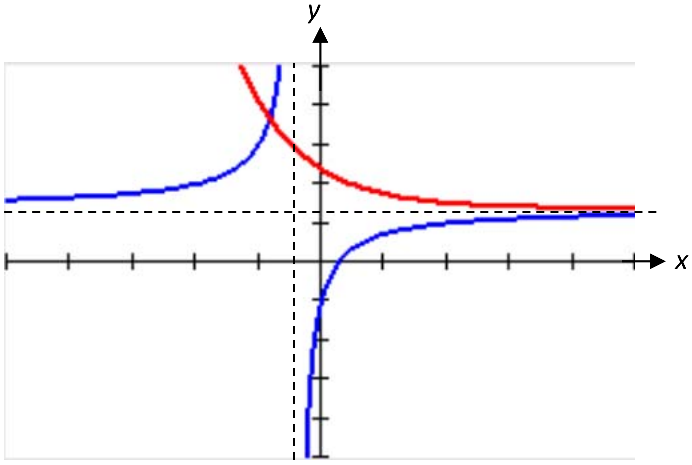
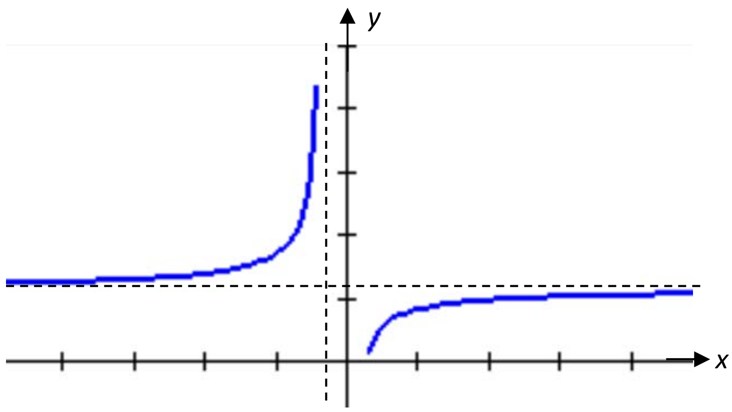
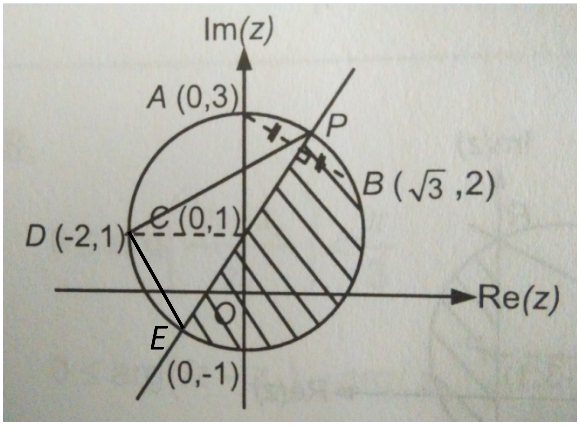
MI PE II 2015: PU3 H2 Paper 2: Marking scheme

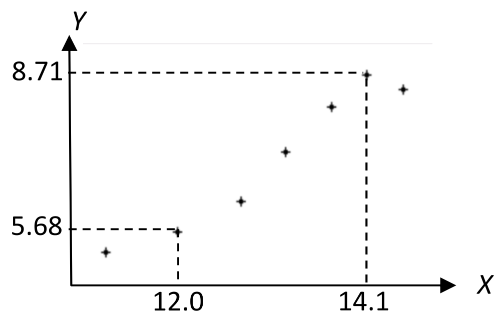
1i	
1ii	$7x-2=(e^{-x}+k)(5x+2)$ $\frac{7x-2}{5x+2}=e^{-x}+k$ By observation, to have only 1 root, $k\geq\frac{7}{5}$
1iii	
2a	$\pi R^2-\pi r^2=20$ $2\pi R\frac{dR}{dt}-2\pi r\frac{dr}{dt}=0$ $\frac{dr}{dt}=\frac{2\pi R}{2\pi r}\times\frac{dR}{dt}=\frac{2R}{r}$ When R = 5, $r=\sqrt{R^2-\frac{20}{\pi}}=\sqrt{25-\frac{20}{\pi}}$ $\frac{dr}{dt}=\frac{2(5)}{\sqrt{25-\frac{20}{\pi}}}=2.32$ cm/s (3 sf)

2bi	$\frac{\mathrm{d}x}{\mathrm{d}t}=2$ $\frac{\mathrm{d}y}{\mathrm{d}t}=-\frac{2}{(2t+1)^2}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{(2t+1)^2}$ From given line, gradient of line $=\frac{4}{9}$. We observe $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{(2t+1)^2}\neq\frac{4}{9}$ for all $t\in\mathbb{R}$ so it is shown there is no such tangent.
2bii	Grad of normal $=(2t+1)^2$ At $t=\frac{1}{2}$, $x=0$, $y=\frac{1}{2}$ and grad = 4 Eqn of normal is $y=4x+\frac{1}{2}$. When normal meets curve again: $\frac{1}{2t+1}=4(2t-1)+\frac{1}{2}$ $t=-\frac{9}{16}$ or $\frac{1}{2}$ (NA, already used) For $t=-\frac{9}{16}$, coordinates of point is $\left(-\frac{17}{8},\ -8\right)$.
3i	$\overrightarrow{AB}=k\overrightarrow{BC}$ $\begin{pmatrix}4-\alpha\\-1\\-2\end{pmatrix}=k\begin{pmatrix}-5\\-5\\\beta\end{pmatrix}$ $k=\frac{1}{5}$ $\therefore\ \alpha=5$ and $\beta=-10$
3ii	Since both triangles share same base with points A, B, C, so ratio is 1 : 5.
3iii	$\mathbf{r}=\begin{pmatrix}4\\-2\\0\end{pmatrix}+\lambda\begin{pmatrix}1\\1\\2\end{pmatrix},\ \lambda\in\mathbb{R}$

3iv	Since $\overrightarrow{OP}=\frac{1}{3}\overrightarrow{PB}$, $\overrightarrow{OP}=\frac{1}{4}\overrightarrow{OB}=\frac{1}{4}\begin{pmatrix}4\\-2\\0\end{pmatrix}=\begin{pmatrix}1\\-0.5\\0\end{pmatrix}$ $\overrightarrow{OQ}=\begin{pmatrix}4+\lambda\\-2+\lambda\\2\lambda\end{pmatrix}$ $\overrightarrow{PQ}=\begin{pmatrix}4+\lambda-1\\-2+\lambda+0.5\\2\lambda\end{pmatrix}=\begin{pmatrix}3+\lambda\\-1.5+\lambda\\2\lambda\end{pmatrix}$ $\therefore\ \frac{\left\|\overrightarrow{PQ}\cdot\overrightarrow{OB}\right\|}{\left\|\overrightarrow{OB}\right\|}=\sqrt{5}$ $\|15+2\lambda\|=10$ So $\lambda=-\frac{5}{2}$ or $\lambda=-\frac{25}{2}$ For $\lambda=-\frac{5}{2}$, $\overrightarrow{OQ}=\begin{pmatrix}1.5\\-4.5\\-5\end{pmatrix}$ and for $\lambda=-\frac{25}{2}$, $\overrightarrow{OQ}=\begin{pmatrix}-8.5\\-14.5\\-25\end{pmatrix}$ Coordinates of Q are (1.5, -4.5, -5) and (-8.5, -14.5, -25).
4i,ii	 Midpoint of A and B = $\frac{\sqrt{3}}{2}+\frac{5}{2}i$
4iii	$h=1$

4iv	From diagram, $\angle PCD = \pi - (\tan^{-1}\sqrt{3}) = \dfrac{2\pi}{3}$ As ΔPDC is isos. Δ, greatest arg = $\dfrac{1}{2}\left(\pi - \dfrac{2\pi}{3}\right) = \dfrac{\pi}{6}$ and smallest arg = $-\left[\dfrac{1}{2}\left(\pi - \dfrac{\pi}{3}\right)\right] = -\dfrac{\pi}{3}$ Range is $-\dfrac{\pi}{3} \le z \le \dfrac{\pi}{6}$.
5i	The group should decide on quota for each male and female stratum, eg 50 males and 50 females. They should then situate themselves at the exit. Select and survey people according to strata until respective quotas are reached.
5ii	Not appropriate. The reason is since the period is June holidays, there is a high likelihood that many school children will visit.
6i	Let X be the random variable for weight of a durian. Let T be the random variable for total weight of a basket of durians. $X_1 + X_2 + X_3 + X_4 + X_5 + 1.52 \sim \mathrm{N}\left(5(2.36) + 1.52,\ 5(0.07^2)\right)$ $T \sim \mathrm{N}(13.32,\ 0.0245)$ $\mathrm{P}(T < 13.2) = 0.222$ (3 sf)
6ii	$\overline{X} \sim \mathrm{N}\left(13.32,\ \dfrac{0.0245}{10}\right)$ $\mathrm{P}(\overline{X} > 13.2) = 0.992$ (3 sf)
7i	Let X be the number of hours of sleep for each student in the recording. $\mathrm{H}_0 : \mu = 6.5$ $\mathrm{H}_1 : \mu < 6.5$ Assuming H_0 is true, since σ is unknown and n is small, we use t-test and assume X follows normal distribution. At 10% level, we reject H_0 if p-value ≤ 0.1 Using GC, p-value = 0.4096 Since p-value = 0.4096 > 0.1, so we cannot reject H_0 and conclude, at 10% level, that there is insufficient evidence to support Mr Lee's claim.

7ii	$$H_0 : \mu = 6.5$$ $$H_1 : \mu < 6.5$$ For $\alpha = 0.03$, z–value = −1.8808 Let Y be the number of hours of sleep for each student in a sample of 60. Since sample size = 60 is large, by Central Limit Theorem, $$\overline{Y} \sim N\left(6.5, \frac{s^2}{60}\right) \text{ approximately.}$$ For H_0 to be rejected, $$\frac{6.42 - 6.5}{s/\sqrt{60}} < -1.8808$$ $$s^2 < 0.109 \text{ (3 sf)}$$
8i	Let X be the number of defective pencils in a packet of 40 pencils. $$X \sim B(40, 0.013)$$ $P(X \le 1) = 0.905$ (3 sf)
8ii	Let Y_1 be the number of defective pencils in a packet of 20 pencils. $$Y_1 \sim B(20, 0.013)$$ Let Y_2 be the number of defective pencils in a packet of 30 pens. $$Y_2 \sim B(30, 0.015)$$ 2 pencils, 0 pen: $P(Y_1 = 2) \times P(Y_2 = 0) = 0.01612$ 1 pencil, 1 pen: $P(Y_1 = 1) \times P(Y_2 = 1) = 0.05887$ 0 pencil, 2 pens: $P(Y_1 = 0) \times P(Y_2 = 2) = 0.04934$ Required prob = 0.01612 + 0.05887 + 0.04934 = 0.124 (3 sf)
8iii	$T \sim \text{Po}[0.015(n)]$ approximately $P(n - T = 50) \le 0.2$ Using GC, the least number is 52.
9i	Let X be the number of customers in a hour during the peak period. $$X \sim \text{Po}(15.6)$$ $P(X \ge 12) = 1 - P(X \le 11)$ $= 0.851$ (3 sf)
9ii	Expected number

	$= 2(15.6) + 0.5(8.2)$ $= 35.3$
9iii	Let A be the number of customers during the peak period. $A \sim \text{Po}(31.2)$ Since $\lambda = 31.2 > 10$, so $A \sim \text{N}(31.2, 31.2)$ approximately Let B be the number of customers during the off-peak period. $B \sim \text{Po}(41.0)$ Since $\lambda = 41 > 10$, so $B \sim \text{N}(41, 41)$ approximately For required condition, we have $\text{P}(B > A) = \text{P}(B - A > 0)$ $B - A \sim \text{N}(9.8, 72.2)$ $\text{P}(B - A > 0) = \text{P}(B - A > 0.5)$ [Continuity correction] $= 0.863$
10i	$r = 0.970$ (3sf) Since r is close to +1, it suggests a strong positive linear correlation between X and Y so a linear model is appropriate.
10ii	
10iii	Simplify $Y = pe^{qX^2}$ to $\ln Y = q(X^2) + \ln p$. Obtain r value = 0.977 (3 sf) The better model is $Y = pe^{qX^2}$ as the r value is higher.
10iv	$\ln Y = 0.00646(X^2) + 0.833$ $Y = 6.85$ (3 sf) Result is reliable as r is close to +1 and it is obtained through interpolation.
10v	r value will not change. A change in the scale of the variables will not affect the scatter / cluster of the points which determines the r value.
11ai	$\frac{{}^{18}C_2 \times {}^{2}C_2}{{}^{20}C_4} = 0.0316$
11aii	Males from basketball: ${}^{3}C_2 \times {}^{7}C_2$ Males from volleyball: ${}^{4}C_2 \times {}^{9}C_2$ Males from badminton: ${}^{2}C_2 \times {}^{9}C_2$

	Prob = $\frac{{}^{3}C_{2}\times{}^{7}C_{2}+{}^{4}C_{2}\times{}^{9}C_{2}+{}^{2}C_{2}\times{}^{9}C_{2}}{{}^{20}C_{4}}=0.0650$
11aiii	$\text{P(3 females} \mid \text{1 netballer)} = \frac{\text{P(3 females and 1 must be netballer)}}{\text{P(1 netballer)}}$ $= \frac{{}^{4}C_{1}\,{}^{6}C_{2}\,{}^{10}C_{1}}{{}^{4}C_{1}\,{}^{16}C_{3}}$ $= 0.268$
11bi	$5! \times 4! \times 3! \times 2! = 34560$
11bii	$(6^2 \times 5^2 \times 4^2 \times 3^2 \times 2^2 \times 1^2) / 6$ $= 86400$