	Class	Index Number
Name:		

Jurong West Secondary School

Preliminary Examinations 2020

/ 80

ADDITIONAL MATHEMATICS 4047/01

Secondary Four Express/ Five Normal Academic **27 August 2020**

Paper 1 **1000 - 1200**

2 hours

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

After checking of answer script		
Checked by Student	**Signature**	**Date**

This document consists of **17** printed pages.

Setter: Ms Sharron Chiam

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\ .$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n\ ,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

1 A cone with base radius $\left(5+2\sqrt{3}\right)$ cm and a slant height l cm has a curved surface area of $\left(51-3\sqrt{3}\right)\pi$ cm^2. Without using a calculator, obtain an expression for l in the form of $\left(a+b\sqrt{3}\right)$, where a and b are integers. [4]

<u>Solution</u>:
Curved surface area of cone $=\pi rl$

$\pi\left(5+2\sqrt{3}\right)l=\left(51-3\sqrt{3}\right)\pi$

$l=\dfrac{\left(51-3\sqrt{3}\right)\pi}{\pi\left(5+2\sqrt{3}\right)}$ [M1] – Correct use of formula

$=\dfrac{51-3\sqrt{3}}{5+2\sqrt{3}}\times\dfrac{5-2\sqrt{3}}{5-2\sqrt{3}}$ [M1] – Correct use of conjugate surds

$=\dfrac{255-102\sqrt{3}-15\sqrt{3}+6(3)}{25-4(3)}$ [M1]

$=\dfrac{273-117\sqrt{3}}{13}$

$=\left(21-9\sqrt{3}\right)$ cm [A1]

2 The acute angles A and B are such that $\tan(A+B)=8$ and $\tan A=\dfrac{1}{5}$. Without using a calculator, find the exact value of $\cos B$. [5]

<u>Solution</u>:

$\tan(A+B)=8$

$\dfrac{\tan A+\tan B}{1-\tan A\tan B}=8$

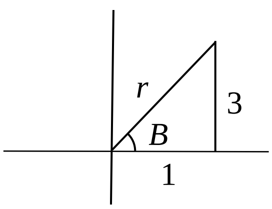
$\dfrac{\frac{1}{5}+\tan B}{1-\frac{1}{5}\tan B}=8$ [M1] – Correct use of double angle formula

$\dfrac{1}{5}+\tan B=8\left(1-\dfrac{1}{5}\tan B\right)$ [M1]

$\dfrac{1}{5}+\tan B=8-\dfrac{8}{5}\tan B$

$\dfrac{13}{5}\tan B=\dfrac{39}{5}$

$\tan B=3$ [M1]



By Pythagoras' Theorem,

$r=\sqrt{1^2+3^2}$ [M1]

$r = \sqrt{10}$

$\cos B = \frac{1}{\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}}$

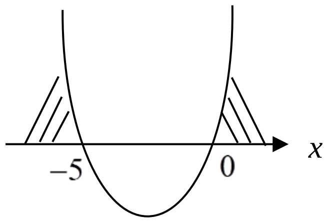
$\cos B = \frac{\sqrt{10}}{10}$ [A1]

3 **(i)** Find the range of values of m for which the curve $y = (4+m)x^2 - 4x + m + 1$ has a maximum point. [1]

Solution:
For curve to have a maximum point,
$4 + m < 0$
$m < -4$ [B1]

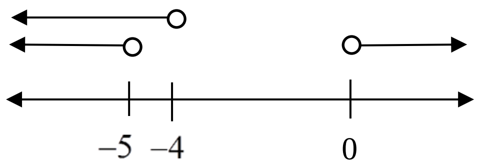
(ii) Find the range of values of m for which the curve $y = (4+m)x^2 - 4x + m + 1$ is always negative for all real values of x. [3]

Solution:
For curve to be always negative,
$b^2 - 4ac < 0$
$(-4)^2 - 4(4+m)(m+1) < 0$ [M1] – Either seen for correct use of discriminant
$16 - 4(m^2 + 5m + 4) < 0$
$16 - 4m^2 - 20m - 16 < 0$
$-4m^2 - 20m < 0$ [M1]
$m^2 + 5m > 0$
$m(m+5) > 0$
$m < -5$ or $m > 0$ [A1]



(iii) Hence state the range of values of m for which the curve has a maximum point and is always negative for all values of x. [1]

Solution:
$m < -5$ [B1]



4 **(a)** It is given that $\int f(x)\,dx = 3x^2 \ln x + kx^2 + c$ where k and c are constants.

(i) If $\int_1^e f(x)\,dx = e^2 + 2$, show that $k = -2$. [3]

Solution:

$\int_1^e f(x)\,dx = e^2 + 2$

$\left[3x^2 \ln x + kx^2 + c\right]_1^e = e^2 + 2$ [M1]

$\left[3e^2 \ln e + ke^2 + c - (3\ln 1 + k + c)\right] = e^2 + 2$

$3e^2 + ke^2 + c - (k + c) = e^2 + 2$

$3e^2 + ke^2 - k = e^2 + 2$ [M1]

$3e^2 - e^2 - 2 = k - ke^2$

$2e^2 - 2 = k - ke^2$

$2(e^2 - 1) = -k(e^2 - 1)$

$\therefore k = -2$ [A1]

(ii) Find $f(x)$. [2]

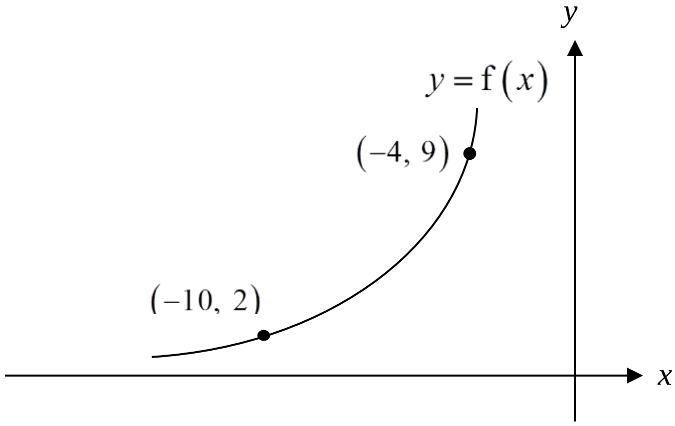
Solution:

$\int f(x)\,dx = 3x^2 \ln x - 2x^2 + c$

$f(x) = 6x \ln x + 3x^2 \left(\frac{1}{x}\right) - 4x$ [M1] – Correct use of product rule

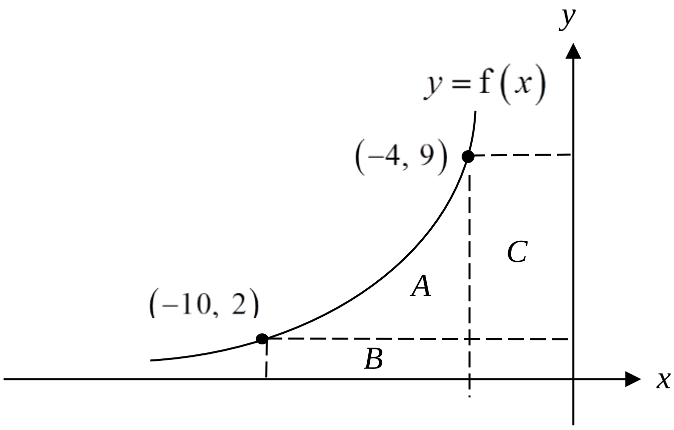
$= 6x \ln x - x$
$= x(6\ln x - 1)$ } [A1] – Either seen

(b) The figure shows part of the curve $y = \mathrm{f}(x)$. $(-10, 2)$ and $(-4, 9)$ are two points on the curve.



Given that $\int_{-10}^{-4} y \, \mathrm{d}x = 30$, find $\int_{2}^{9} x \, \mathrm{d}y$. [2]

Solution:



Area of $A + B = 30$

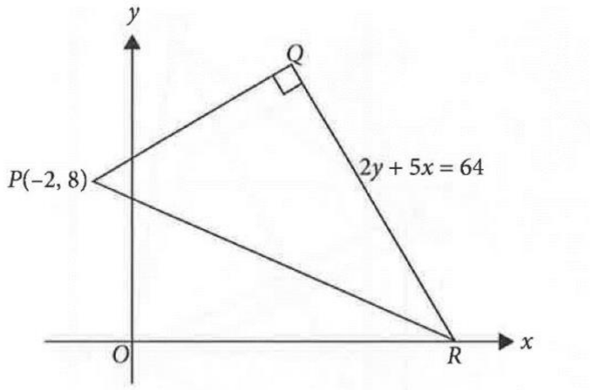
Area of $B = 6(2)$
$= 12$

Area of $A = 30 - 12$ [M1]
$= 18$

Area of $C = 7(4)$
$= 28$

$\int_{2}^{9} x \, \mathrm{d}y = \text{Area of } A + C$
$= 28 + 18$
$= 46$ [A1]

5



The diagram shows a triangle *PQR* in which the point *P* is $(-2, 8)$, the point *R* lies on the *x*-axis and angle *PQR* is $90°$. The equation of *QR* is $2y+5x=64$.

(i) Find the coordinates of *Q*. [5]

Solution:

$2y+5x=64$

$2y=-5x+64$

$y=-\frac{5}{2}x+32$

Gradient of *QR* $=-\frac{5}{2}$

Gradient of *PQ* $=\frac{2}{5}$ [B1]

Equation of *PQ:*

$8=\frac{2}{5}(-2)+c$ [M1] – Attempt to find *c*

$8=\frac{2}{5}(-2)+c$

$c=\frac{44}{5}$

$\therefore$ Equation of *PQ* is $y=\frac{2}{5}x+\frac{44}{5}$. [A1]

$y=-\frac{5}{2}x+32$ ---- (1)

$y=\frac{2}{5}x+\frac{44}{5}$ ---- (2)

(1) = (2)

$-\frac{5}{2}x+32=\frac{2}{5}x+\frac{44}{5}$ [M1]

$-\frac{29}{10}x=\frac{-116}{5}$

$x=8$

Subst. $x = 8$ into (1)

$y = -\frac{5}{2}(8) + 32$

$\quad = 12$

$\therefore \; Q(8, 12)$ [A1]

(ii) Given that *M* is the midpoint of *PR* and that *PQRS* is a rectangle, find the coordinates of *M* and of *S*. [3]

Solution:

When $y = 0$,

$2(0) + 5x = 64$

$x = 12.8$ [M1]

$\therefore \; R(12.8, 0)$

Coordinates of $M = \left(\frac{-2 + 12.8}{2}, \frac{8+0}{2} \right)$

$\quad = (5.4, 4)$ [A1]

Let $S(x, y)$

$\left(\frac{x+8}{2}, \frac{y+12}{2} \right) = (5.4, 4)$

$\frac{x+8}{2} = 5.4$ and $\frac{y+12}{2} = 4$

$x + 8 = 10.8$ $\quad$ $y + 12 = 8$

$x = 2.8$ $\quad$ $y = -4$

$\therefore \; S(2.8, -4)$ [A1]

6 **(a)** Solve the equation $\lg(20+5x)-\lg(10-x)=1$. [3]

Solution:

$\lg(20+5x)-\lg(10-x)=1$

$\lg\left(\dfrac{20+5x}{10-x}\right)=1$ [M1]

$\dfrac{20+5x}{10-x}=10$ [M1]

$20+5x=100-10x$

$15x=80$

$x=5\dfrac{1}{3}$ [A1]

(b) Given that $\log_a p = x$ and $\log_a q = y$, express in terms of x and y,

(i) $\log_{pq} a$, [2]

Solution:

$\log_{pq} a = \dfrac{\log_a a}{\log_a pq}$ [M1]

$= \dfrac{1}{1og_a p + \log_a q}$

$= \dfrac{1}{x+y}$ [A1]

(ii) $\log_p aq$. [3]

Solution:

$\log_p aq = \log_p a + \log_p q$ [M1]

$= \dfrac{\log_a a}{\log_a p} + \dfrac{\log_a q}{\log_a p}$ [M1]

$= \dfrac{1+y}{x}$ [A1]

Or

$\log_p aq = \dfrac{\log_a aq}{\log_a p}$ [M1]

$= \dfrac{\log_a a + \log_a q}{\log_a p}$ [M1]

$= \dfrac{1+y}{x}$ [A1]

7 **(i)** Express $4x^2-12x+5$ in the form of $a(x-b)^2-c$. [2]

Solution:

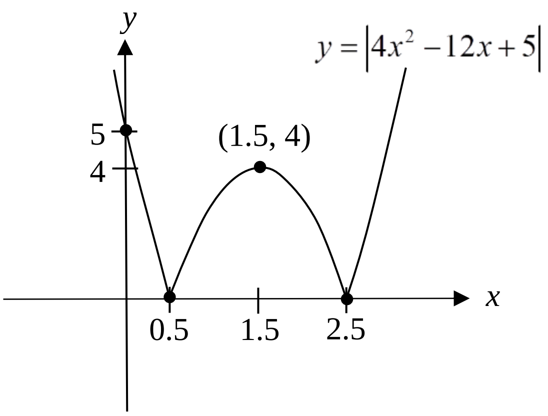
$$4x^2-12x+5=4\left(x^2-3x+\frac{5}{4}\right)$$
$$=4\left[x^2-3x+\left(-\frac{3}{2}\right)^2-\left(-\frac{3}{2}\right)^2+\frac{5}{4}\right]$$
$$=4\left[\left(x-\frac{3}{2}\right)^2-1\right]$$
$$=4\left(x-\frac{3}{2}\right)^2-4$$

[B1] – For $4\left(x-\frac{3}{2}\right)^2$ seen

[B1] – For -4 seen

(ii) Sketch the graph of $y=\left|4x^2-12x+5\right|$, indicating the intercepts of both axes and the coordinates of the turning point. [3]

Solution:



[B1] – Correct shape
[B1] – Turning point at (1.5, 4)
[B1] – Correct intercepts

(iii) Determine the set of values of m for which the equation $\left|4x^2-12x+5\right|=m$ has 4 solutions. [1]

Solution:

$0<m<4$ [B1]

(iv) Solve $\left|4x^2-12x+5\right|=5$. [2]

Solution:

$\left|4x^2-12x+5\right|=5$

$4x^2-12x+5=5$ or $4x^2-12x+5=-5$ [M1]

$4x^2-12x=0$ $\qquad$ $4x^2-12x+10=0$ (N.A)

$4x(x-3)=0$ $\qquad$ $b^2-4ac=(-12)^2-4(4)(10)$

$x=0$ or $x=3$ $\qquad\qquad\qquad\qquad =-16$ $\qquad\qquad$ [A1]

8 A curve is such that $\dfrac{d^2y}{dx^2}=2+\dfrac{16}{x^3}$ and $(2,7)$ is a minimum point on the curve.

(i) Find the equation of the curve. [5]

<u>Solution</u>

$$\frac{d^2y}{dx^2}=2+\frac{16}{x^3}$$

$$\frac{dy}{dx}=\int 2+16x^{-3}\ \mathrm{d}x$$

$$=2x+\frac{16x^{-2}}{-2}+c$$ [M1] – Correct differentiation

$$=2x-\frac{8}{x^2}+c$$

At $(2,7)$, $\dfrac{dy}{dx}=0$

$$2(2)-\frac{8}{(2)^2}+c=0$$ [M1] – Attempt to find c

$$4-2+c=0$$
$$c=-2$$
$$\frac{dy}{dx}=2x-\frac{8}{x^2}-2$$

$$y=\int 2x-8x^{-2}-2\ \mathrm{d}x$$

$$=\frac{2x^2}{2}-\frac{8x^{-1}}{(-1)}-2x+c$$ [M1] – Correct differentiation

$$=x^2+\frac{8}{x}-2x+c$$

At $(2,7)$,

$$7=(2)^2+\frac{8}{2}-2(2)+c$$ [M1] – Attempt to find c

$$7=4+4-4+c$$
$$c=3$$

$$\therefore y=x^2+\frac{8}{x}-2x+3$$ [A1]

(ii) Find the value of x for which the curve has a maximum gradient, and find this maximum gradient. [3]
(You are not required to show that gradient is maximum.)

Solution:
For maximum gradient,

$$\frac{d^2y}{dx^2} = 0$$

$$2 + \frac{16}{x^3} = 0$$

[M1] – Either seen

$$2x^3 + 16 = 0$$

$$2x^3 = -16$$

$$x^3 = -8$$

$$x = -2$$ [A1]

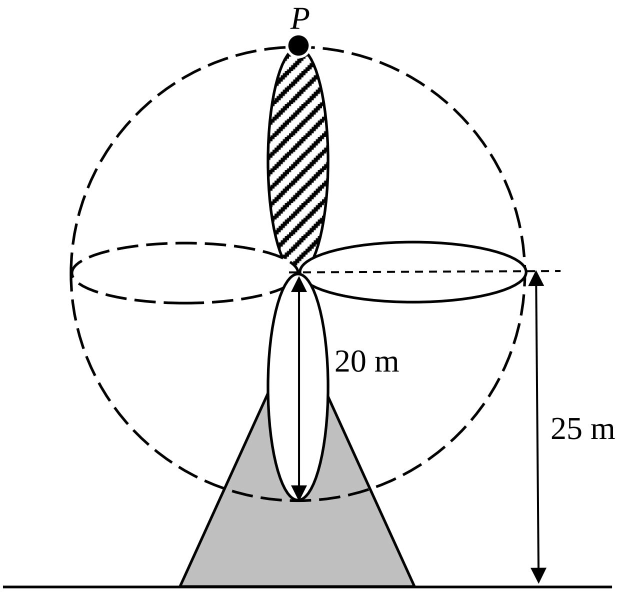
When $x = -2$,

$$\frac{dy}{dx} = 2(-2) - \frac{8}{(-2)^2} - 2$$

$$= -8$$ [A1]

$\therefore$ Maximum gradient $= -8$

9 The diagram shows a windmill with blades 20 m in length. The centre of their circular motion is a point 25 m above the ground. One of the blades has been painted with stripes and the tip of the stripped blade is currently at point P, 45 m above the ground. When in operation, the windmill takes 6 seconds to complete one revolution.



(i) The height above the ground, y m, of the tip of the stripped blade is modelled by the equation $y = a\cos kt + b$, where t in the time in seconds after leaving point P. Find the value of a and of b. [2]

Solution:

$$\text{Amplitude} = \frac{45-5}{2}$$
$$= 20$$

$\therefore\ a = 20$ [B1]

$b = 25$ [B1]

(ii) Show that the value of k is $\frac{\pi}{3}$ radians per second. [1]

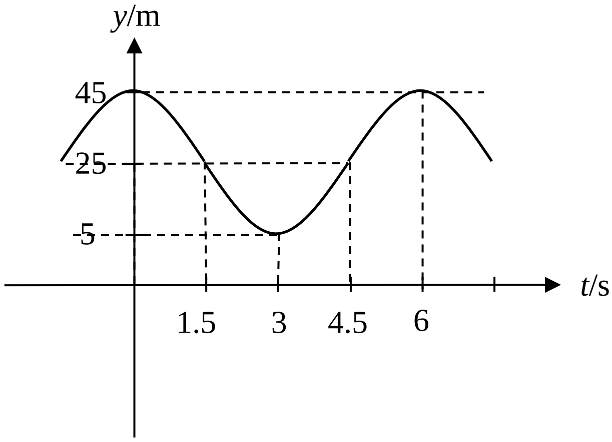
Solution:

$$\frac{2\pi}{k} = 6$$

$k = \frac{\pi}{3}$ (shown) [B1]

(iii) Sketch the graph of $y = a\cos kt + b$ for $0 \le t \le 6$. [2]

Solution:



[B1] – 1 cosine

[B1] – Correct amplitude and positioned correctly

(iv) Find the length of time for which the tip of the stripped blade is at most 15 m above the ground. [3]

Solution:

$20\cos\frac{\pi}{3}t + 25 = 15$ [M1]

$20\cos\frac{\pi}{3}t = -10$

$\cos\frac{\pi}{3}t = -0.5$

Basic angle $= \cos^{-1}(0.5)$

$= \frac{\pi}{3}$ [M1]

$\frac{\pi}{3}t = \pi - \frac{\pi}{3}, \quad \pi + \frac{\pi}{3}$

$t = 2, \quad 4$

$\therefore$ Length of time $= 2\text{ s}$ [A1]

10 **(i)** Express $\dfrac{x+1}{x(2x-1)^2}$ in partial fractions. [5]

Solution:

$$\frac{x+1}{x(2x-1)^2}=\frac{A}{x}+\frac{B}{2x-1}+\frac{C}{(2x-1)^2}$$

$$x+1=A(2x-1)^2+Bx(2x-1)+Cx$$

[M1] – Either seen

Let $x=0$

$A=1$ [M1] – For evaluating A and C

Let $x=\frac{1}{2}$

$\frac{3}{2}=\frac{1}{2}C$

$C=3$

Let $x=1$

$2=1(1)^2+B(1)+3$ [M1] – For evaluating B

$B+4=2$

$B=-2$

$$\therefore \frac{x+1}{x(2x-1)^2}=\frac{1}{x}-\frac{2}{2x-1}+\frac{3}{(2x-1)^2}$$

[A1] – At least 1 term correct

[A1] – Remaining 2 terms correct

(ii) Hence find $\int \dfrac{x+1}{x(2x-1)^2}\,\mathrm{d}x$. [4]

Solution:

$$\int \frac{x+1}{x(2x-1)^2}\,\mathrm{d}x=\int \frac{1}{x}-\frac{2}{2x-1}+\frac{3}{(2x-1)^2}\,\mathrm{d}x$$ [M1]

$$=\int \frac{1}{x}-\frac{2}{2x-1}+3(2x-1)^{-2}\,\mathrm{d}x$$

$$=\ln x-\ln(2x-1)+\frac{3(2x-1)^{-1}}{(-1)(2)}+c$$

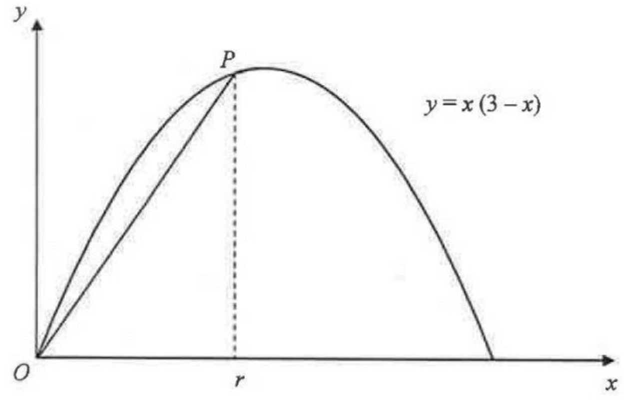
$$=\ln x-\ln(2x-1)-\frac{3}{2(2x-1)}+c$$

$$=\ln\frac{x}{2x-1}-\frac{3}{2(2x-1)}+c$$

[A3] – Either seen

Minus 1 for each error including + c

11



The diagram shows the trajectory of an athlete during a long jump which can be represented by the equation $y = x(3-x)$ where *x* and *y* are the horizontal distance and vertical height of the jump respectively. *O* is the point where the athlete takes-off from the ground. When $x = r$ m, the athlete is at point *P*.

(i) Show that the distance, *s* m, between *O* and *P* is given by $s = \sqrt{r^4 - 6r^3 + 10r^2}$. [3]

Solution:
When $x = r$,
$y = r(3-r)$ [B1]

$s^2 = r^2 + \left[r(3-r)\right]^2$ [M1] – Use of Pythagoras' Theorem
$s^2 = r^2 + \left(3r - r^2\right)^2$
$s^2 = r^2 + 9r^2 - 6r^3 + r^4$
$s^2 = 10r^2 - 6r^3 + r^4$
$s = \sqrt{r^4 - 6r^3 + 10r^2}$ (shown) [A1]

(ii) Show that $\dfrac{ds}{dr} = \dfrac{r\left(2r^2 - 9r + 10\right)}{\sqrt{r^4 - 6r^3 + 10r^2}}$. [2]

Solution:
$s = \sqrt{r^4 - 6r^3 + 10r^2}$

$s = \left(r^4 - 6r^3 + 10r^2\right)^{\frac{1}{2}}$

$\dfrac{ds}{dr} = \dfrac{1}{2}\left(r^4 - 6r^3 + 10r^2\right)^{-\frac{1}{2}}\left(4r^3 - 18r^2 + 20r\right)$ [M1]

$= \left(r^4 - 6r^3 + 10r^2\right)^{-\frac{1}{2}}\left(2r^3 - 9r^2 + 10r\right)$

$= \dfrac{r\left(2r^2 - 9r + 10\right)}{\sqrt{r^4 - 6r^3 + 10r^2}}$ [A1]

(iii) Given that r can vary, find the values of r for which s is stationary. [3]

Solution:

For stationary value of s, $\dfrac{ds}{dr} = 0$.

$\dfrac{r\left(2r^2 - 9r + 10\right)}{\sqrt{r^4 - 6r^3 + 10r^2}} = 0$ [M1] – Knowledge of stationary value

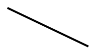
$r\left(2r^2 - 9r + 10\right) = 0$

$r(2r - 5)(r - 2) = 0$

$r = 0$ (N.A) or $r = 2$ or $r = 2.5$ [A2] – Minus 1 for each error

(iv) By using the first derivative test, determine whether the smaller of these values of r will give a maximum or a minimum value of s. [2]

Solution:

r	< 2	2	> 2
$\dfrac{ds}{dr}$	> 0	$= 0$	< 0
Sketch of tangent			
Outline of graph			

[M1]

$\therefore$ s is maximum when $r = 2$. [A1]

End of Paper