurement when we take an average from a single measurement of multiple, similar objects, compared to taking the average of repeated measurements of a single item. Whenever possible in practicals, we should try to "stack" items for a single measurement (it is not always possible to have access to multiple items!).

(a) State the precision of a 30 cm rule: 0.1 cm

(b) **(i)** Given 1 wooden block, measure and record the thickness for one wooden block, *t*.

t_1 / cm	t_2 / cm	t_3 / cm	$\langle t \rangle$ / cm
1.0	1.0	1.0	1.0

t = 1.0 cm (1 d.p.)

(ii) Let $z = t_1 + t_2 + t_3$. Find the uncertainty Δz.

$$\Delta z = \Delta t_1 + \Delta t_2 + \Delta t_3 = 0.3 \text{ cm}$$

$\Delta z =$ 0.3 cm (1 d.p.)

(iii) The average thickness $\langle t \rangle = \frac{1}{3}z$. Find $\Delta\langle t \rangle$.

$$\Delta\langle t \rangle = \frac{1}{3}(0.3) = 0.1 \text{ cm}$$

$\Delta\langle t \rangle =$ 0.1 cm (1 s.f.)

(iv) Express *t* with its associated uncertainty: *t* = (1.0 ± 0.1) cm

Note: In **(b)(i)-(ii)**, we apply d.p. rule and the final answers follow the lowest d.p. of the component values. In **(b)(iii),** the number 3 is a counting number and is not a measurement. It is regarded as a constant and does not carry an uncertainty. Hence s.f of answer follows s.f. of *z*.

(c) **(i)** Given 12 wooden blocks, measure and record the thickness of 1 wooden block, *t*.

Let *T* be total thickness of 12 stacked blocks.

$$T = 12.1 \text{ cm}, \qquad t = \frac{T}{12} = \frac{12.1}{12} = 1.00833 = 1.01 \text{ cm}$$

t = 1.01 cm (3 s.f.)

(ii) Determine the uncertainty Δt.

$$\Delta t = \frac{1}{12}(\Delta T) = \frac{1}{12}(0.1) = 0.0083 \text{ cm } (2 \text{ s.f.})$$

$\Delta t =$ 0.0083 cm (2 s.f.)

(iii) Express *t* with its associated uncertainty: *t* = (1.008 ± 0.008) cm

Notes: In **(c)(i)-(ii)**, t the number 12 is a counting number and is not a measurement. It is regarded as a constant and does not carry an uncertainty. Hence s.f of answer follows s.f. of *z*.

1.4 Scalars and Vectors

A **scalar** is a physical quantity that has magnitude only.	A **vector** is a physical quantity that has both magnitude and direction.

Examples:

- mass
- energy
- pressure
- electric current.

- velocity
- acceleration
- weight
- electric field strength

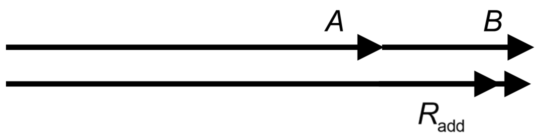
1.4.1 Addition of Vectors

Adding 2 vectors *A* and *B* that are acting in the same direction results in a vector *R*

- with a magnitude that is equal to the sum of the individual magnitudes of the two

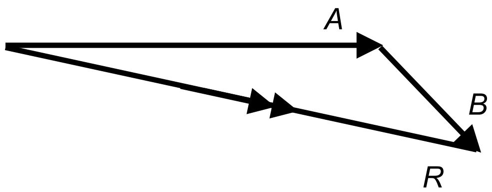
$$|R| = |A| + |B|$$

- acting in the same direction



adding 2 vectors along the same direction

If 2 vectors are acting at an angle to each other, we apply trigonometry.



adding 2 vectors acting at an angle to each other

An example of an application involving the addition of vectors is to determine the resultant force acting on an object when multiple external forces are acting on it.

In general, we “line up” vectors to be summed in a “head-to-tail” order. We then link the starting point to the tail of the last summed vector to obtain (geometrically) the resultant vector.

1.4.2 Subtraction of Vectors

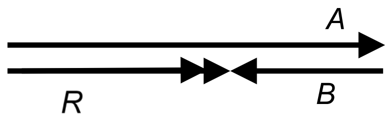
If vector *B* is to be subtracted from vector *A*, this operation is equivalent to adding the negative vector of *B* to *A*. If 2 parallel vectors are acting in opposite directions, adding them will result in a vector

- with a magnitude that is equal to the difference between the individual magnitudes of the two

$$|R| = |A| - |B|$$

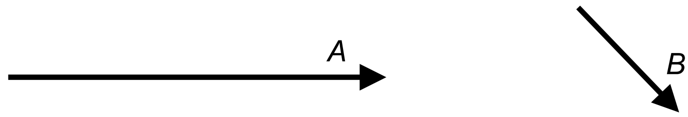
- acting in the same direction as the vector with a larger magnitude

because $|A| > |B|$, direction of resultant *R* is in the direction of *A*

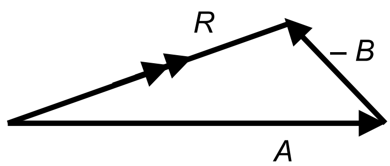


Adding 2 vectors in opposite direction / subtracting colinear vectors

If the vectors are not parallel to each other, we add the negative vector of *B* to *A* and align them "head to tail" once again:

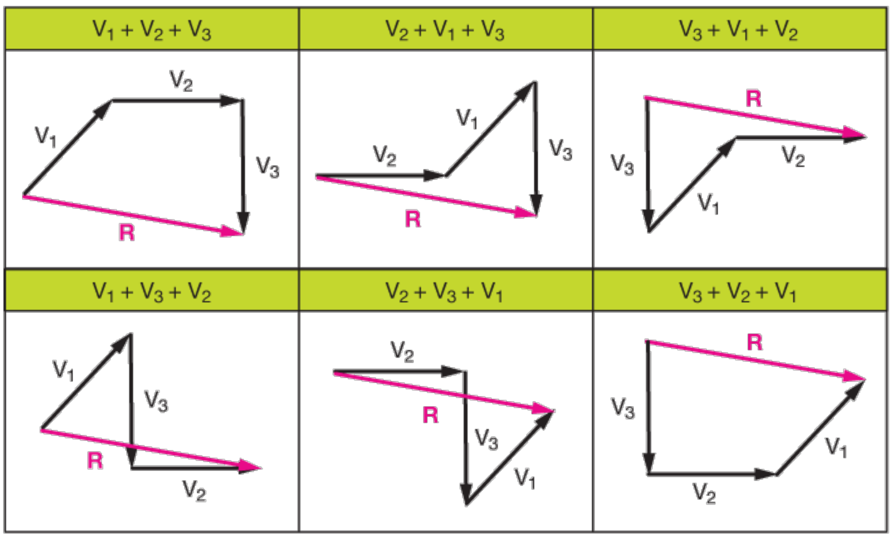


$$A - B = A + (-B)$$



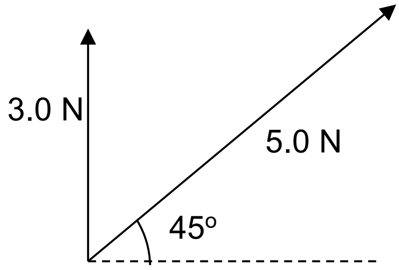
subtracting 2 vectors at an angle

An example of an application involving the subtraction of vectors is the determination of the change of velocity of a body. Mathematically, the order in which we add vectors does not change the resultant. We say that vector addition is *commutative.*

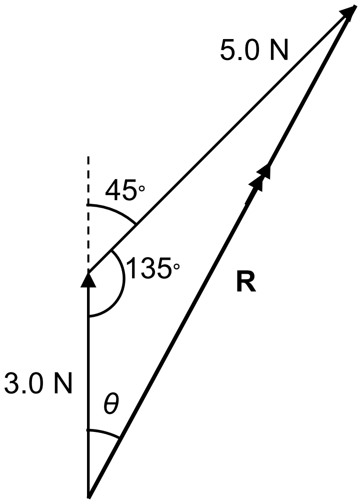


Example 13

Find the resultant force due to the 2 coplanar forces.



Solution



by cosine rule:

$$R^2 = (3.0)^2 + (5.0)^2 - 2(3.0)(5.0)\cos 135^\circ$$
$$R = \sqrt{55.213}$$
$$= 7.43 \text{ N}$$

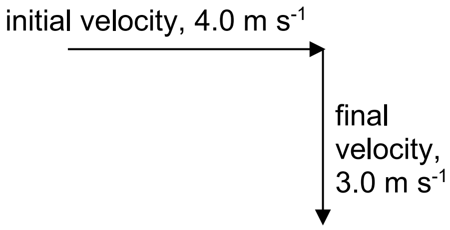
by sine rule:

$$\frac{5.0}{\sin\theta} = \frac{\sqrt{55.213}}{\sin 135^\circ}$$
$$\theta = 28.4^\circ$$

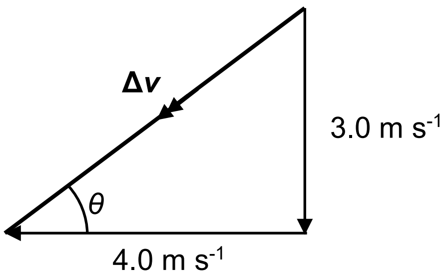
Example 14

A ball hits a surface and its velocity changes as shown.

Find the change in velocity.



Solution



opposite direction to "initial velocity"

change in velocity $\Delta v = v_{\text{final}} - v_{\text{initial}}$

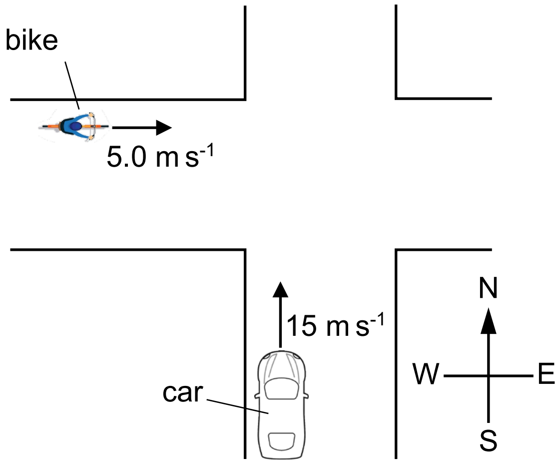
by Pythagoras Theorem:

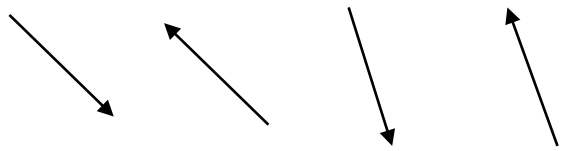
$$(\Delta v)^2 = (3.0)^2 + (4.0)^2$$
$$\Delta v = 5.00 \text{ m s}^{-1}$$

$$\tan\theta = \frac{3.0}{4.0}$$
$$\theta = 36.9^\circ$$

Example 15

A car (moving north at 15 m s^{-1}) and a bike (moving east at 5.0 m s^{-1}) are equidistant from a crossroads. At this moment, which arrow represents the velocity of the car relative to the bike?



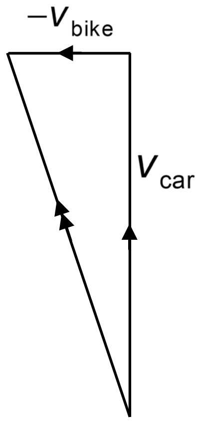


Solution

velocity of car relative to bike $v = v_{car} - v_{bike}$

$= v_{car} + (-v_{bike})$

answer : D



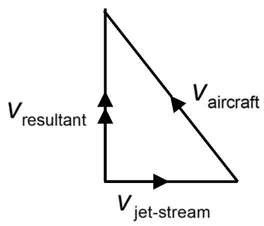
Example 16

An aircraft flies with an airspeed of 700 km h^{-1} through a 250 km h^{-1} jet-stream wind from the west. The pilot wishes to fly directly north from South Sumatra towards Changi airport. Find the

(i) speed of the aircraft in the direction of north relative to the ground, and

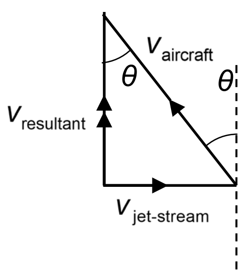
(ii) the direction that the aircraft is actually pointing at.

Solution



(i) speed of aircraft relative to ground:

$$v_{resultant} = \sqrt{v_{aircraft}^2 - v_{jet\text{-}stream}^2} = \sqrt{700^2 - 250^2} = 654 \text{ km h}^{-1}$$



(ii) Let direction of aircraft be θ west of north.

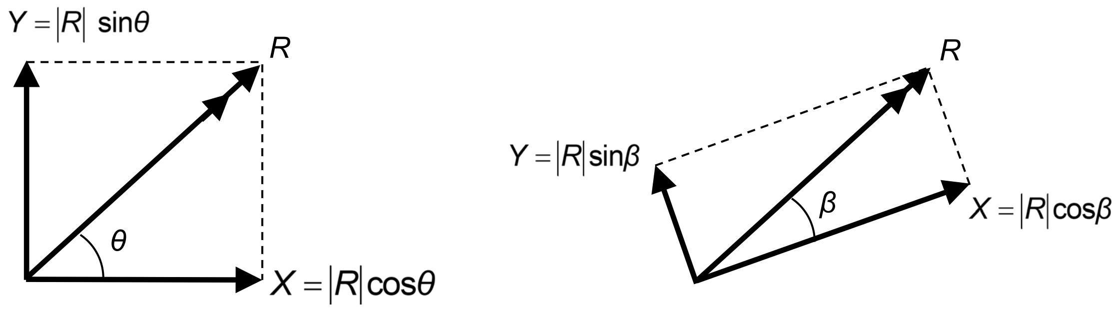
$$(v_{aircraft})\sin\theta = v_{jet\text{-}stream}$$

$$\theta = \sin^{-1}\left(\frac{v_{jet\text{-}stream}}{v_{aircraft}}\right) = \sin^{-1}\left(\frac{250}{700}\right) = 20.9^\circ$$

Notes: Do apply logic to check results. The pilot is "expending" some of the aircraft's capability in generating speed to "balance" out the wind (jet-stream) so the resultant speed should be less than what the aircraft is capable of in still air.

1.4.3 Resolution of Vectors

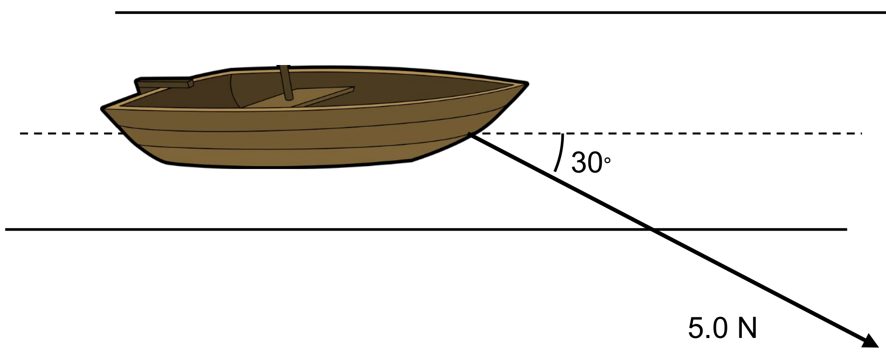
A vector *R* can be resolved (broken down into components) into any perpendicular pair of vectors *X* and *Y*.



resolving a vector R along 2 perpendicular directions

Example 17

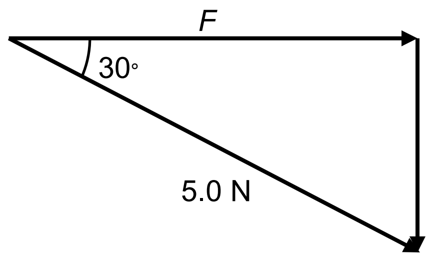
A boat is pulled along straight river by a force of 5.0 N acting at 30° to the bank.
Find the magnitude of force that is accelerating the boat along the axis of the river.



Solution

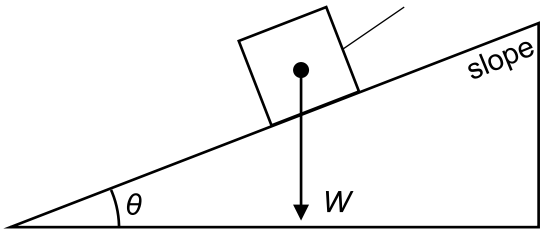
component of force along river :

$$F = (5.0)\cos 30^\circ$$
$$= 4.33 \text{ N}$$



Example 18

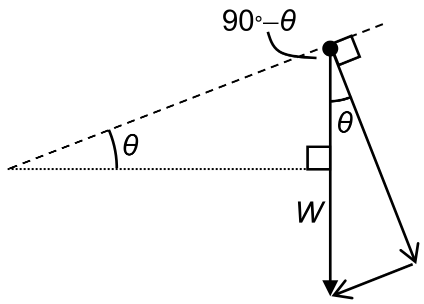
Find the magnitude of the component of the block's weight *W* causing it to slide down the slope.



Solution

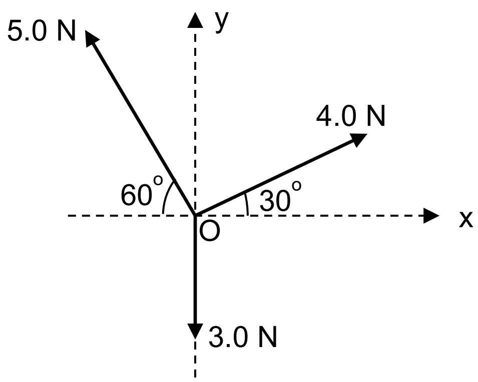
We resolve the weight of the block (keep as hypotenuse of force triangle) into two component vectors; one parallel to the slope surface and another normal to the slope.

$W_{\text{along slope}} = W \sin\theta$



Example 19

Three coplanar forces act at the point O as shown in the diagram. Find the resultant force.



Solution

vector sum of forces along x-direction

$F_x = (-5.0\cos 60°) + 4.0\cos 30°$

$= 0.96410 \text{ N}$

vector sum of forces along y-direction

$F_y = (5.0\sin 60°) + 4.0\sin 30° + (-3.0)$

$= 3.3301 \text{ N}$

[**magnitude**]

$|F| = \sqrt{F_x^2 + F_y^2}$

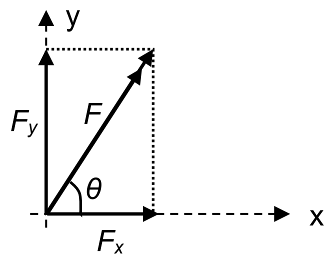
$= \sqrt{0.96410^2 + 3.3301^2}$

$= 3.4669 = 3.47 \text{ N}$

[**direction**]

$\tan\theta = \dfrac{F_y}{F_x} = \dfrac{3.3301}{0.96410}$

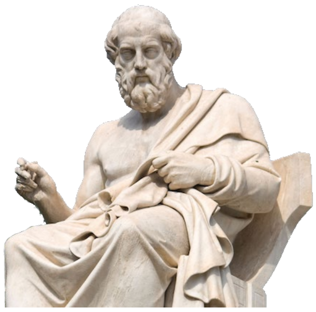
$\theta = 73.9°$



1.5 Ending Notes: The Necessity of Measurement in Our Everyday Life

"If someone separated the art of counting and measuring and weighing from all the other arts, what was left of each (of the others) would be, so to speak, insignificant."

- Plato



Plato. *Athenian philosopher during the Classical period in ancient Greece.*

In a world in which no measurement tools exist, there can be no modern medicine, healthcare, surgery, pharmacology, radiology, dentistry, optometry, or audiology. Therefore, in a measureless world, there can be no Science.

You can use the space below to do up your own mind-map to summarise this topic.

APPENDIX 1

Showing Uncertainty Propagation via the Max-Min Method

Consider 2 measurements with their respective absolute uncertainties: $x \pm \Delta x$ and $y \pm \Delta y$
Let z be the calculated quantity.

- If $z = x - y$:

$$\begin{aligned} z_{max} &= x_{max} - y_{min} \\ &= x + \Delta x - (y - \Delta y) \\ &= (x - y) + (\Delta x + \Delta y) \end{aligned}$$

$$\begin{aligned} z_{min} &= x_{min} - y_{max} \\ &= x - \Delta x - (y + \Delta y) \\ &= (x - y) - (\Delta x + \Delta y) \end{aligned}$$

$$\begin{aligned} \text{then } \Delta z &\approx \frac{1}{2}(z_{max} - z_{min}) \\ &= \Delta x + \Delta y \end{aligned}$$

- If $z = xy$:

$$\begin{aligned} z_{max} &= (x_{max})(y_{max}) \\ &= (x + \Delta x)(y + \Delta y) \\ &= xy + x(\Delta y) + \Delta x(y) + (\Delta x)(\Delta y) \end{aligned}$$

$$\begin{aligned} z_{min} &= (x_{min})(y_{min}) \\ \text{s} \quad &= (x - \Delta x)(y - \Delta y) \\ &= xy - x(\Delta y) - \Delta x(y) + (\Delta x)(\Delta y) \end{aligned}$$

$$\begin{aligned} \text{then } \Delta z &\approx \frac{1}{2}(z_{max} - z_{min}) \\ &= x(\Delta y) + \Delta x(y) \end{aligned}$$

dividing both sides by z

$$\begin{aligned} \frac{\Delta z}{z} &= \frac{x(\Delta y)}{z} + \frac{\Delta x(y)}{z} \\ &= \frac{x(\Delta y)}{xy} + \frac{\Delta x(y)}{xy} \end{aligned}$$

$$\therefore \quad \frac{\Delta z}{z} = \frac{\Delta y}{y} + \frac{\Delta x}{x}$$

- if $z = kxy$, find $\dfrac{\Delta z}{z}$.

$$z = kxy$$

$$\begin{aligned} z_{\max} &= k\left[(x_{\max})(y_{\max})\right] \\ &= k\left[(x+\Delta x)(y+\Delta y)\right] \\ &= k\left[xy + x(\Delta y) + \Delta x(y) + (\Delta x)(\Delta y)\right] \end{aligned}$$

$$\begin{aligned} z_{\min} &= k\left[(x_{\min})(y_{\min})\right] \\ &= k\left[(x-\Delta x)(y-\Delta y)\right] \\ &= k\left[xy - x(\Delta y) - \Delta x(y) + (\Delta x)(\Delta y)\right] \end{aligned}$$

$$\begin{aligned} \text{then } \Delta z &\approx \frac{1}{2}(z_{\max} - z_{\min}) \\ &= k\left[x(\Delta y) + \Delta x(y)\right] \end{aligned}$$

dividing both sides by z

$$\begin{aligned} \frac{\Delta z}{z} &= \frac{x(\Delta y)}{z} + \frac{\Delta x(y)}{z} \\ &= \frac{x(\Delta y)}{xy} + \frac{\Delta x(y)}{xy} \\ &= \frac{\Delta y}{y} + \frac{\Delta x}{x} \end{aligned}$$