Candidate Name: ________________________ Class: _____________

JC2 PRELIMINARY EXAM
Higher 2

MATHEMATICS
Paper 2

9740/02
21 Sept 2015
3 hours

Additional Materials: Cover page
Answer papers
List of Formulae (MF15)

READ THESE INSTRUCTIONS FIRST

Write your full name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

Section A : Pure Mathematics [40 marks]

1 The function f is defined by

$$\mathrm{f} : x \mapsto \lambda x - x^2, \quad x \in \mathbb{R}, \quad x > \frac{\lambda}{2},$$

where $\lambda > 2$.

(i) Find, in terms of λ, $\mathrm{f}^{-1}(x)$, stating the domain of f^{-1}. [4]

(ii) On the same diagram, sketch the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$, showing their graphical relationship clearly. [2]

(iii) Find, in terms of λ, the solution of the equation $\mathrm{f}(x) = \mathrm{f}^{-1}(x)$. [3]

2 By sketching the graphs of $y = \dfrac{1}{x^3}$ and $y = \sqrt{x}$, solve the inequality

$$\frac{1}{x^3} > \sqrt{x}\,. \qquad [2]$$

Hence, without using a calculator, evaluate

$$\int_{\frac{1}{4}}^{4} \left| \frac{1}{x^3} - \sqrt{x} \right| \mathrm{d}x\,. \qquad [4]$$

The area bounded by the curves $y = \dfrac{1}{x^3}$, $y = \sqrt{x}$, the line $x = 2$ and the x-axis is rotated completely about the y-axis to form a solid of revolution of volume V. Find the numerical value of V, giving your answer correct to 3 decimal places. [3]

3 Sketch, on a single Argand diagram, the set of points representing all complex numbers satisfying both of the following inequalities:

$$\left|z-\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)\right| \le \frac{\sqrt{3}}{2} \quad \text{and} \quad -\frac{\pi}{3} \le \arg(z-2\mathrm{i}) \le 0 .$$ [4]

Hence find

(i) the minimum value of $|\mathrm{i}-2z|$, [3]

(ii) the exact value of the complex number z such that $\arg(z)$ is minimum. [3]

4 The parametric equations of a curve are

$$x=t^2, \quad y=t^2-t .$$

(i) The point P on the curve has parameter p. Show that the equation of the tangent at P is $2py=(2p-1)x-p^2$. [3]

(ii) The tangent at P meets the x- and y- axes at the points Q and R respectively. Find the coordinates of Q and R. [2]

(iii) Find a cartesian equation of the locus of the midpoint of QR as p varies. [3]

(iv) Find the equation of the tangent at the point $(4,6)$ and determine if this tangent meets the curve again. [4]

Section B : Statistics [60 marks]

5 There are 8 red cards, and a single letter is printed on each of them. Together they can be arranged to form the words “GOOD LUCK”. The digits 1 to 9 are printed on 9 white cards, with each card having a single digit. A code is formed by laying out 4 red cards and 4 white cards in a row. Find the number of codes that can be formed if the red and white cards must alternate. [5]

6 A popular brand of titbits is holding a "Win as You Eat" promotion as its marketing strategy. 2% of all the standard size packets produced during the promotion period contain prize winning coupons, with each packet containing at most 1 prize winning coupon. The titbits are sold as a family pack, each containing 10 randomly chosen standard size packets.

(i) Find the probability that a randomly chosen family pack contains no winning coupon. [1]

(ii) The family packs are delivered to supermarkets in cartons. Each carton contains 20 family packs. Find the probability that a randomly chosen carton contains more than 16 family packs with no winning coupon. [2]

(iii) A particular family buys one family pack every week, for 5 consecutive weeks. Find the probability that the fifth week is the second week that the family does not get any winning coupon. [3]

(iv) An event organiser stocks up 1000 standard size packets for an event. The probability of having at least k winning coupons is more than 0.15. Using a suitable approximation, find the largest possible value of k. [4]

7 The duration of a patient's consultation, in minutes, with a general practitioner (GP) and a specialist are modelled as having independent normal distributions with mean and standard deviation as given in the table.

	Mean	Standard Deviation
Consultation with GP	6.2	1.9
Consultation with specialist	10.7	2.8

(i) Find the probability that the total duration of 3 patients' consultation with the GP is shorter than twice the duration of a patient's consultation with the specialist. [3]

The consultation fee charged by the GP is made up of 2 components: a fixed component of \$10 and a variable component of \$1 per minute. Similarly, the consultation fee charged by the specialist has a fixed component of \$25 and a variable component of \$2 per minute.

(ii) Find the probability that the consultation fee of a patient visiting the specialist is at most 3 times that of a patient visiting the GP. [4]

8 A beverage company claims that the vitamin C content of orange juice produced by the company is the same as that in freshly squeezed orange juice. A random sample of 90 packets of orange juice produced by the company is taken and the vitamin C content, *x* mg per 100 ml, is measured. The results are summarised by

$$\sum x = 8993, \ \sum x^2 = 900240.$$

It is known that the vitamin C content of freshly squeezed orange juice has a mean of 101 mg per 100 ml.

Test, at the 5% significance level, whether the company's claim is valid. [5]

Another random sample of 10 packets of orange juice produced by the company is taken. Assuming that the standard deviation of vitamin C content of orange juice produced by the company is now known to be 4 mg per 100ml, find the set of values within which the mean mass of this sample must lie for the company's claim to be valid at the 5% significance level. Give your answer to 2 decimal places, and state any necessary assumption for your calculations to be valid. [3]

9 On average, a travel agency receives 1.2 complaints daily.

(i) State, in this context, two conditions that must be met for the number of complaints to be well modelled by a Poisson distribution. Explain why one of your conditions may not be met. [3]

For the remainder of this question assume that these conditions are met.

(ii) Find the probability that, in a period of 10 days, the total number of complaints received is below the expected value. [2]

(iii) Find the probability that there is at least 1 complaint received daily for 2 consecutive days. [2]

(iv) Find the least number of consecutive days for which the probability of at least 1 complaint received exceeds 0.999. [4]

(v) Find the probability that the average number of complaint received per day over a period of one year (365 days) is less than 1.3. [2]

10 The age in months (m) and average head circumference from the time of birth measured in centimetres (h) of a random sample of female babies in a certain country are given in the table below.

m	1	2	3	4	5	6	7	8	9	10
h	36.1	37.9	39.2	40.6	41.5	42.4	43.1	43.9	44.5	45.1

(i) Find the value of the product moment correlation coefficient, and explain why its value does not necessarily mean that the best model for the relationship between m and h is $h = a + bm$, where a and b are constants. [2]

It is thought that the average head circumference of female babies after m months could also be modelled by one of the formulae

$$h^2 = c + dm \qquad \text{or} \qquad \ln h = e + fm$$

where c, d, e and f are constants.

(ii) By calculating the product moment correlation coefficients, determine whether $h^2 = c + dm$ or $\ln h = e + fm$ is a better model than $h = a + bm$. [2]

(iii) It is required to estimate the value of m for which $h = 39.9$. Find the equation of a suitable regression line and use it to find the required estimate. Explain why the regression line of m on h, the regression line of m on h^2 and the regression line of m on $\ln h$ should not be used to find the required estimate. [4]

11 For events A and B, it is given that $\mathrm{P}(B) = \frac{7}{27}$, $\mathrm{P}(B \mid A) = \frac{2}{9}$ and $\mathrm{P}(A \mid B') = \frac{7}{10}$.

(i) State, with reason, whether A and B are independent. [1]

(ii) Find $\mathrm{P}(A \cup B)$. [3]

(iii) Find $\mathrm{P}(A \cap B)$. [3]

For a third event C, it is given that $\mathrm{P}(B \cap C) = \frac{1}{18}$ and that A and C are mutually exclusive.

(iv) Find the range of $\mathrm{P}(C)$. [2]

Pioneer Junior College
H2 Mathematics **JC2-2015**
Prelim Exam Paper 2 (Solution)

Q1

(i)

Let $f(x) = y$

$\lambda x - x^2 = y$ **or** $x^2 - \lambda x + y = 0$

$\frac{\lambda^2}{4} - \left(x - \frac{\lambda}{2}\right)^2 = y$ $x = \frac{\lambda \pm \sqrt{\lambda^2 - 4y}}{2}$

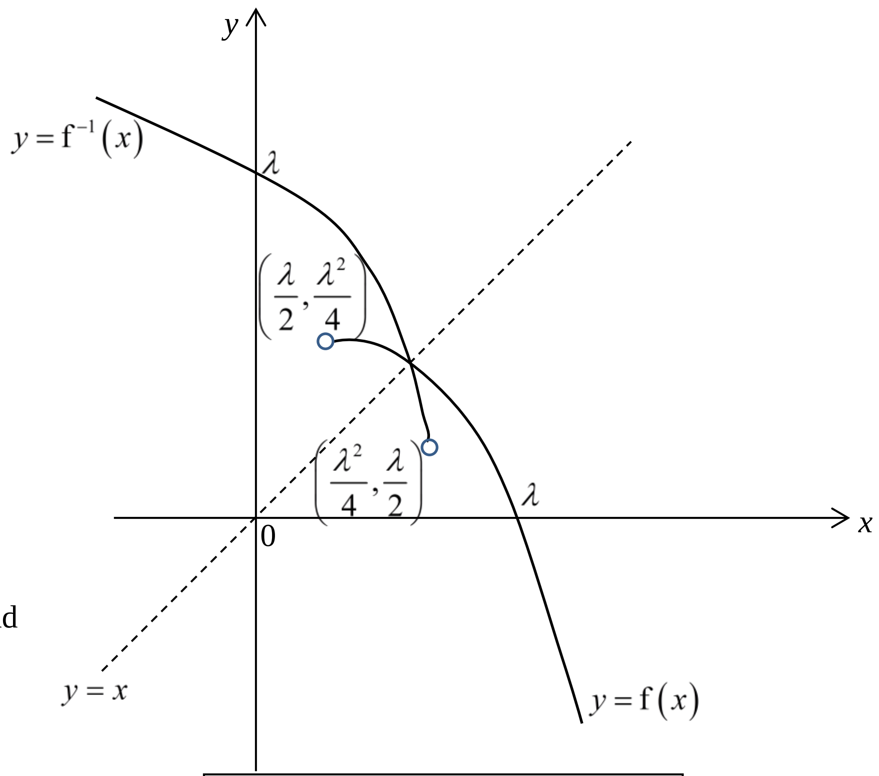
$\left(x - \frac{\lambda}{2}\right)^2 = \frac{\lambda^2}{4} - y$

$x = \frac{\lambda}{2} \pm \sqrt{\frac{\lambda^2}{4} - y}$

Since $x > \frac{\lambda}{2}$, $x = \frac{\lambda}{2} + \sqrt{\frac{\lambda^2}{4} - y}$

Therefore, $f^{-1}(x) = \frac{\lambda}{2} + \frac{\sqrt{\lambda^2 - 4x}}{2}$, $x < \frac{\lambda^2}{4}$

(ii)



(iii)

$f(x) = f^{-1}(x)$

Solve $f(x) = x$ instead

$\lambda x - x^2 = x$

$x^2 - \lambda x + x = 0$

$x^2 - (\lambda - 1)x = 0$

$x(x - (\lambda - 1)) = 0$

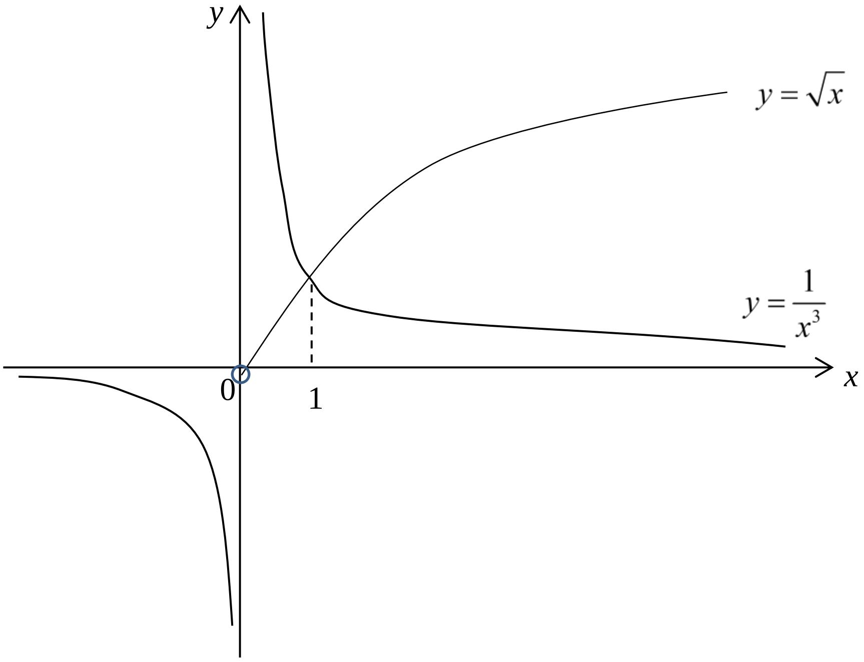
$x = 0$ or $x = \lambda - 1$

Since $x > \frac{\lambda}{2}$, $x = \lambda - 1$

Note :

The intersection between $y = f(x)$ and $y = f^{-1}(x)$ is the same as the intersection between $y = f(x)$ and $y = x$

Q2



From graph, the solution for $\frac{1}{x^3} > \sqrt{x}$ is $0 < x < 1$

Note :

For questions involving $\int \left| f(x) \right| \mathrm{d}x$, consider the original graph $y = f(x)$, i.e without modulus.

$$\int_{\frac{1}{4}}^{4} \left| \frac{1}{x^3} - \sqrt{x} \right| \mathrm{d}x$$

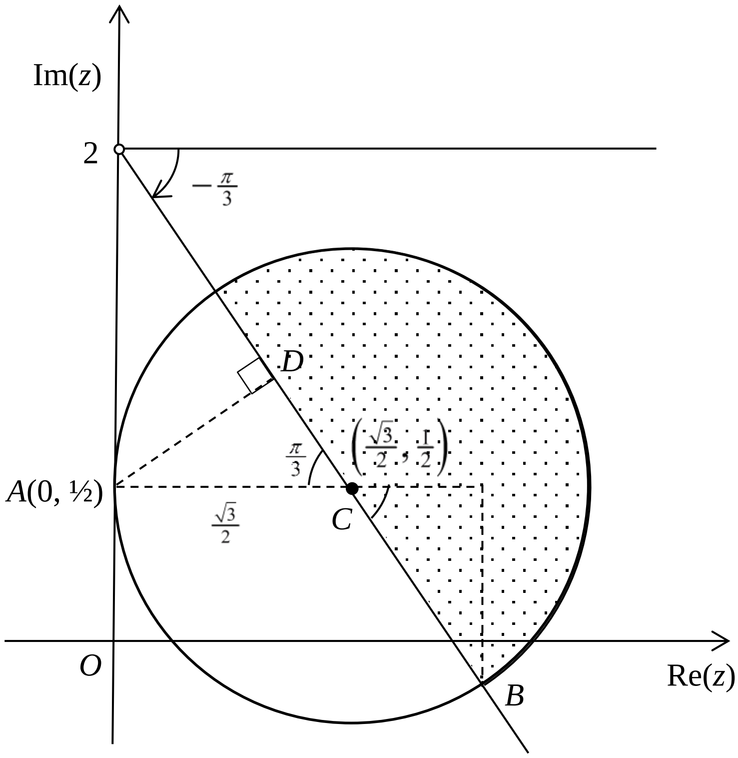
$$= \int_{\frac{1}{4}}^{1} \frac{1}{x^3} - \sqrt{x} \ \mathrm{d}x - \int_{1}^{4} \frac{1}{x^3} - \sqrt{x} \ \mathrm{d}x$$

$$= \left[-\frac{1}{2x^2} - \frac{2}{3}x^{\frac{3}{2}} \right]_{\frac{1}{4}}^{1} - \left[-\frac{1}{2x^2} - \frac{2}{3}x^{\frac{3}{2}} \right]_{1}^{4}$$

$$= \frac{1067}{96}$$

$$V = \pi(2)^2\left(\frac{1}{8}\right) + \pi\int_{\frac{1}{8}}^{1}\left(\frac{1}{y}\right)^{\frac{2}{3}} \ \mathrm{d}y - \pi\int_{0}^{1} y^4 \ \mathrm{d}y = 5.655$$

Q3



(i)

$|\mathrm{i}-2z| = \left|(-2)(z-\frac{1}{2}\mathrm{i})\right| = 2\left|z-\frac{1}{2}\mathrm{i}\right|$

Min $\left|z-\frac{1}{2}\mathrm{i}\right| = AD = \frac{\sqrt{3}}{2}\sin\frac{\pi}{3} = \frac{3}{4}$

Hence min value of $|\mathrm{i}-2z|$ is $\frac{3}{2}$

(ii)

Let z_B be the complex number such that arg (z) is least.

$$z_B = \left(\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}\cos\frac{\pi}{3}\right)+\left(\frac{1}{2}-\frac{\sqrt{3}}{2}\sin\frac{\pi}{3}\right)\mathrm{i} = \frac{3\sqrt{3}}{4}-\frac{1}{4}\mathrm{i}$$

Q4

(i)

$x = t^2 \qquad\qquad y = t^2 - t$

$\dfrac{\mathrm{d}x}{\mathrm{d}t} = 2t \qquad\qquad \dfrac{\mathrm{d}y}{\mathrm{d}t} = 2t - 1$

$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{2t-1}{2t}$

At the point P, $t = p$

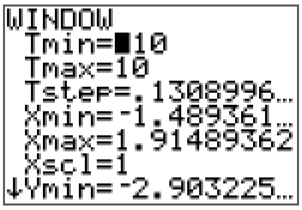
$$y - \left(p^2 - p\right) = \frac{2p-1}{2p}\left(x - p^2\right)$$

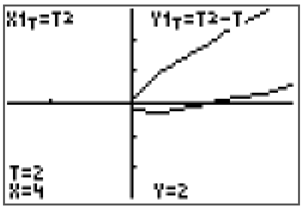
$$2py - \left(2p^3 - 2p^2\right) = (2p-1)\left(x - p^2\right)$$

$$2py - 2p^3 + 2p^2 = (2p-1)x - 2p^3 + p^2$$

$$2py = (2p-1)x - p^2$$

Note :
It is good to sketch the graph for visualising. Remember to include negative t values in the window settings.





(ii)

$$2py = (2p-1)x - p^2$$

At Q, $y = 0$

$$0 = (2p-1)x - p^2$$

$$x = \frac{p^2}{2p-1}$$

At R, $x = 0$

$$2py = (2p-1)x - p^2$$

$$y = -\frac{p}{2}$$

Coordinates of Q and R are $\left(\dfrac{p^2}{2p-1}, 0\right)$ and $\left(0, -\dfrac{p}{2}\right)$

(iii)

Coordinates of Midpoint of QR are $\left(\dfrac{p^2}{4p-2}, -\dfrac{p}{4}\right)$

$x = \dfrac{p^2}{4p-2} \qquad\qquad$ Eqn(1)

$y = -\dfrac{p}{4}$

$p = -4y \qquad\qquad$ Eqn(2)

Eqn (2) into Eqn (1)

$$x = \frac{(-4y)^2}{4(-4y) - 2}$$

$$-x(8y+1) = 8y^2$$

$$8y^2 + 8xy + x = 0$$

(iv)
At (4, 6),
$x = 4 = t^2 \Rightarrow t = 2 \quad or \quad -2$
When $t = 2,\quad y = (2)^2 - 2 = 2$ (rejected as it does not satisfy the given coordinates)
When $t = -2,\quad y = (-2)^2 - (-2) = 6$
Hence $t = p = -2$
$2py = (2p-1)x - p^2$
At $t = p = -2$,
Equation of tangent is $4y = 5x + 4$.

$4(t^2 - t) = 5t^2 + 4$
$t^2 + 4t + 4 = 0$
$(t+2)^2 = 0$
$t = -2$
Since there is only one solution for *t*, which is the given point, the tangent at $(4,6)$ does not meet the curve again.

Q5

Case 1 : 2 O is chosen

No of codes = ${}^9C_4 \times 4! \times {}^6C_2 \times \frac{4!}{2!} \times 2 = 1088640$

Note : Inserting Method, i.e ${}^5C_4$ to insert 4 Letters or 4 Digits cannot be done here, as it may result in situation such as LDLDDLDL

Case 2 : 1 O or no O is chosen

No of codes = ${}^9C_4 \times 4! \times {}^7C_4 \times 4! \times 2 = 5080320$

Total No of codes = $1088640 + 5080320 = 6168960$

Q6

(i)

X ~ number of winning coupon out of 10.
X ~ B(10, 0.02)
$P(X = 0) = 0.81707 \approx 0.817$ or $(0.98)^{10}$

(ii)

Y ~ number of family packs with no winning coupon out of 20.
Y ~ B(20, 0.81707)
$P(Y > 16) = 1 - P(Y \le 16) = 0.48882 \approx 0.489$

Note : Use more decimal places for intermediate working.

(iii)

Method 1

$\left[P\left(X \geq 1\right)\right]^3 P\left(X = 0\right)\dfrac{4!}{3!}P\left(X = 0\right)$

$= \left[1 - P\left(X = 0\right)\right]^3 \left[P\left(X = 0\right)\right]^2 \dfrac{4!}{3!}$

$= \left(1 - 0.81707\right)^3 \left(0.81707\right)^2 \dfrac{4!}{3!}$

$= 0.0163$

Note : Out of the first 4 weeks,
3 weeks have winning coupon,
1 week has no winning coupon.

Method 2
Consider the first 4 weeks.
Let W ~ number of weeks with no winning coupon out of 4.
W ~ B(4, 0.81707)
$P(W = 1) = 0.020007$
Required probability = $P(W = 1)P(X = 0) = 0.020007 \times 0.81707 = 0.0163$

(iv)

T ~ number of winning coupon out of 1000.
T ~ B(1000, 0.02)
Since $n = 1000 > 50$, $np = 1000(0.02) = 20 > 5$ and $n\left(1 - p\right) = 1000(0.98) = 980 > 5$
T ~ N(20, 19.6) approx.

$P\left(T \geq k\right) > 0.15$

$P\left(T < k\right) < 0.85$

$P\left(T < k - 0.5\right) < 0.85$ c.c

$k - 0.5 < 24.59$
$k < 25.09$
$k = 25$

Note :

1) There is no need to standardise

Alternative

$P\left(T \geq k\right) > 0.15$

$P\left(T > k - 0.5\right) > 0.15$ c.c

$k = 24$	$P\left(T > k - 0.5\right) = 0.215 > 0.15$
$k = 25$	$P\left(T > k - 0.5\right) = 0.155 < 0.15$
$k = 26$	$P\left(T > k - 0.5\right) = 0.107 < 0.15$

k = 25

Note :

1) Enter into GC,
Y1 = normalcdf (X−0.5, 10^99, 20, $\sqrt{19.6}$)
2) Present 3 sets of probability values, including one before and one after the correct answer and show the comparison with 0.15.

Q7

(i)

Let G ~ duration of consultation with GP
Let S ~ duration of consultation with specialist
$G_1 + G_2 + G_3 - 2S \sim \mathrm{N}(-2.8,\ 42.19)$
$\mathrm{P}\left(G_1 + G_2 + G_3 - 2S < 0\right) = 0.667$

(ii)
$2S + 25 \le 3(1G + 10)$
$2S - 3G \le 5$
$2S - 3G \sim \mathrm{N}(2.8,\ 63.85)$
$\mathrm{P}\left(2S - 3G \le 5\right) = 0.608$

Alternative
$X = 10 + G \sim N\left(16.2, 1.9^2\right)$
$Y = 25 + 2S \sim N(46.4, 31.36)$
$Y - 3X \sim N\left(-2.2, 63.85\right)$
$\mathrm{P}(Y - 3X \le 0) = 0.608$

Q8

$\bar{x} = \frac{\sum x}{n} = \frac{8993}{90} = 99.922$
$s^2 = \frac{1}{n-1}\left\{\sum x^2 - \frac{(\sum x)^2}{n}\right\} = \frac{1}{89}\left\{900240 - \frac{(8993)^2}{90}\right\} = 18.421$
Test $\mathrm{H}_0:\ \mu = 101$ vs $\mathrm{H}_1:\ \mu \ne 101$
Since n is large, by CLT, $\bar{X} \sim \mathrm{N}\left(101, \frac{18.421}{90}\right)$ approximately
Level of significance: 5%
Critical region: $z < -1.96$ or $z > 1.96$
Test Statistic $z = \frac{\bar{x} - \text{"claimed value"}}{\frac{s}{\sqrt{n}}} = \frac{99.922 - 101}{\sqrt{\frac{18.421}{90}}} = -2.3828 < -1.9600$
From GC, p-value = 0.0172 (which is less than 0.05)
As the p-value is less than the level of significance, we reject H_0. There is sufficient evidence, at the 5% level, to conclude that the company's claim is invalid.
In order to not reject H_0 at 5 % level of significance, the test statistic should fall outside the critical region.
$-1.96 < z < 1.96$
$-1.96 < \frac{\bar{x} - 101}{\frac{4}{\sqrt{10}}} < 1.96$
$98.52 < \bar{x} < 103.48$
It is assumed that the vitamin C content of orange juice produced by the company follows a normal distribution.

Q9

(i)
Condition 1: The complaints received are independent of each other.
Condition 2: The average number of complaints received is constant at 1.2 daily over a period of time.

The complaints received may not independent of each other as they may be from the same tour group.
OR
The number of complaints received may be higher during the peak holiday periods.

(ii)
X ~ no. of complaints received in 10 days.
$X \sim \text{Po}(12)$
$P(X < 12) = P(X \leq 11) = 0.462$

(iii)

Y ~ no. of complaints received in a day.
$Y \sim \text{Po}(1.2)$
$\left[P(Y \geq 1)\right]^2 = \left[1 - P(Y = 0)\right]^2 = 0.488$

(iv)
W ~ no. of complaints received in n days.
$W \sim \text{Po}(1.2n)$
$P(W \geq 1) > 0.999$
$P(W = 0) < 0.001$
Method 1
Using GC,
When $n = 5$, $P(W = 0) = 0.00248 (> 0.001)$
When $n = 6$, $P(W = 0) = 0.00075 (< 0.001)$
When $n = 7$, $P(W = 0) = 0.00022 (< 0.001)$
Hence, least number of days is 6.

Method 2
$\mathrm{e}^{-1.2n} < 0.001$
$n > \dfrac{\ln 0.001}{-1.2}$
$n > 5.7565$
Hence, least number of days is 6.

(v)
Since n is large, by CLT,
$\bar{Y} \sim \text{N}\left(1.2, \dfrac{1.2}{365}\right)$ approximately
$P\left(\bar{Y} < 1.3\right) = 0.959$

Q10

(i)
By GC, $r = 0.984$
Although the r value of 0.984 suggests that a linear model may be appropriate, there may be another model that fits the data better.
For every increase of 1 unit in m, the corresponding increase in h is not constant, which would not be the case if the data follows a linear model. (The increase in h should be approximately constant for a linear model)

(ii)
By GC,
For $h^2 = c + dm$, $r = 0.989 > 0.984$
For $\ln h = e + fm$, $r = 0.978 < 0.984$
Since r for the model $h^2 = c + dm$ is closer to 1 compared to 0.984, $h^2 = c + dm$ is a better model than $h = a + bm$.

(iii)

By GC,
$h^2 = 1289.7 + 79.031m$
$h^2 = 1290 + 79.0m$ (3 sig fig)
When $h = 39.9$,
$39.9^2 = 1289.7 + 79.031m$
$m = 3.83$

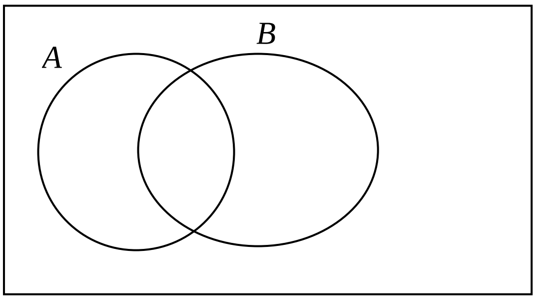
The age in months (m) is the independent variable and the average head circumference from the time of birth measured in centimetres (h) is the dependent variable in the question.
In general, the regression line of dependent variable on independent variable should always be used regardless of estimating m or h.
Therefore, the regression line of m on h, the regression line of m on h^2 and the regression line of m on $\ln h$ should not be used to find the required estimate.

Q11

(i)

Since $\mathrm{P}(B\,|\,A)=\dfrac{2}{9}\neq \mathrm{P}(B)=\dfrac{7}{27}$, A and B are not independent.

(ii)



Method 1

Given $\mathrm{P}(A\,|\,B')=\dfrac{7}{10}$

$$\frac{\mathrm{P}(A\cap B')}{\mathrm{P}(B')}=\frac{7}{10}$$

$$\mathrm{P}(A\cap B')=\frac{7}{10}\mathrm{P}(B') \qquad \text{Eqn (1)}$$

Using the fact $\mathrm{P}\left(A\cup B\right)=\mathrm{P}(A\cap B')+\mathrm{P}\left(B\right)$ Eqn (2)

Eqn (1) into (2)

$$\mathrm{P}\left(A\cup B\right)=\frac{7}{10}\mathrm{P}(B')+\mathrm{P}\left(B\right)$$

$$\mathrm{P}\left(A\cup B\right)=\frac{7}{10}\left(\frac{20}{27}\right)+\frac{7}{27}=\frac{7}{9}$$

Method 2

Given $\mathrm{P}(A\,|\,B')=\dfrac{7}{10}$

$$\frac{\mathrm{P}(A\cap B')}{\mathrm{P}(B')}=\frac{7}{10}$$

$$\mathrm{P}(A\cap B')=\frac{7}{10}\mathrm{P}(B')$$

$$\mathrm{P}(A)-\mathrm{P}(A\cap B)=\frac{7}{10}\mathrm{P}(B') \qquad \text{Eqn (1)}$$

Using the fact $\mathrm{P}\left(A\cup B\right)=\mathrm{P}\left(B\right)+\mathrm{P}\left(A\right)-\mathrm{P}\left(A\cap B\right)$ Eqn (2)

Eqn (1) into (2)

$$\mathrm{P}\left(A\cup B\right)=\frac{7}{10}\mathrm{P}(B')+\mathrm{P}\left(B\right)$$

$$\mathrm{P}\left(A\cup B\right)=\frac{7}{10}\left(\frac{20}{27}\right)+\frac{7}{27}=\frac{7}{9}$$

Method 3

$$\mathrm{P}(A'\,|\,B')=1-\mathrm{P}(A\,|\,B')=1-\frac{7}{10}=\frac{3}{10}$$

$$\frac{\mathrm{P}(A'\cap B')}{\mathrm{P}(B')}=\frac{3}{10}$$

$$\frac{1-\mathrm{P}(A\cup B)}{\mathrm{P}(B')}=\frac{3}{10}$$

$$\mathrm{P}(A\cup B)=1-\frac{3}{10}\mathrm{P}(B')=1-\frac{3}{10}\left(\frac{20}{27}\right)=\frac{7}{9}$$

(iii)

Given $\mathrm{P}(B \mid A) = \dfrac{2}{9}$

$\dfrac{\mathrm{P}(B \cap A)}{\mathrm{P}(A)} = \dfrac{2}{9}$

$\mathrm{P}(A) = \dfrac{9}{2}\mathrm{P}(B \cap A)$ Eqn (3)

Using $\mathrm{P}(A \cup B) = \mathrm{P}(B) + \mathrm{P}(A) - \mathrm{P}(A \cap B)$ Eqn (4)

Eqn (3) into (4)

$\mathrm{P}(A \cup B) = \mathrm{P}(B) + \dfrac{9}{2}\mathrm{P}(B \cap A) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cup B) = \mathrm{P}(B) + \dfrac{7}{2}\mathrm{P}(A \cap B)$

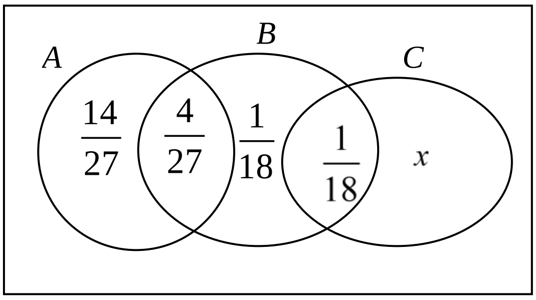
$\mathrm{P}(A \cap B) = \dfrac{2}{7}\left[\mathrm{P}(A \cup B) - \mathrm{P}(B)\right]$

$\mathrm{P}(A \cap B) = \dfrac{2}{7}\left[\dfrac{7}{9} - \dfrac{7}{27}\right] = \dfrac{4}{27}$

(iv)

Let x be $\mathrm{P}(B' \cap C)$.

C does not overlap A. Therefore C is drawn as shown below.



Since probability cannot exceed 1,

$\mathrm{P}(A \cup B) + x \le 1$

$\dfrac{7}{9} + x \le 1$

$x \le \dfrac{2}{9}$

$x + \dfrac{1}{18} \le \dfrac{2}{9} + \dfrac{1}{18}$

$\mathrm{P}(C) \le \dfrac{5}{18}$

$\dfrac{1}{18} \le \mathrm{P}(C) \le \dfrac{5}{18}$