Chapter 18 ALTERNATING CURRENTS

Content

- Characteristics of alternating currents
- The transformer
- Rectification with a diode

Learning Outcomes

Candidates should be able to:

(a) show an understanding of and use the terms period, frequency, peak value and root-mean-square (r.m.s.) value as applied to an alternating current or voltage.

(b) deduce that the mean power in a resistive load is half the maximum power for a sinusoidal alternating current.

(c) represent an alternating current or an alternating voltage by an equation of the form $x = x_0 \sin \omega t$.

(d) distinguish between r.m.s. and peak values and recall and solve problems using the relationship $I_{\text{rms}} = \dfrac{I_0}{\sqrt{2}}$ for the sinusoidal case.

(e) show an understanding of the principle of operation of a simple iron-core transformer and recall and solve problems using $\dfrac{N_s}{N_p} = \dfrac{V_s}{V_p} = \dfrac{I_p}{I_s}$ for an ideal transformer.

(f) explain the use of a single diode for the half-wave rectification of an alternating current.

Ace gryphon

18 Introduction

The rise of Alternating Current

During the 1880s in the US, there was a heated debate between two inventors over the best method of electric-power distribution. Thomas Edison favoured direct current (d.c.), while George Westinghouse favoured alternating current (a.c.). Westinghouse argued that transformers could be used to step up and down voltages with a.c. but not with d.c.; high voltages and correspondingly low currents are best for long-distance power transmission to minimize power lost across the cables and low voltages are safer for consumer use. This was an advantage over d.c. systems which had limited range due to the inherently low voltages used.

Despite Edison's numerous public attempts to show that high voltage transmission was unsafe, Westinghouse eventually won the 'fight'. Most present-day power distribution systems operate with a.c.. Each time you turn on an electrical appliance at home, you are using alternating current to provide the power to operate it.

18.1 Characteristics of Alternating Currents

Examples of a.c. waveforms

An alternating current varies periodically with time in magnitude and direction

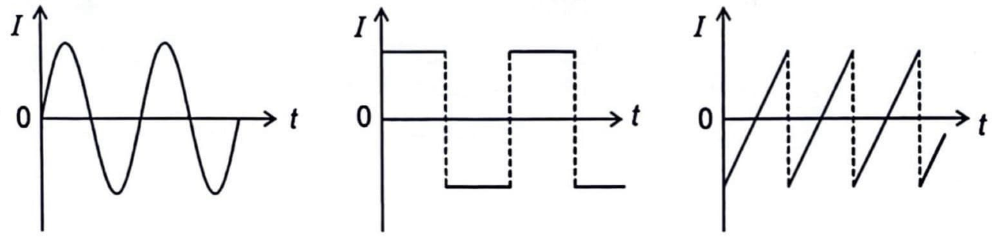


Fig. 1

Term	Definition
Period, T	Time taken for one complete cycle.
Frequency, f	Number of cycles per unit time.
Peak current, I_0	Amplitude of the current.

Sinusoidal a.c.

According to Faraday's law of electromagnetic induction, an induced e.m.f. is generated when there is a changing magnetic flux. To generate a.c., a coil is made to rotate in the presence of a magnetic field such that the induced e.m.f. produced varies sinusoidally with time which in turn leads to a sinusoidal alternating current.

The sinusoidal alternating current and alternating voltage can be represented by the following equations and graphs:

Note:
I_0 and V_0 are the <u>peak</u> current and voltage
ω is the <u>angular frequency</u>, where $\omega = 2\pi f$

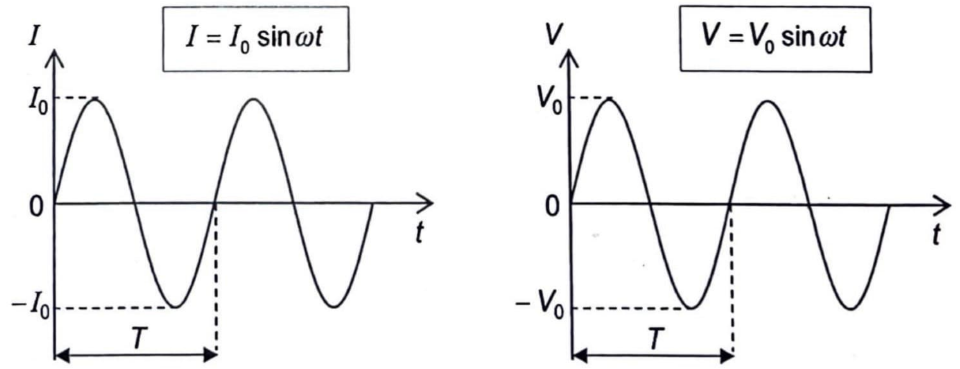


Fig. 2

18.2 Power in an a.c. circuit

In a simple resistive a.c. circuit, the resistor heats up when an alternating current passes through it, in the same way as when a direct current is used. This indicates that some power is dissipated in the resistor.

Although temperature increase depends on the magnitude of the current, it is **independent of the direction of current.**

The circuit in Fig. 3 shows a resistor R connected to an (sinusoidal) a.c. voltage source. The instantaneous current flowing through and the potential difference across the resistor R is I_{ac} and V_{ac} respectively such that at any time,

$$V_{ac} = I_{ac}R \quad \text{and} \quad P_{ac} = I_{ac}V_{ac} = I_{ac}^2 R = \frac{V_{ac}^2}{R}$$

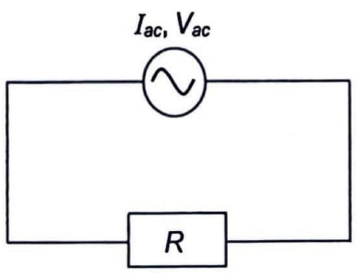


Fig. 3

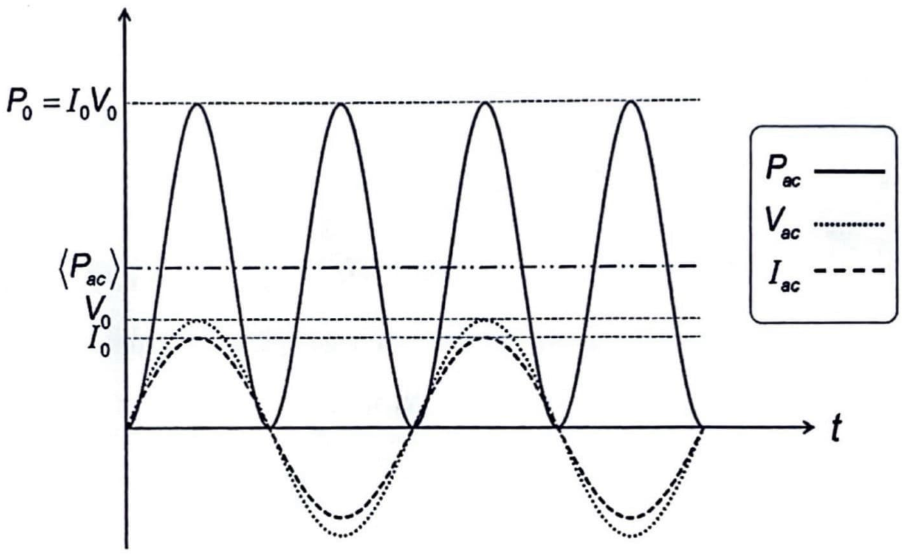


Fig. 4

The variation of I_{ac}, V_{ac} and P_{ac} with time is represented graphically as in Fig. 4. The area under the $P_{ac} - t$ graph gives the total energy dissipated in the resistor for a time *t*.

[Recall that $P = \frac{dW}{dt}$. Hence $W = \int P \, dt$. Integrating power *P* over a period of time d*t* will give the amount of work done *W* or energy converted during the time. Hence the amount of energy converted equal to the area under the *power-time* graph.]

Mean Power Since instantaneous power P_{ac} changes with time, a more meaningful quantity - mean power $\langle P_{ac} \rangle$ (i.e. the average power dissipated in the resistor) can be defined:

$$\langle P_{ac} \rangle = \frac{\text{Total Energy dissipated in time } t}{t}$$

Graphically,
$$\langle P_{ac} \rangle = \frac{\text{area under the } P_{ac} - t \text{ graph in time } t}{t}$$

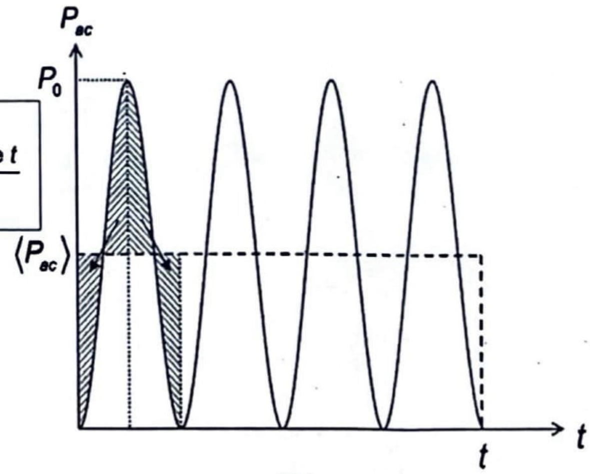


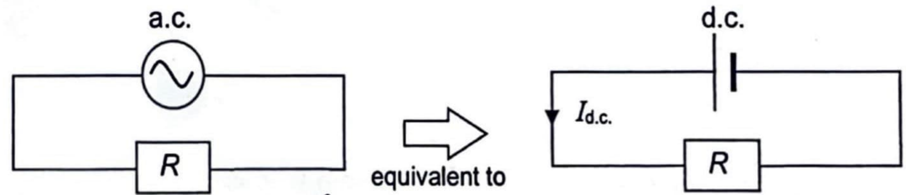
Fig. 5

From Fig. 5, the area under the $P_{ac} - t$ graph is the same as the area of $\langle P_{ac} \rangle \times t$.

$$\langle P_{ac} \rangle = \langle I_{ac}^2 R \rangle = \langle I_{ac}^2 \rangle R = \left(\sqrt{\langle I_{ac}^2 \rangle}\right)^2 R = (I_{rms})^2 R$$

The value I_{rms} is the **root** of the average **(mean)** of the **square** value of current I_{ac}.

In Fig. 6, the diagram on the left shows a resistor R connected to an a.c. source, while the diagram on the right shows the same resistor connected to a d.c. source. P_R is the power dissipated in R.



$P_R = \langle P_{ac} \rangle = \langle I_{ac}{}^2 \rangle R = (I_{rms})^2 R$

Fig. 6

$P_R = \langle P_{dc} \rangle = I_{dc}{}^2 R$

Suppose the resistor in both circuits dissipates heat at the same rate, we can then equate the average power dissipated as:

$$\langle P_{ac} \rangle = P_{dc}$$

$$(I_{rms})^2 R = I_{dc}^2 R$$

$$\therefore I_{rms} = I_{dc}$$

Note: a d.c. source produces constant power so $\langle P_{dc} \rangle = P_{dc}$

Definition

The r.m.s. value of the alternating current or voltage is that value of the direct current or voltage that would produce thermal energy at the same rate in a resistor.

Formula

Hence, the average power dissipated in the resistor

$$\langle P \rangle = I_{rms}^2 R \quad \text{or} \quad \frac{V_{rms}^2}{R} \quad \text{or} \quad I_{rms} V_{rms}$$

r.m.s. value of current

To find the r.m.s. value of current, start the procedure from the right of the letters

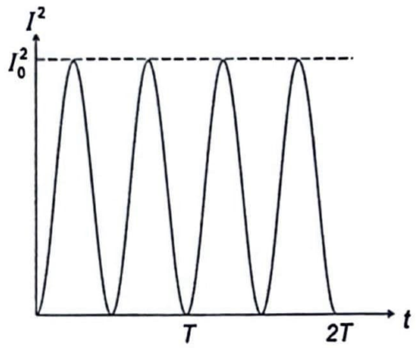
r m s

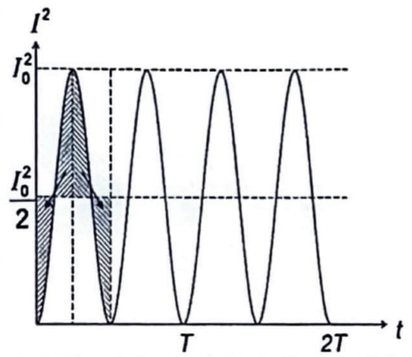
Step 1: We square the instantaneous current I

Step 2: Find the mean (or average) value of I^2
(area under I^2–t graph over one period divided by the period)

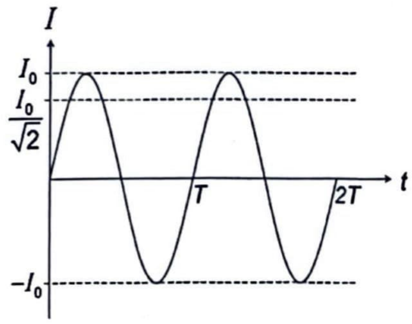
Step 3: Take the square root of that mean value.

1. Square	2. Mean	3. Root
$I^2 = I_0^2 \sin^2 \omega t$	$\langle I^2 \rangle = \dfrac{I_0^2}{2}$	$I_{rms} = \sqrt{\langle I^2 \rangle} = \sqrt{\dfrac{I_0^2}{2}} = \dfrac{I_0}{\sqrt{2}}$





Note that the shaded regions above and below the $\dfrac{I_0^2}{2}$ line have the same area



Formula

Thus for a sinusoidal current and sinusoidal voltage:

$$I_{rms} = \frac{I_0}{\sqrt{2}}$$

$$V_{rms} = \frac{V_0}{\sqrt{2}}$$

Mean power for a sinusoidal a.c.

$$\langle P \rangle = I_{rms} V_{rms} = \left(\frac{I_0}{\sqrt{2}}\right)\left(\frac{V_0}{\sqrt{2}}\right) = \frac{P_0}{2}$$

Note

It can be seen from the graph that the frequency of the power is twice the frequency of the sinusoidal a.c..

F.Y.I. (not in syllabus)

Mathematically,

$$I_{rms} = \sqrt{\frac{\int_0^T I^2 dt}{T}} = \sqrt{\langle I^2 \rangle}$$

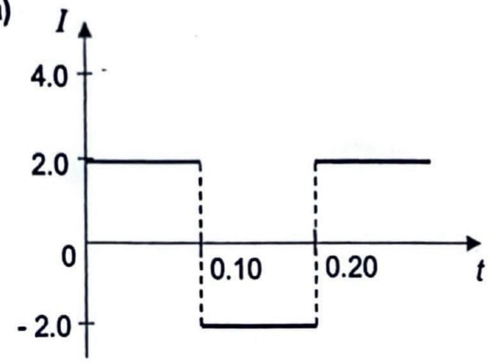
and for sinusoidal current,

$$\int_0^T I^2 dt = \int_0^T \left(I_0^2 \sin^2 \omega t\right) dt = I_0^2 \int_0^T \frac{1}{2}(1 - \cos 2\omega t)\, dt = I_0^2 \left[t - \frac{1}{2\omega} \sin 2\omega t\right]_0^T = I_0^2 [T]$$

$$I_{rms} = \sqrt{\frac{\int_0^T I^2 dt}{T}} = \sqrt{\frac{I_0^2 [T]}{2T}} = \sqrt{\frac{I_0^2}{2}} = \frac{I_0}{\sqrt{2}}$$

Example 1 Calculate the r.m.s. value of the alternating currents shown.

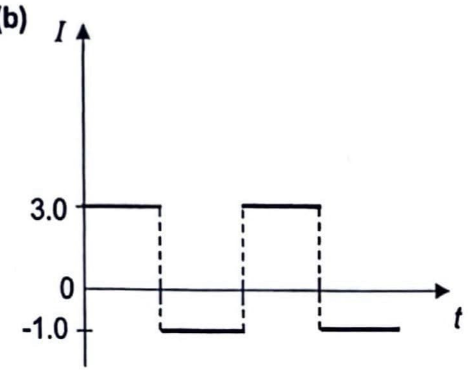
(a)



$$\text{Mean} = \frac{4.0 \times 0.20}{0.20} = 4.0$$

$$I_{rms} = \sqrt{4.0} = 2.0 \text{ A}$$

(b)



$$\text{Mean} = \frac{9.0 \times \frac{T}{2} + 1.0 \times \frac{T}{2}}{T} = 5.0$$

$$I_{\text{rms}} = \sqrt{5.0} = 2.2 \text{ A}$$

Example 2 When a domestic electric heater is operated from a 240 V a.c. supply, an r.m.s. current of 8.0 A flows. Assume that the heater is purely resistive, calculate

(a) its resistance
(b) the mean power output
(c) the maximum instantaneous power

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(If the question does not specify whether the given value of the source is r.m.s. or peak value, assume that it is r.m.s. value and the a.c. supply is sinusoidal.)

(a) resistance of heater: $R = \frac{V_{\text{rms}}}{I_{\text{rms}}}$

$$= \frac{240}{8}$$

$$= 30\ \Omega$$

(b) Mean power output

$$\langle P \rangle = I_{rms} V_{rms} = (8.0)(240) = 1920 \text{ W}$$

(c) Max instantaneous power

$$\langle P \rangle = \frac{P_o}{2}$$

$$\Rightarrow P_o = 2\langle P \rangle = 3840 \text{ W}$$

18.3 The Transformer

Transformers

Function: A device that uses mutual electromagnetic induction to vary (either step up or step down) an alternating voltage.

Structure: Fig. 7 shows a schematic of a simple step up transformer.

It consists of two coils wound around the same soft iron core.

The coil that is connected to the a.c. voltage source is known as the primary coil and has N_P turns.

The coil connected to the load is known as the secondary coil has N_S turns.

The common iron core concentrates the magnetic flux through both the primary and secondary coils which is typically made from 'E' and 'I' shaped laminations.

Principle: The a.c. voltage source (primary voltage) V_P causes an alternating current I_P to flow through the primary coil.

This sets up a varying magnetic field in the iron core which links the primary coil with the secondary coil.

In accordance to Faraday's law of electromagnetic induction, the changing magnetic field induces an alternating e.m.f. across each turn of wire in both the primary and secondary coils.

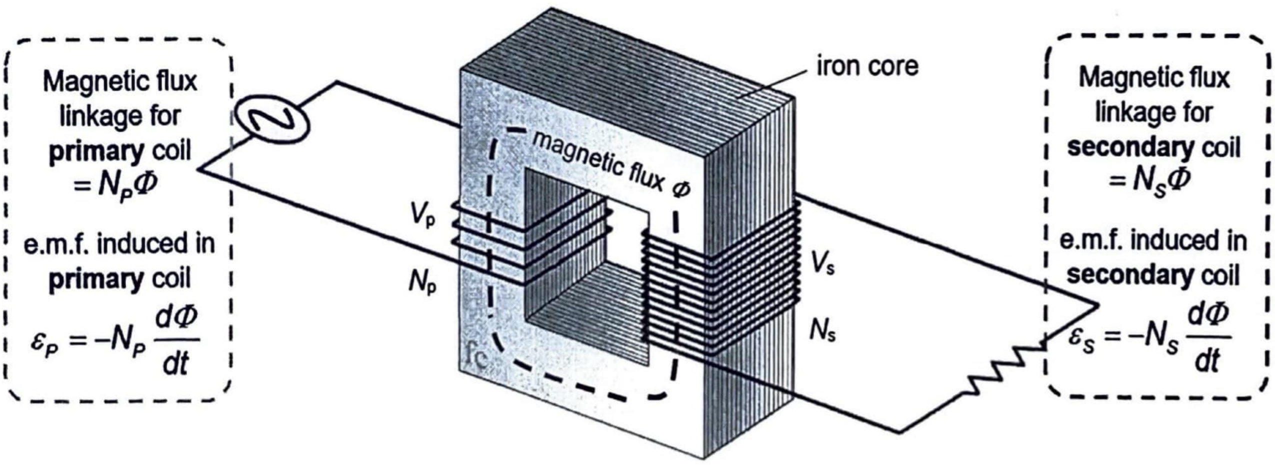


Fig. 7

For an ideal transformer (i.e.100% efficient),

- there is no energy loss in the coils and the core during operation.
- It also has no flux leakage and
- hence at any instant, **the magnetic flux Φ through each turn is the same in both the primary and secondary coils.**

Since magnetic flux Φ through each turn is the same in both the primary and secondary coils, the induced e.m.f. per turn is the same.

$$\frac{\varepsilon_P}{N_P} = \frac{\varepsilon_S}{N_S} \Rightarrow \frac{\varepsilon_S}{\varepsilon_P} = \frac{N_S}{N_P}$$

Since resistance of the coils of an ideal transformer is zero, ε_S and ε_P are equal to their terminal voltages V_S and V_P respectively.

$$\frac{V_S}{V_P} = \frac{N_S}{N_P}$$

By selecting the appropriate turns ratio $\frac{N_S}{N_P}$, the desired output or secondary voltage can be obtained from a given input or primary voltage.

Step-up transformer: $N_S > N_P$, $V_S > V_P$
Step-down transformer: $N_S < N_P$, $V_S < V_P$

Since there is no energy dissipation in both the core and the coils for an ideal transformer:

input power = output power

$$I_P V_P = I_S V_S$$

Formula

$$\frac{V_S}{V_P} = \frac{I_P}{I_S} = \frac{N_S}{N_P}$$

Example 3 An ideal transformer has 550 turns and 30 turns in the primary and secondary coils respectively.

(a) What is the maximum output potential difference if V_p is 3.3 kV?

(b) What is the maximum primary current required if a maximum current of 11 A is drawn by a resistive load? *(note: the load is connected to the secondary coil)*

(a) $$\frac{V_s}{V_p} = \frac{N_s}{N_p} \Rightarrow V_s = \frac{30}{550}(3.3) = 0.18 \text{ kV}$$

$$\text{Maximum } V_s = 0.18\sqrt{2} = 0.25 \text{ kV}$$

(b) $$\frac{N_s}{N_p} = \frac{I_p}{I_s} \Rightarrow \text{Maximum } I_p = \frac{30}{550}(11) = 0.60 \text{ A}$$

Power Loss in transformers

Cause of power loss in a transformer	**Design features to reduce power loss**
Joule Heating of copper wires of the coils.	Thick copper wire of low resistance is used, particularly for the coil carrying high current at low voltage.
Heating effect due to **Eddy currents** induced in the iron core.	The iron core is made of laminated sheets, cutting across the path of eddy currents, to increase resistance to current flow.
Hysteresis Loss Energy is used in the process of magnetising the iron and reversing this magnetisation every time the current reverses. This heats up the iron core.	The iron core is made of soft iron ("annealed iron" which has high values of permeability and lower reluctance), which can be easily magnetised and demagnetised by the magnetic field of the primary coil.
Magnetic Flux leakage Some of the magnetic field lines produced by the primary coil do not link with the secondary coil, reducing the e.m.f. induced in the secondary coil.	The presence of iron core maximises the flux linkage between the primary and secondary coils. Flux leakage can also be minimized by having a good design. In the 'E-I' shaped iron core, the secondary coil is wound on top of the primary coil and the iron core forms a closed loop.

(More information can be found in the Appendix)

Transmission of electrical power

When it comes to heating and lighting applications, a.c. has no practical advantage over d.c. because the heating effect of a current is independent of its direction. In fact, many practical systems like planes, light rail system, computers and consumer electronics are powered by direct current.

The main advantage of using a.c. is that it can be stepped up or down easily using a transformer compared to d.c. for high voltage transmission. High voltage transmission of electrical power is more efficient due to low power losses in the cables.

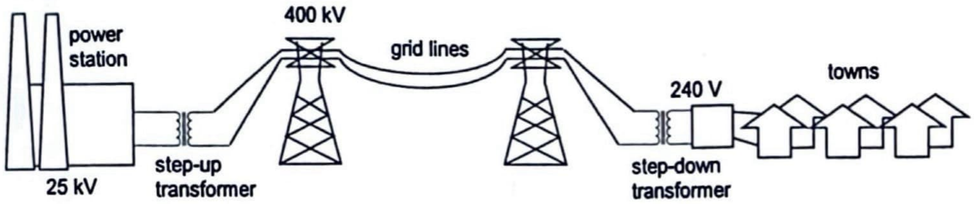


Fig. 8

Consider a transmission line supplying electrical power P from a generating (power) station to consumers (e.g. homes, commercial buildings, factories). Fig. 9 shows a simplified schematic of the power transmission system.

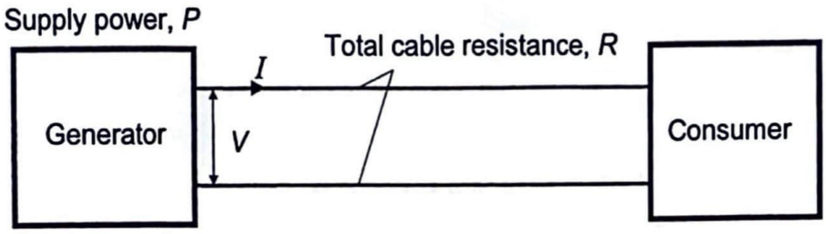


Fig. 9

Since the transmission cables run for kilometers, some power loss due to Joule heating is expected.

For a constant power supplied by the generator P and transmission voltage V, the current in the cables is $I = \dfrac{P}{V}$

If total resistance of the cables is R, the power loss across the cables is

$$P_{loss} = I^2R = \left(\frac{P}{V}\right)^2 R$$

Hence **electrical power is transmitted (over long distances) at high voltage so as to minimise power lost in the cables.**

Note

The potential drop across the cable is calculated by $V_{cable} = IR = \left(\dfrac{P}{V}\right)R$.

$$V_{cable} \neq V \text{, and } P_{loss} = I^2R = \frac{V_{cable}^2}{R}$$

Example 4 An electricity-generating station delivers energy at a rate of 20 MW to a city 1.0 km away. A common voltage for commercial power generators is 22 kV, but a step-up transformer is used to boost the voltage to 230 kV before transmission.

(a) If the resistance of the wires is 2.0 Ω and energy costs $0.20 per kWh, estimate the cost of the energy converted to thermal energy in the wires in one day.
(b) Find the cost in (a) if the energy is delivered at its original voltage of 22 kV

(a) $I_{rms} = \frac{P}{V_{rms}} = \frac{20\times10^6}{230\times10^3} = 87.0\text{ A} \Rightarrow P_{loss} = I_{rms}{}^2R = (87.0)^2(2.0) = 15.1\text{ kW}$

Energy converted to thermal energy in one day = $15.1\times 24 = 362\text{kWh}$

Cost = $362\times\$0.20 = \$72.40$

(b) $I_{rms} = \frac{P}{V_{rms}} = \frac{20\times10^6}{22\times10^3} = 909\text{ A} \Rightarrow P_{loss} = I_{rms}{}^2R = (909)^2(2.0) = 1652.9\text{kW}$

Energy converted to thermal energy in one day = $1652.9\times 24 = 39670\text{kWh}$

Cost = $39670\times\$0.20 = \$7930$

18.3 Rectification with a Diode

Diodes Rectification is the conversion of alternating current to direct current. Diodes are used for rectification. A diode is a device that conducts current predominantly in one direction. The circuit symbol for a diode is shown in Fig. 10 with the arrow head indicating the forward direction of conventional current flow.

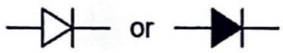
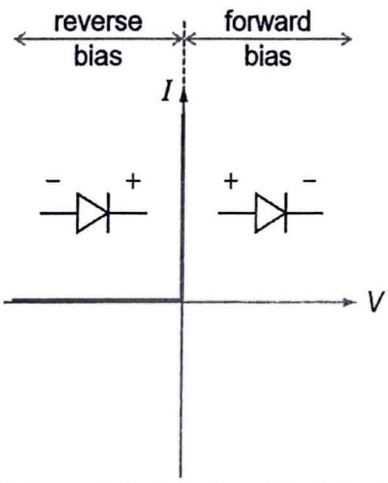


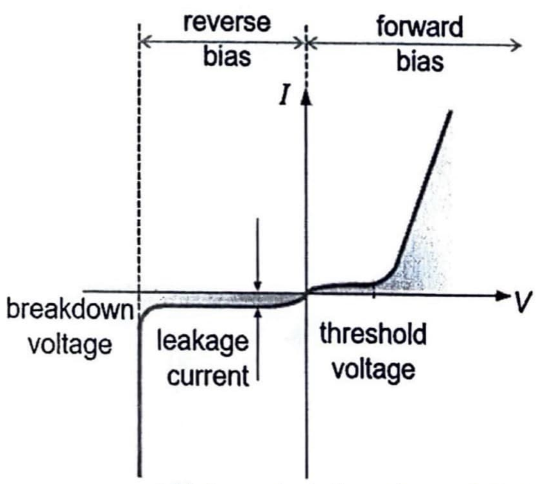
Fig. 10

When connection is made to a supply so that a diode conducts, it is said to be in *forward* bias and in the non-conducting state it is in *reverse* bias.

An ideal diode has zero resistance in forward bias and infinite resistance in reverse bias. The *I-V* characteristics are shown in Fig. 11.



I-V characteristics of an **ideal** diode



I-V characteristics of a **real** diode

Fig. 11

Half-wave rectification The circuit in Fig. 12 consists of an ideal diode in series with an a.c. input to be rectified and a load requiring d.c. output. For simplicity, the load is represented by a resistor R, but it may be any other electronic component.

The corresponding input and output voltages are shown on the right.

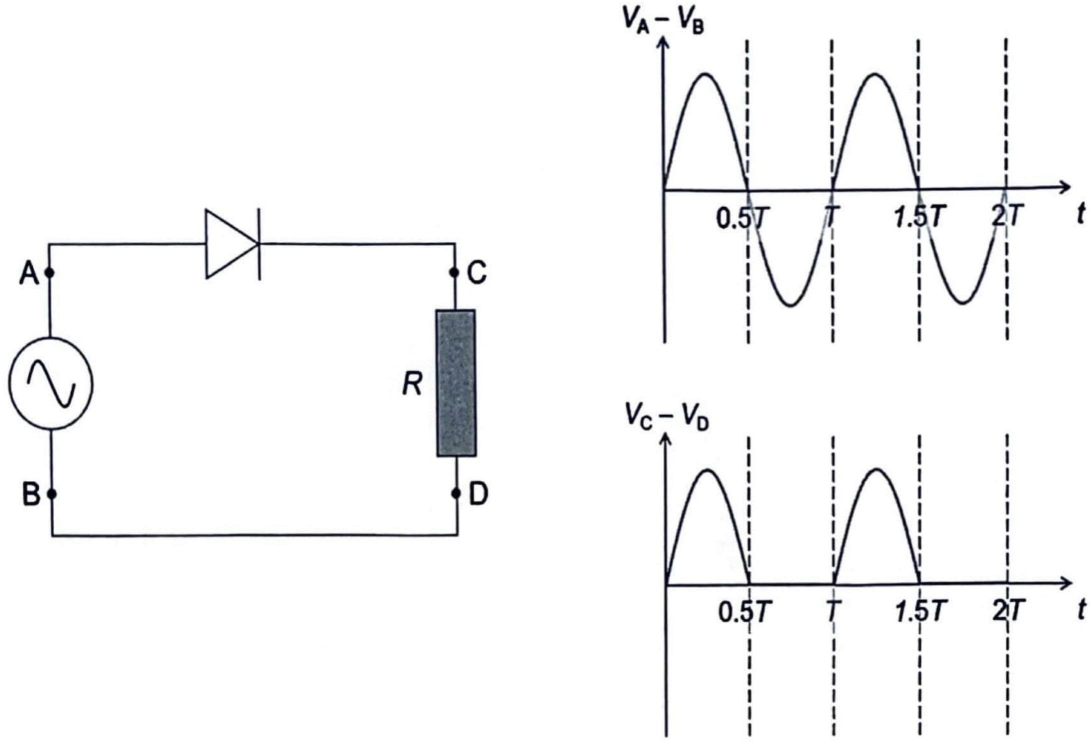


Fig. 12

First ½ cycle: The diode is in **forward bias**.

$t = 0$ to $t = 0.5T$ The current flows round the circuit as the diode acts like a closed switch. The a.c. source applies a potential difference across the load and drives a current through the load. The p.d. across R is the same as the voltage of the a.c. supply given that the diode is ideal.

$V_A > V_B$

Second½ cycle: The diode is in **reverse bias**.

$t = 0.5T$ to $t = T$ No current flows as the diode acts like an open switch. The p.d. across R is zero.

$V_A < V_B$ This is repeated for each cycle of a.c. input.

Example 5 If the peak current of the sinusoidal a.c. supply in a half-wave rectifier circuit is I_0, what is the value of the root mean square current in R? Assume the diode is ideal.

$$\text{mean} = \frac{\frac{I_0^2}{2} \times \frac{T}{2}}{T}$$

$$= \frac{I_0^2}{4}$$

$$I_{rms} = \sqrt{\frac{I_0^2}{4}} = \frac{I_0}{2}$$

Appendix Full Wave Rectification

Full Wave Bridge Rectifier

Fig.15 shows four diodes arranged in a bridge network.

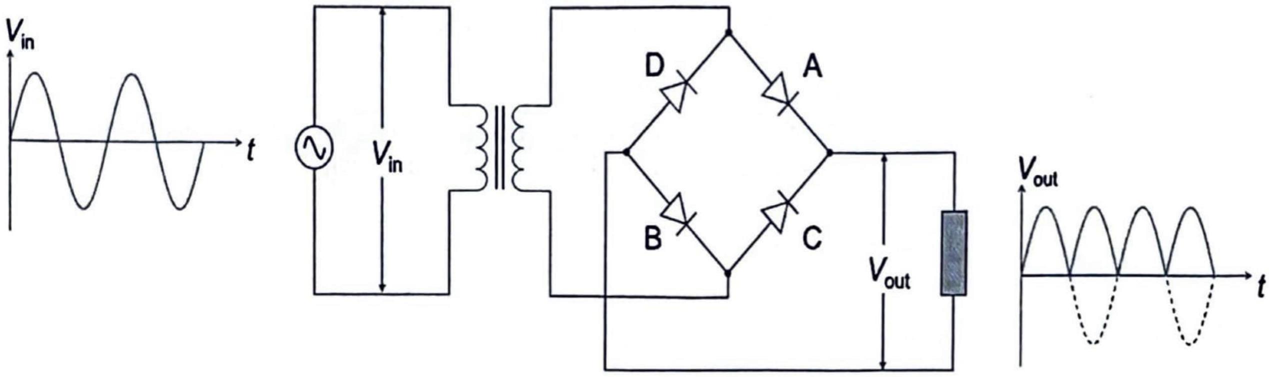
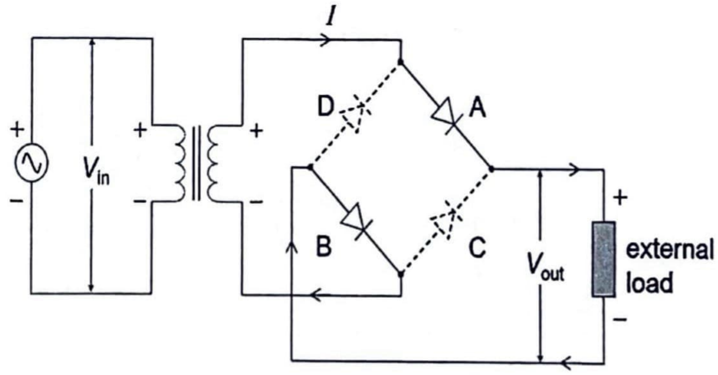


Fig. 13

The full wave bridge rectifier uses four individual rectifying diodes connected in a closed loop "bridge" configuration to produce the desired output. The main advantage of this bridge circuit is that it does not require a special centre tapped transformer, thereby reducing its size and cost.

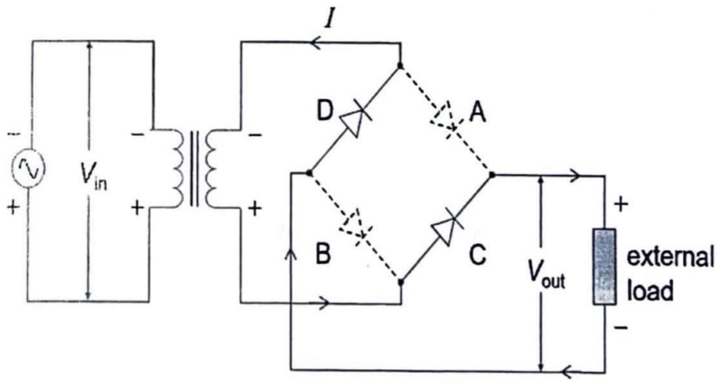
The four diodes labelled A to B are arranged in "series pairs" with only two diodes conducting current during each half cycle.

Positive half-cycle



During the positive half cycle of the supply, diodes A and B conduct in series (C and D are in reverse bias)

Negative half-cycle



During the negative half cycle of the supply, diodes C and D conduct in series (A and B are in reverse bias)

Power Loss in transformers

Power Loss in transformers

The power loss in transformers are solely electrical as transformers are static devices (no moving parts), thus there is no loss of mechanical power (e.g. due to friction of moving parts).

The causes of power loss in transformers are as listed below in detail.

Cause: **1. Joule Heating of the copper wires**

Joule heating, also known as Ohmic heating and resistive heating, is the process by which the passage of an electric current through a conductor produces heat.

For transformers, the copper wires used for the primary and secondary coils heat up when current passes through them, resulting in a power loss.

Design Feature: Thick copper wire is used particularly for coils carrying large current at low voltage.

Cause: **2. Magnetic flux leakage**

Magnetic flux leakage happens when not all of the magnetic flux is used to do useful work.

In an ideal transformer, all the flux will link with both primary and secondary windings but in reality, it is impossible to link all the flux in transformer with both primary and secondary windings. Fig. 14 illustrates the magnetic flux of a core type transformer.

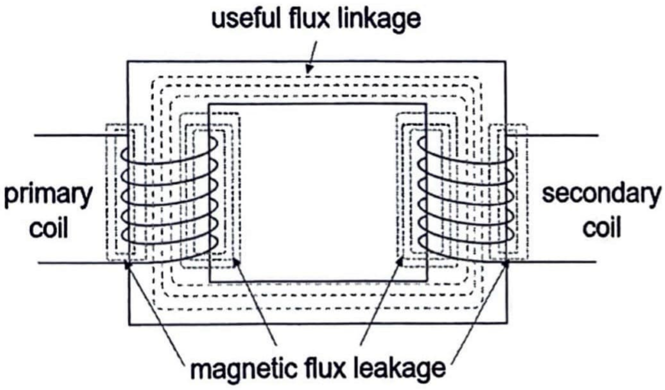


Fig. 14

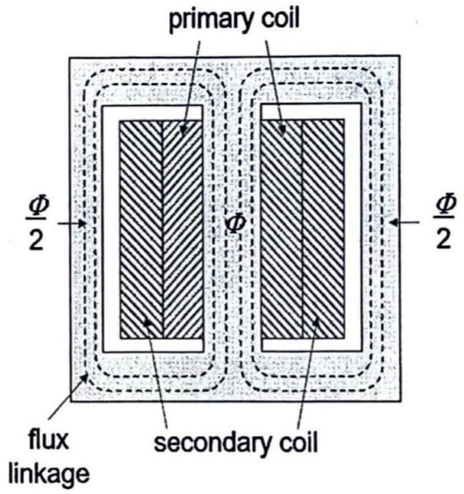


Fig. 15

Design Feature: The presence of an iron core increases the magnetic flux and reduces flux leakage between both coils.

Flux leakage can be minimised if both coils could be made to occupy the same space by placing secondary and primary in a concentric manner can solve the problem to a good extent as shown in Fig. 15.

Cause: **3. Hysteresis loss**

Hysteresis loss results in power lost as heat due to reversal of magnetization in the transformer core every time the current reverses.

This loss depends upon the volume and grade of the iron, frequency of magnetic reversals and value of flux density.

Design Feature: The core is made of soft iron, which has low hysteresis – it can be easily magnetised and demagnetised by the magnetic field caused by alternating current in the primary winding.

Cause: **4. Eddy Current losses**

The alternating current supplied to the primary coil sets up an alternating magnetizing flux. This flux links with secondary coil as well as surrounding conducting parts like the iron core of the transformer, and induces an e.m.f. in these parts.

The induced e.m.f. causes small circulating currents called as eddy currents, which in turn causes power loss due to heat.

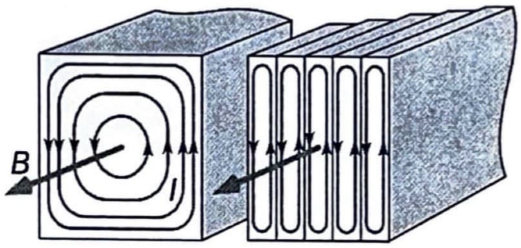
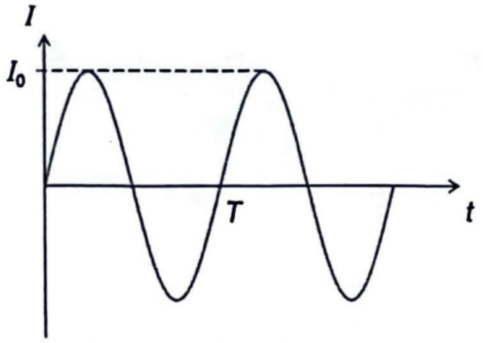
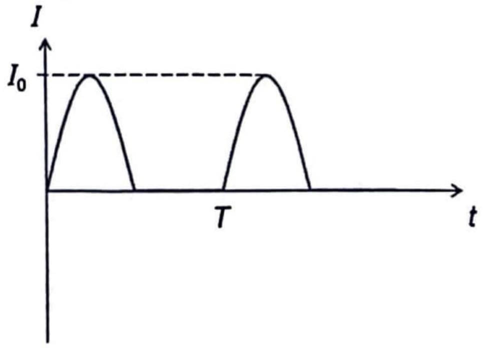
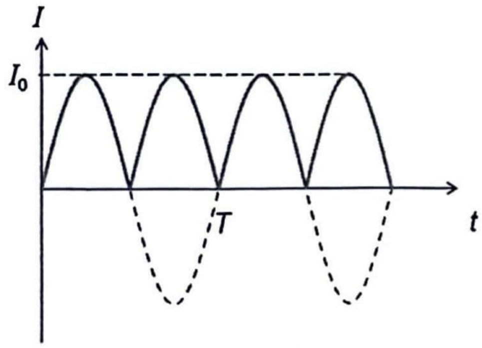
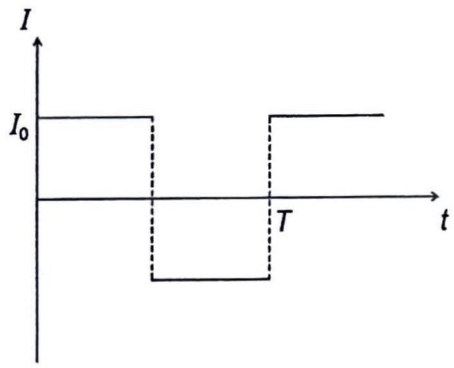


Fig. 16

Design Feature: The effect is minimised by the use of a *laminated* core, i.e., one built up of thin sheets or laminae.

The large electrical surface resistance of each lamina, due to an insulating varnish, effectively confines the eddy currents to individual laminae. The possible eddy-current paths are narrower (the effective resistance of eddy-current paths is considerably increased), and the eddy-currents and their heating effect are greatly reduced. This is illustrated in Fig. 16.

Comparing r.m.s. current and average current

Variation of current *I* with time *t*	Waveform	I_{rms} (memorise)	$\langle I \rangle$ (don't need to memorise)
	Sinuisoidal	$\frac{I_0}{\sqrt{2}}$	0
	Half-wave rectified	$\frac{I_0}{2}$	$\frac{I_0}{\pi}$
	Full-wave rectified	$\frac{I_0}{\sqrt{2}}$	$\frac{2I_0}{\pi}$
	Rectangle/ Square	I_0	0

Tutorial 18 ALTERNATING CURRENTS

Self-Check Questions

S1. What is meant by period, frequency, peak value and root-mean square current as applied to an alternating current?

S2. A sinusoidal current is represented by the equation $I = I_o \sin(\omega t)$. What is its period, frequency, peak value and root-mean square value?

S3. Deduce that the mean power in a resistive load is half the maximum power for a sinusoidal alternating current.

S4. What is the relationship between r.m.s. and peak values for a sinusoidal current?

S5. Explain the principle of operation of a simple iron-core transformer.

S6. For an ideal transformer, what is the relationship between the turns ratio of windings, the input and output potential differences, and the primary and secondary currents?

S7. Explain how a diode is used for rectification of an alternating current.

Self-Practice Questions

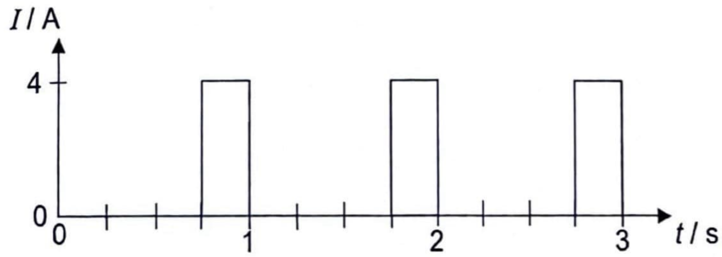
SP1. **(a)** What is meant by the r.m.s. value of an alternating current?

(b) Calculate the peak value of a 240 V mains electricity supply.

(c) A sinusoidal alternating current of r.m.s. value 5.0 A passes through a 4.0 Ω resistor. Calculate

- **(i)** the peak value of the current,
- **(ii)** the mean power in the resistor,
- **(iii)** the maximum power in the resistor.

SP2.



(a) The figure above shows the variation with time t of a periodic current I. Determine

- **(i)** the average value of the current,
- **(ii)** the root-mean-square current.

(b) The periodic current passes through a resistor, producing heat at a certain rate. State the steady current, when passed through the same resistor, would have an identical heating effect. (J83/I/10)

SP3. A mains electricity supply has a r.m.s. voltage of 240 V and a peak voltage of 340 V. When connected to this supply, a heater dissipates energy at a rate of 1000 W. The heater is then connected to a 340 V d.c. supply and its resistance remains the same. At what rate does the heater now dissipate energy? (N2000/I/21)

SP4. The primary coil of a transformer is connected to an alternating voltage supply. The secondary coil is connected across a variable resistor. Suggest ways in which you can decrease the p.d. across the secondary coil.

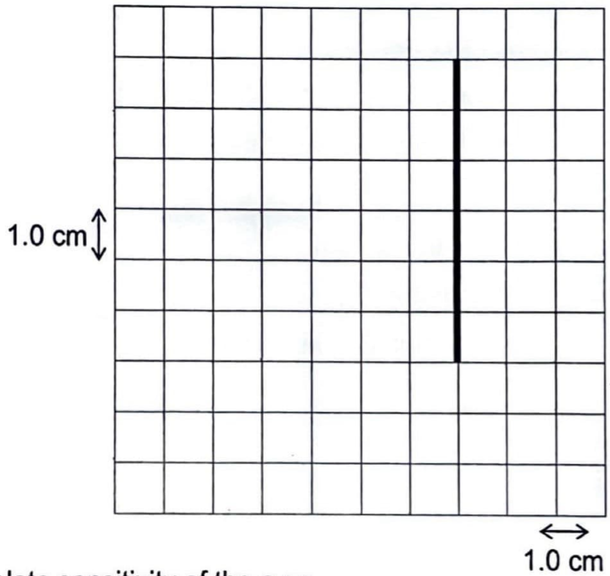
SP5. In a laboratory experiment to test a transformer, a student obtained the following results.

V_p / V	I_p / mA	N_p turns	V_s / V	I_s / mA	N_s turns
240	2.0	?	?	50	50

Assuming the transformer is 100% efficient, what are the missing entries?

(Modified from J93/I/20)

SP6. The diagram shows the display on a cathode-ray oscilloscope (c.r.o.) when a sinusoidal p.d. is applied to the Y-input. The r.m.s. value of the applied p.d. is 4.24 V. The time-base is switched off.

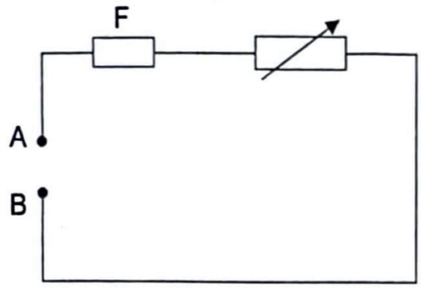


Calculate the Y-plate sensitivity of the c.r.o.

Discussion Questions

Characteristics of Alternating Currents

D1. When an a.c. supply of 240 V r.m.s. is connected to the terminals AB in the circuit shown below, the fuse F breaks the circuit if the current just exceeds 13 A r.m.s.



When the a.c. supply is replaced with a 120 V d.c. source, an identical fuse breaks the circuit if the current just exceeds

A $\frac{13}{2}$ A **B** $\frac{13}{\sqrt{2}}$ A **C** 13 A **D** $13\sqrt{2}$ A

(J88/I/23)

D2. A sinusoidal potential V_1 shown in Fig. 2a is applied across a resistor R which produces heat at a mean rate P.

Determine the mean rate of heat production when the square-wave of potential V_2 shown in Fig. 2b is applied across the same resistor.

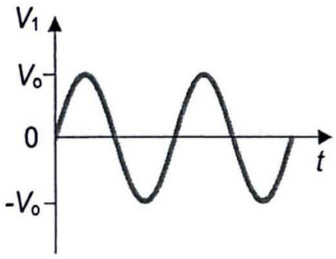


Fig. 2a

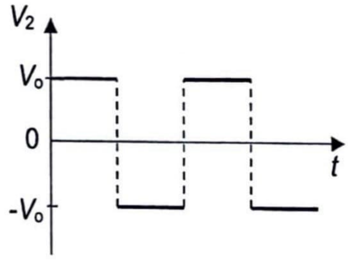
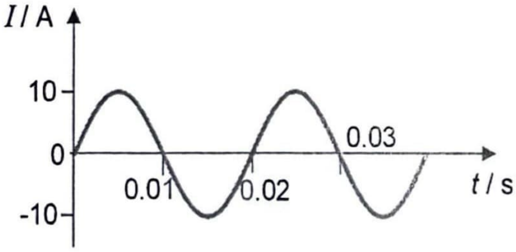


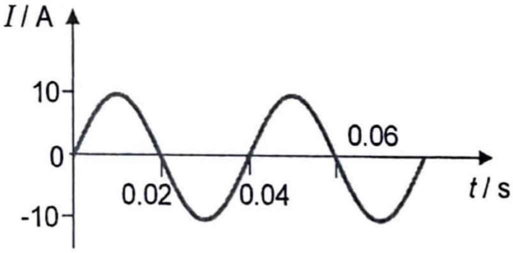
Fig. 2b

(J82/II/18) [2]

D3. **(a)** Calculate the r.m.s. current of (i) and (ii). [2]



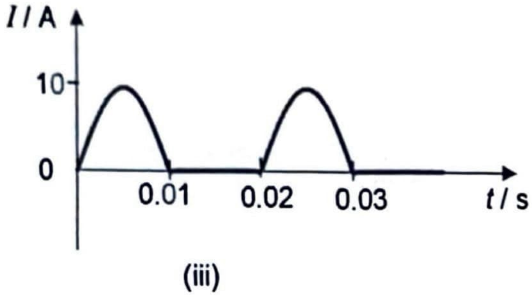
(i)

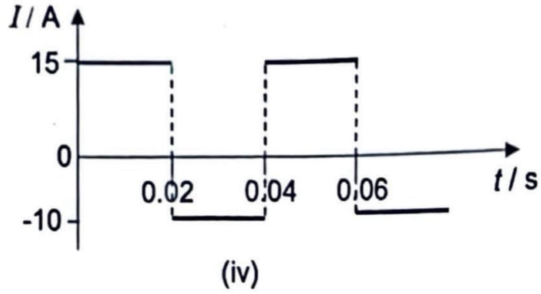


(ii)

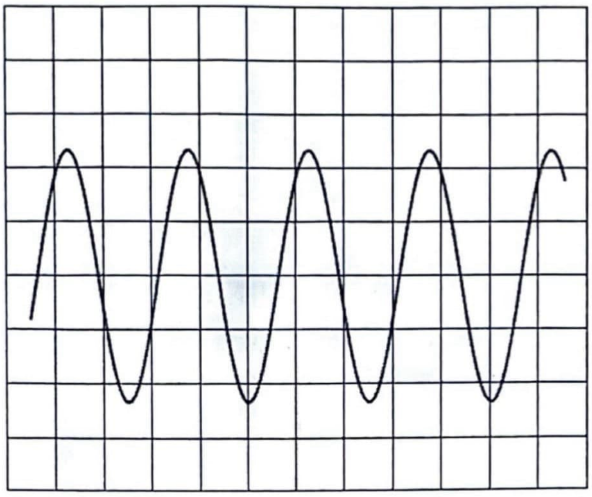
(b) What can you conclude from your answers to (i) and (ii)? [1]

(c) Determine the r.m.s. current of (iii) and (iv). [4]





D4. A cathode-ray oscilloscope (c.r.o.) is connected across the output of a transformer and the screen is as shown below. The squares on the screen have sides of one centimetre.



The Y-plate sensitivity is set at 2.0 V cm^{-1} and the timebase is set so that the horizontal deflection is 0.50 ms cm^{-1}.

For the alternating potential difference applied to the Y-plates, deduce the values of the following quantities:

(a) period, [1]

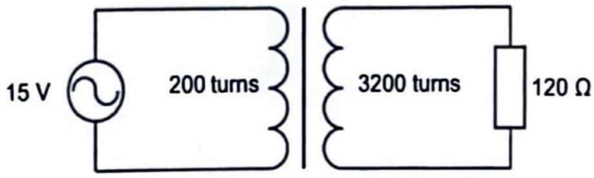
(b) frequency, [1]

(c) peak value of potential difference, [1]

(d) root-mean-square value of potential difference. [1]

The transformer

D5. The primary of an ideal transformer has 200 turns and is connected to a 15 V supply. The secondary has 3200 turns and is connected to a resistor of resistance 120 Ω, as shown in the diagram.

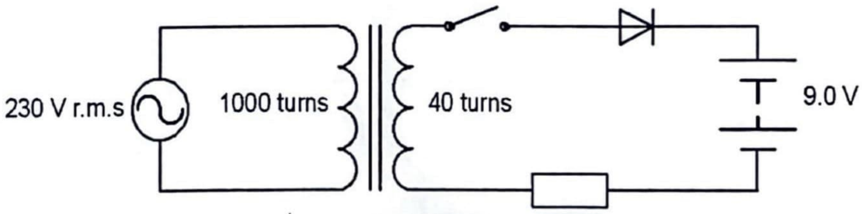


What are possible values of the secondary voltage, the secondary current and the mean power dissipated in the resistor?

	secondary voltage / V r.m.s.	secondary current / A r.m.s.	resistor power / W
A	24	0.020	4.8
B	24	0.20	48
C	240	0.50	120
D	240	2.0	480

(N2011/1/33)

D6. The primary coil of a transformer has 1000 turns and is connected to a 230 V r.m.s. supply. The secondary coil has 40 turns and may be connected, through a switch and a diode, to a 9.0 V rechargeable battery, as illustrated in the figure below.



(a) Initially the switch is open. Considering both the transformer and the diode to be ideal, calculate

(i) the r.m.s. potential difference across the secondary, [1]

(ii) the peak potential difference across the secondary. [1]

(b) The switch is now closed so that the battery is being recharged.

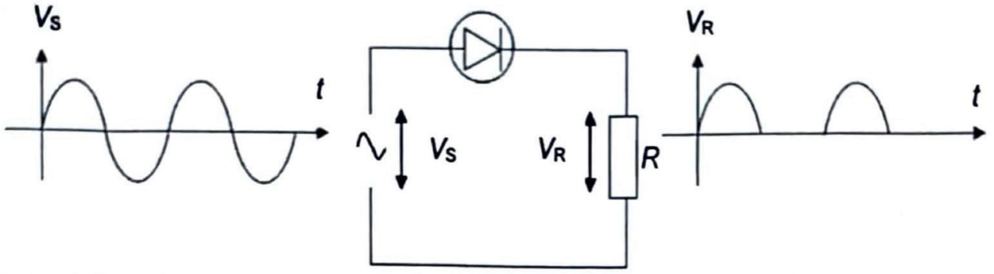
(i) Suggest why the diode is necessary in the secondary circuit. [1]

(ii) Suggest why the resistor is necessary in the circuit. [1]

(J2000/II/5)

Rectification

D7. An alternating supply is applied to a resistor R with a diode connected in series with it. The time *t* variation of the supply voltage V_S and the p.d. V_R across the resistor are shown in the diagram.



Which of the following statements is *incorrect*?

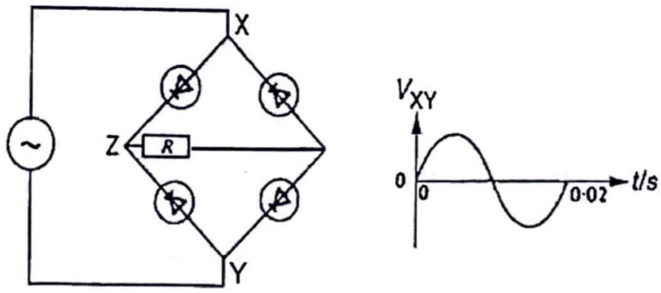
A Half-wave rectification takes place so that the current supplied to the resistor is one-directional.

B The period of the V_R-t graph is half of that of the V_S-t graph.

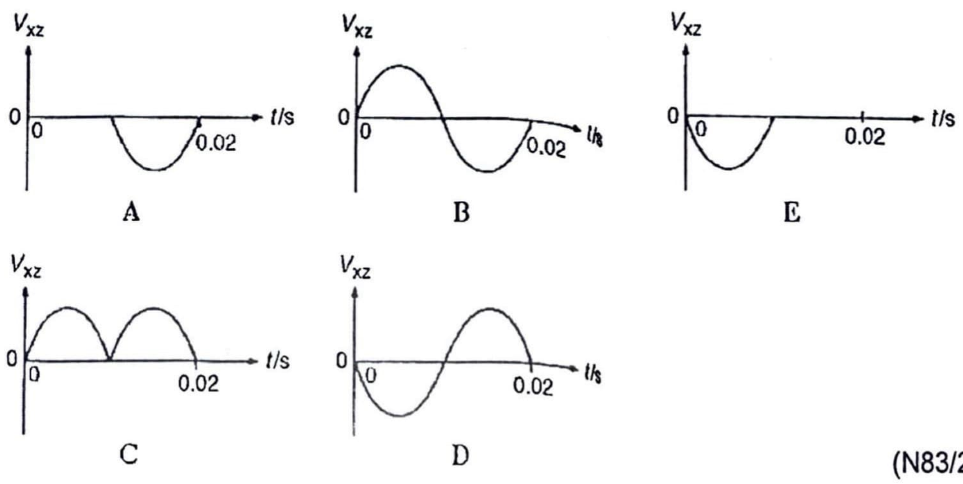
C The time intervals corresponding to zero voltage across resistor R in the graph represent the stages when the diode is reverse-biased.

D Root-mean-square voltage across resistor R is half of the peak voltage applied across it.

D8. The circuit represents a bridge rectifier arrangement. The graph below shows the variation over one cycle of the potential of X with respect to Y.



Which one of the graphs best represents the corresponding variation of the potential of X with respect to Z?



(N83/2/24)

Challenging Questions

C1. Two air-core solenoids P and Q which are identical in length and area of cross-section, have 10 and 50 turns respectively. They are placed near each other.

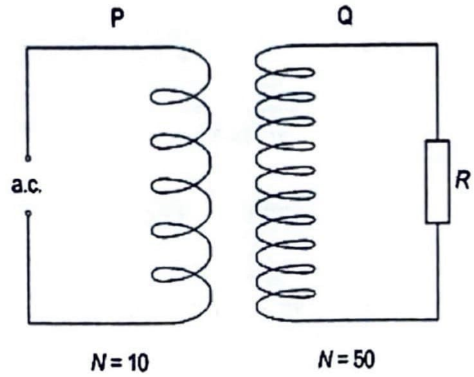


Fig 23.1

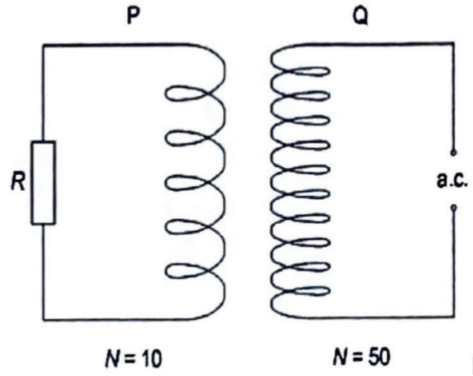


Fig 23.2

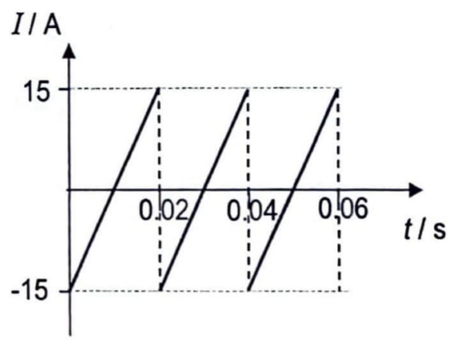
In Fig. 23.1, when the current in P changes at a rate of 5.0 A s^{-1}, an e.m.f. of 2.0 mV is induced in Q. The current in P is then switched off. In Fig. 23.2, a current which is changing at 2.0 A s^{-1} is then sent through Q. What e.m.f. will be induced in P?

[Magnetic flux density in a solenoid, $B = \mu_0 n I$ where μ_0 is the permeability of free space, n is the number of turns per unit length and I is the current in the solenoid.]

*Knowledge of mutual inductance is required in this question.

(RJC 2004 Prelim P1 Q23)

C2. Determine the r.m.s. current shown in the figure.



Ans:

SP1.	339 V, 7.07 A, 100 W, 200 W
SP2.	1.0 A, 2.0 A, 2.0 A
SP3.	2000 W
SP5.	1250, 9.6 V
SP6.	2.0 V cm^{-1}
D2.	2*P*
D3.	7.07 A, 5.00 A, 12.7 A
D4.	1.25 ms, 800 Hz, 4.7 V, 3.3 V
D6.	9.2 V, 13.0 V
C1:	8.0×10^{-4} V
C2:	8.66 A

TUTORIAL 18 SUGGESTED SOLUTIONS

SELF-CHECK QUESTIONS:

S1.

Term	Definition
Period, T	of an alternating current is the time taken for one complete cycle.
Frequency, f	of an alternating current is the number of complete cycles per unit time.
Peak current, I_0	of an alternating current is the amplitude of the current.
Root-mean square current, $I_{r.m.s.}$	of the alternating current is that value of direct current that would produce thermal energy at the same rate in a resistor.

S2.

Period $T = \dfrac{2\pi}{\omega}$

Frequency $f = \dfrac{1}{T} = \dfrac{\omega}{2\pi}$

Peak value is I_0

Root-mean square value $I_{r.m.s.} = \dfrac{I_0}{\sqrt{2}}$

S3. The instantaneous power delivered to R at time t is

$$P = IV_R = (I_0 \sin\omega t)(V_0 \sin\omega t) = I_0 V_0 \sin^2 \omega t$$

The mean power is given by

$$\langle P \rangle = \langle I_0 V_0 \sin^2 \omega t \rangle = I_0 V_0 \langle \sin^2 \omega t \rangle = I_0 V_0 \left(\frac{1}{2}\right) = \frac{P_0}{2}$$

i.e. the mean power in a resistive load is half the maximum power for a sinusoidal a.c.

*Prove for yourself that $\langle \sin^2 \omega t \rangle = \dfrac{1}{2}$

S4. $I_{rms} = \dfrac{I_0}{\sqrt{2}}$

S5. The principle of operation of a simple iron-core transformer:

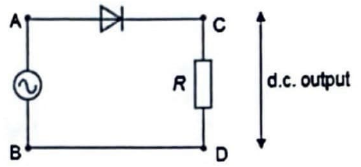
- a transformer works by electromagnetic induction
- the a.c. source causes an alternating current to flow in the primary coil, which sets up an alternating magnetic flux in the iron core
- this induces an alternating e.m.f. in the secondary coil, in accordance with Faraday's law of electromagnetic induction
- the induced e.m.f. in the secondary coil give rise to an alternating current which delivers energy to the device to which the secondary is connected
- all currents and e.m.f.s have the same frequency as the a.c. source

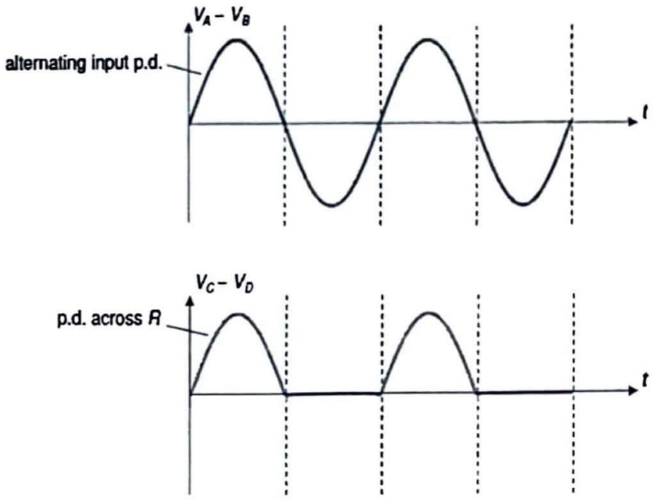
S6. For an ideal transformer:

$$\frac{N_s}{N_p} = \frac{V_s}{V_p} = \frac{I_p}{I_s}$$

S7.

1. Connect a diode in circuit as shown below.
2. Diode allows current to flow during one-half of a cycle when it is forward biased.
3. When it is reverse bias, no current flows through *R*.





SELF-PRACTICE QUESTIONS:

SP1. **(a)** The r.m.s. value of the alternating current is the value of the direct current that would produce thermal energy at the same rate in a resistor.

(b) For a 240 V mains electricity supply, $V_{rms} = 240$ V.

The peak value of the 240 V mains, $V_0 = \sqrt{2}\ V_{rms} = \sqrt{2}(240\text{ V}) = 339\text{ V}$.

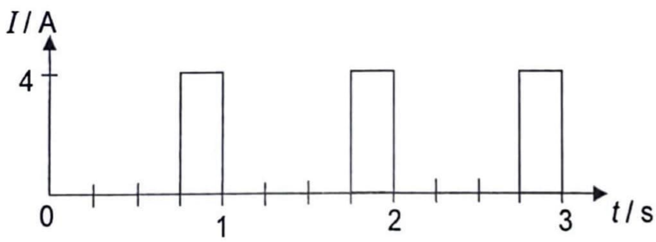
(c) A sinusoidal alternating current of r.m.s. value 5.0 A passes through a 4.0 Ω resistor.

(i) Peak value of the current $I_0 = \sqrt{2}\ I_{rms} = \sqrt{2}(5.0\text{ A}) = 7.07\text{ A}$

(ii) Mean power in the resistor $\langle P \rangle = I_{rms}{}^2 R = 5.0^2 \times 4.0 = 100\text{ W}$

(iii) Maximum power $P_0 = 2\langle P \rangle = 2 \times 100 = 200\text{ W}$

SP2.



(a)(i) Average value of the current $= \dfrac{0 \times 0.75 + 4 \times 0.25}{1} = 1.0\text{ A}$

(ii) Root-mean-square current $= \sqrt{\dfrac{0 \times 0.75 + 4^2 \times 0.25}{1}} = 2.0\text{ A}$

(b) A steady current of 2.0 A will produce an identical heating effect as the alternating current. This is according to the definition of r.m.s. value of an alternating current.

SP3. $\langle P\rangle = \dfrac{V_{rms}^{\ 2}}{R} \Rightarrow R = \dfrac{V_{rms}^{\ 2}}{\langle P\rangle} = \dfrac{240^2}{1000}$

Power dissipated in 340 V d.c. source $= \dfrac{V_{d.c.}^{\ 2}}{R} = \dfrac{340^2}{\left(\dfrac{240^2}{1000}\right)} = 2000$ W

<u>OR</u> we can treat 340 V d.c. as V_o, then $P_o = 2\langle P\rangle = 2000$ W

SP4. Since $\dfrac{V_s}{V_p} = \dfrac{N_s}{N_p}$, to decrease the p.d. V_s across the secondary coil, we can

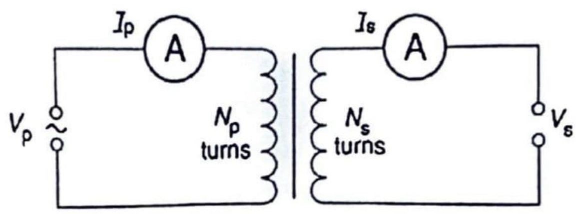
- increase N_p, the number of turns in the primary coil
- decrease N_s, the number of turns in the secondary coil
- decrease V_p, the supply voltage that is connected to the primary coil

*changing the resistance of the variable resistor will not affect the p.d. across the secondary coil.

SP5. $\dfrac{V_s}{V_p} = \dfrac{N_s}{N_p} = \dfrac{I_p}{I_s} = \dfrac{2.0}{50}$

$N_p = \dfrac{50}{2.0} \times N_s = \dfrac{50}{2.0} \times 50 = 1250$

$V_s = \dfrac{2.0}{50} \times V_p = \dfrac{2.0}{50} \times 240 = 9.6$ V



SP6. For a sinusoidal p.d.,

$V_0 = \sqrt{2}\ V_{r.m.s.} = \sqrt{2}(4.24\text{ V}) = 6.0\text{ V}$

From the diagram, peak to peak p.d., or $2V_o$ has a span of 6.0 cm

Hence V_o has a span of 3.0 cm

$\therefore$ Y-plate sensitivity $= \dfrac{6.0\text{ V}}{3.0\text{ cm}} = 2.0\text{ V cm}^{-1}$