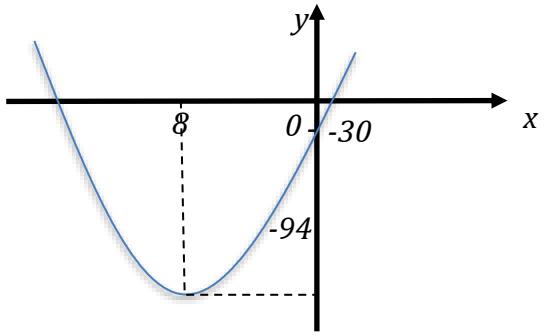
Fairfield Methodist School (Secondary)
Secondary 4 Express / 5 Normal (Academic)
Mathematics Paper 1
Marking Scheme
Preliminary Examinations 2023

Qn No.	Workings	Description	Mark Allocation	AO
1(a)	$540 = 2^2 \times 3^3 \times 5$		B1	AO1
1(b)	$m = 2$ $n = 5$		 B1	AO1
2(a)	901.16		B1	AO1
2(b)	9.0116×10^2 or 9.01×10^2 (3 s.f.)		B1	AO1
3(a)	37		B1	AO1
3(b)	$9 + 4n$		B1	AO1
3(c)	Let $318 = 9 + 4n$ $4n = 309$ $n = 77\frac{1}{4}$ Since n is not a positive integer, 318 is not a term of the sequence.		 B1	AO3
4(a)	$72 + 2x + 75 + 69 + x = 360$ $3x + 216 = 360$ $3x = 144$ $x = 48$		 B1	AO2
4(b)	No. of adults $= \frac{75}{360} \times 240$ $= 50$		 B1	AO1
5	y-intercept = $5(\frac{2}{5}) = 2$ Gradient of line graph $= -\frac{vert}{horizontal}$ $= -\frac{2}{\frac{2}{5}}$ $= -5$ Equation of line is y = - 5x + 2		M1 M1 A1	AO2

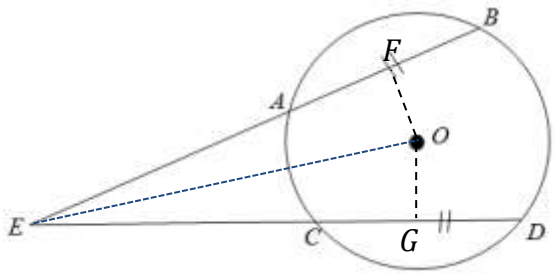
Qn No.	Workings	Description	Mark Allocation	AO
6	Distance travelled by fuel in 24 mins $= 2\times24\times60=2880\text{m}$ Capacity of fuel tank $=2880\times\pi(\frac{8/2}{100})^2$ $=14.476$ $=14.5m^3(3sf)$ $=145000l(3sf)$		M1 M1 A1	AO2
7a	$\left(81x^4\right)^{-\frac{3}{4}}$ $=\frac{1}{81^{\frac{3}{4}}}(x)^{-3}$ $=\frac{1}{(3^4)^{\frac{3}{4}}}\times\frac{1}{x^3}$ $=\frac{1}{3^3}\times\frac{1}{x^3}$ $=\frac{1}{27x^3}$	Apply $(a^m)^n=a^{mn}$	M1 A1	AO1
7b	$32^{\frac{1}{5}}\times2^x=8^{\frac{1}{4}}$ $(2^5)^{\frac{1}{5}}\times2^x=(2^3)^{\frac{1}{4}}$ $2\times2^x=2^{\frac{3}{4}}$ $2^{1+x}=2^{\frac{3}{4}}$ $1+x=\frac{3}{4}$ $x=\frac{3}{4}-1$ $=-\frac{1}{4}$	Apply $(a^m)^n=a^{mn}$	M1 A1	AO1
8(a)(i)	{3, 6, 9, 12, 15, 18, 21}	No { } will result to no marks awarded	B1	AO1
8(a)(ii)	{4, 8, 10, 14, 16, 20, 22}		B1	AO1
8(b)(i)	$A'\cap(B\cap C)=\varnothing$ $5\in A'\cap C$	B1 for each correct answer	B1 B1	AO1
8(b)(ii)	n[$B'\cap(A\cup C)$] = 5		B1	AO1

Qn No.	Workings	Description	Mark Allocation	AO
9	$\frac{4m^2-20mn+16n^2}{3m-12n}$ $=\frac{4(m^2-5mn+4n^2)}{3(m-4n)}$ $=\frac{4(m-4n)(m-n)}{3(m-4n)}$ $=\frac{4(m-n)}{3}$	M1 for factorising out either common factor correctly M1 for factorising $m^2 - 5mn + 4n^2$ correctly	M1 M1 A1	AO1
10	Let Ching's and Lex's original savings be $\$3u$ and $\$5u$ respectively. $\frac{3u-60}{5u-60}=\frac{4}{7}$ $7(3u-60)=4(5u-60)$ $21u-420=20u-240$ $21u-20u=420-240$ $u=180$ $\therefore$ Total amount of money now $=180(3+5)-2(60)$ $=\$1320$	M1 for converting to linear equation correctly M1 for expanding either side correctly M1 for simplifying either algebraic terms or constants correctly	M1 M1 M1 A1	AO2
11a	$\cos x=-\cos 65°$ $x=180°-65°=115°$		B1	AO1
11b	Area of $\triangle PQR=\frac{1}{2}(PQ)(PR)\sin\angle QPR$ $15=\frac{1}{2}(10)(6)\sin\angle QPR$ $15=30\sin\angle QPR$ $\sin\angle QPR=\frac{15}{30}$ $\angle QPR=\sin^{-1}(\frac{15}{30})$ $=30°$ or $150°$		M1 M1 A1 for both ans	AO1
12a	A^2 $=\begin{pmatrix}4 & 6\\ 0 & -2\end{pmatrix}\begin{pmatrix}4 & 6\\ 0 & -2\end{pmatrix}$ $=\begin{pmatrix}16 & 12\\ 0 & 4\end{pmatrix}$		B1	AO1

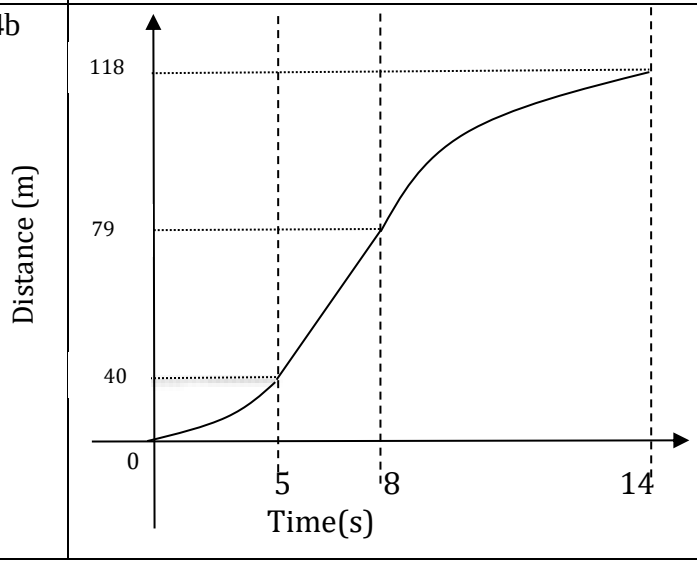
Qn No.	Workings	Description	Mark Allocation	AO
12b	$\mathbf{A} = 2\mathbf{B}$ $\begin{pmatrix} 4 & 6 \\ 0 & -2 \end{pmatrix} = 2\begin{pmatrix} 2 & k \\ 0 & -1 \end{pmatrix}$ $\begin{pmatrix} 4 & 6 \\ 0 & -2 \end{pmatrix} = \begin{pmatrix} 4 & 2k \\ 0 & -2 \end{pmatrix}$ $2k = 6$ $k = 3$		B1	AO1
13a	$S = \begin{pmatrix} 34000 & 20100 \\ 14500 & 30000 \end{pmatrix}$		B1	AO1
13b	$\begin{pmatrix} 1 & 1 \end{pmatrix}\begin{pmatrix} 34000 & 20100 \\ 14500 & 30000 \end{pmatrix}$ $= \begin{pmatrix} 48500 & 50100 \end{pmatrix}$		B1	AO1
13c	The total/combined sales in 2021 and 2022 (or for the 2 years) for IMic and Lenovo Laptops respectively	• Keywords: Total/Combined , respectively, • 2021 and 2022 or past 2 years	B1	AO2
14a	*Angle x* = 180° – 112°(int. angles, AB//CD) = 68°	Deduct 1 mark – wrong or missing reasons	B1	AO1
14b	*Angle z* = 85° (corr. angles, AB//CD) *Angle y* = 180° – 85°(adj. on a str. line) = 95°		M1 A1-	AO1
15a	$y = x^2 + 3$		B1	AO1
15b	$y = x^3 + 3$		B1	AO1
15c	$y = 3^x + 3$		B1	AO1
15d	$y = 3x^{-2}$		B1	AO1
16a	Refer to last page			AO1
16b	Refer to last page			AO1
16c	Refer to last page			AO2

Qn No.	Workings	Description	Mark Allocation	AO
17(a)	$3ax + 16by - 12ay - 4bx$ $= 3ax - 12ay - 4bx + 16by$ $= 3a(x - 4y) - 4b(x - 4y)$ $= (3a - 4b)(x - 4y)$ or $(4b - 3a)(4y - x)$	M1 for factorising any 2 terms correctly	M1 A1	AO1
17(b)	$3mn - 243mn^5$ $= 3mn(1 - 81n^4)$ $= 3mn(1 + 9n^2)(1 - 9n^2)$ $= 3mn(1 + 9n^2)(1 + 3n)(1 - 3n)$	M1 for factorising out $3mn$ M1 for applying difference of squares to $1 - 81n^4$	M1 M1 A1	AO1
18a	$x^2 + 16x - 30$ $= x^2 + 16x + (\frac{16}{2})^2 - (\frac{16}{2})^2 - 30$ $= (x+8)^2 - 94$		B1	AO1
18b	$x^2 + 16x - 30 = 0$ $(x+8)^2 - 94 = 0$ $(x+8)^2 = 94$ $x + 8 = \pm\sqrt{94}$ $x = \sqrt{94} - 8$ or $-\sqrt{94} - 8$ $x = 1.70$(2d.p.) or -17.70(2d.p.)		M1 A1 for both ans	AO1
18c	 Turning Point = (-8, -94) When $x = 0$, $y = (0 + 8)^2 - 94 = -30$	Correct Shape Correct y-intercept and turning point	C1 P1	AO1

Qn No.	Workings	Description	Mark Allocation	AO
19a	Size of 1 int angle from octagon $=\frac{(8-2)\times 180°}{8}=135°$ Size of 1 int angle from Hexagon $=\frac{(6-2)\times 180°}{6}=120°$ $\angle a=360°-135°-120°$(sum of $\angle$s at a pt.) $=105°$		M1 M1 A1	AO1
19b	$\frac{(n-2)\times 180°}{n}=105°$ $n-2=\frac{105°n}{180°}$ $n-\frac{105°n}{180°}=2$ $0.41667n=2$ $n=4.8$ Since n is not a positive integer, that polygon does not exist.		M1 M1 A1	AO3
20a	% of student with revision > 12 hours $=\frac{10+12}{41}\times 100\%=53.7\%(3s.f)$		B1	AO1
20b	Median position = 21st position Median = 12 h – 16hr or 12 hr $\leq$ time $\leq$ 16 hr		B1	AO1
20ci	Est Mean $=\frac{5(2)+6(6)+8(10)+10(14)+12(18)}{41}$ $=11.756$h $=11.8$h		B1	AO1
20cii	Std Deviation = 5.4495h = 5.45 h		B1	AO1

Qn No.	Workings	Description	Mark Allocation	AO
21a	 Since AB=CD, OF = OE(equal chords) ---- S AF = FB = CG = GD (⊥ bisector of chord) $\angle AFO = \angle CGO = 90°$ ---------- R EO is shared side of $\triangle OFE$ and $\triangle OGE$ ----H By RHS Congruency Test, $\triangle OFE \equiv \triangle OGE$ As EF = CE, EA = EF – AF EC = EG – CG Since EF = CE and AF = CG, EA = EC (proven)		M2 for all Evidences M1 for 2 evidences M1 for RHS Test AG1	AO3
21a	Alternative Solution: $EA \times EB = EC \times ED$ (Intersecting Secant Theorem) $EA(EA + AB) = EC(EC + CD)$ $\frac{EA}{EC} = \frac{EA+AB}{EC+CD}$ $\frac{EA}{EC} = \frac{EA+AB}{EC+AB}$, given that $CD = AB$ $\therefore EA = EC$ (proven)	State the theorem	M1 M1 M1 AG1	
21b	$\angle BEO = \angle DEO = \frac{30°}{2}$ (OE is ∠bisector of $\angle BED$) $= 15°$ $AF = \frac{40}{2} = 20cm$ $\tan \angle OEF = \frac{OF}{EF} = \frac{15}{EF}$ $\tan 15° = \frac{15}{EF}$ $EF = \frac{15}{\tan 15°} = 55.981cm$ $AE = EF - 20$ $= 55.981 - 20$ $= 36.0$cm (3 s.f)		M1 M1 A1	AO2

Qn No.	Workings	Description	Mark Allocation	AO
21b	Alternative solution: $\angle EBD = \frac{180° - 30°}{2}$ $= 75°$(base $\angle$s of an isos. Δ,EB=ED) $\angle ABO = \cos^{-1}\left(\frac{40^2 + 25^2 - 25^2}{2(40)(25)}\right)$ $= 36.870°(5sf)$ $\angle OBD = 75° - 36.870° = 38.13°$ $BD = \frac{25}{\sin 38.13°} \times \sin(180° - (38.13° \times 2))$ $= 39.331cm(3sf)$ $EB = \frac{39.331°}{\sin 30°} \times \sin 75°$ $= 75.982cm(3sf)$ $AE = 75.982 - 40$ $= 36.0cm(3sf)$		M1 M1 A1	
22	Perimeter of major sector = 3 × Perimeter of minor sector $x(2\pi - \theta) + 2x = 3(x\theta + 2x)$ $2x\pi - x\theta + 2x = 3x\theta + 6x$ $x(2\pi - \theta + 2) = x(3\theta + 6)$ $4\theta = 2\pi - 6 + 2$ $4\theta = 2\pi - 4$ $\theta = \frac{\pi - 2}{2} = 0.571radians(3dp)$		M1 for each correct perimeter M1 for making θ as the subject A1	AO2
23a	Volume of cone $= \frac{1}{3}$(Base area)($\perp$ height) $259.44 = \frac{1}{3}\pi(5.6)^2(\perp \text{height})$ $\perp \text{height} = \frac{259.44 \times 3}{(5.6)^2\pi}$ $= 7.9001cm$ By Pythagoras' Theorem, Slant height of cylinder $\sqrt{7.9001^2 + 5.6^2}$ $= 9.68357cm$ = radius of sector OAB Arc length AB = $2\pi(5.6)$		M1 M1	AO2

Qn No.	Workings	Description	Mark Allocation	AO
	$9.68357(x) = 2\pi(5.6)$ $x = \dfrac{2\pi(5.6)}{9.68357}$ $= 3.63$ radian $= 208.2°(1dp)$		A1	
23b	Volume of hemisphere $= \dfrac{1}{2}\left(\dfrac{4}{3}\pi r^3\right)$ $= \dfrac{2}{3}\pi(5.6)^3$ $= 367.809$ Total volume of ornament = 367.809 + 259.44 = 627.249 = 627 cm^3(3*sf*)		M1 M1 A1	AO1
24a	$2 \times 5 = 10$ Hence *v* = 3 + 10 = 13 m/s		B1	AO1
24b			B1 for 2 parts drawn correctly. B2 for all 3 parts drawn correctly with correct distance	AO2

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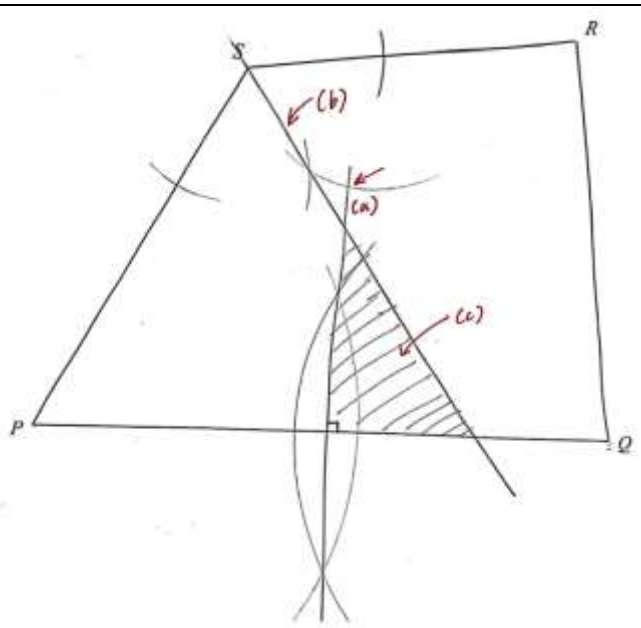
16		Minus 1 mark for missing construction lines, incomplete perpendicular bisector.	B1 for part (a), (b) and (c) respectively	