CATHOLIC JUNIOR COLLEGE
General Certificate of Education Advanced Level
Higher 2
JC2 Preliminary Examination

MATHEMATICS

9740/02

Paper 2

30 August 2016

3 hours

Additional Materials: List of Formulae (MF15)

Name: ______________________________ **Class:** ________________

READ THESE INSTRUCTIONS FIRST

Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, arrange your answers in NUMERICAL ORDER.

The number of marks is given in brackets [ ] at the end of each question or part question.

Section A: Pure Mathematics [40 marks]

1 **(i)** Prove that the substitution $u = x^2 + y^2$ reduces the differential equation $y\frac{\mathrm{d}y}{\mathrm{d}x} + x = \sqrt{x^2 + y^2}$ to

$$\frac{\mathrm{d}u}{\mathrm{d}x} = 2\sqrt{u}.$$

Hence, show that the general solution of the differential equation $y\frac{\mathrm{d}y}{\mathrm{d}x} + x = \sqrt{x^2 + y^2}$ is given by $\sqrt{x^2 + y^2} = x + D$, where D is an arbitrary constant. **[4]**

(ii) The result in part **(i)** represents a family of curves. On a single diagram, sketch a non-linear member of the family which passes through the point $(-2, 0)$.

You should state the equation of the graph and axial intercepts clearly on the diagram. **[3]**

(iii) State an equation of the line of symmetry for the curve in part **(ii)**. **[1]**

2 The three distinct roots of the equation $x^3 - 1 = 0$ are denoted by $1, \omega$ and ω^2.

(a) Without first finding ω explicitly, show that $1 + \omega + \omega^2 = 0$. **[2]**

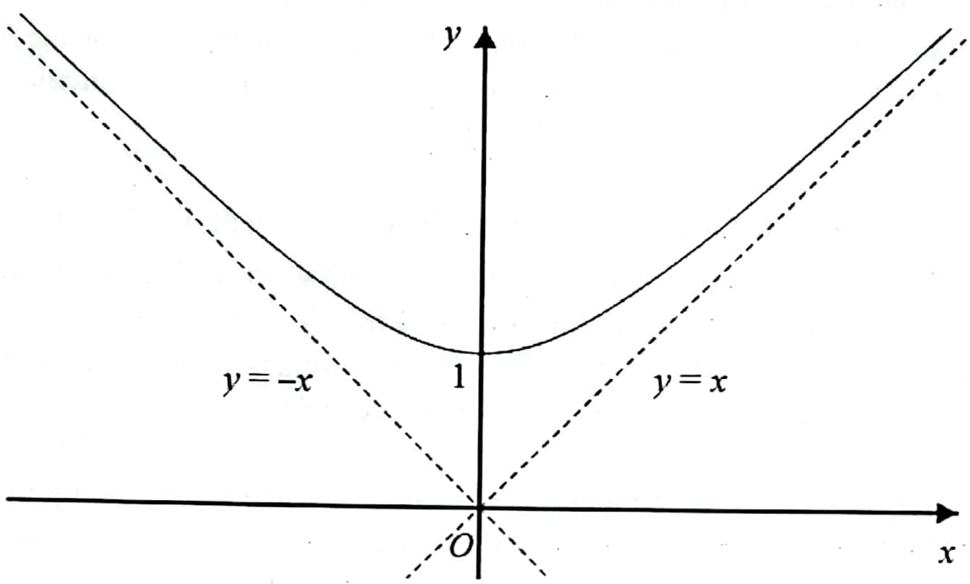
(b) Given now that $0 < \arg(\omega) < \pi$, sketch, on a single Argand diagram, the loci given by

(i) $|z - \omega| = |\omega|$ and **[3]**

(ii) $\arg(z + 1) = \pi + \arg(\omega^2)$. **[2]**

Hence, find the complex number that satisfies both loci, expressing your answer exactly in the form $a + ib$, where a and b are real numbers. **[2]**

3 The diagram shows the graph of curve C represented by $y = \mathrm{f}(x)$, with oblique asymptotes $y = x$ and $y = -x$.



(a) On a separate diagram, sketch a graph of $y = \mathrm{f}'(x)$, clearly indicating the equation(s) of the asymptote(s) and axial-intercept(s). **[2]**

(b) The above curve C is represented by the parametric equations

$$x = \tan\theta,\ y = \sec\theta, \text{ where } -\frac{\pi}{2} < \theta < \frac{\pi}{2}.$$

(i) Show that the normal to the curve at point P, with coordinates $(\tan\theta,\ \sec\theta)$, for $0 < \theta < \frac{\pi}{2}$, is given by $y = -x\ \operatorname{cosec}\theta + 2\sec\theta$. **[2]**

(ii) The normal to the curve at point P intersects the x-axis at point N. Find the coordinates of the mid-point M of PN, in terms of θ. Hence find a cartesian equation of the locus of M, as θ varies. **[4]**

(iii) Taking O as the origin, show that the area of triangle OPN is $\tan\theta\sec\theta$.

Point P moves along the curve such that the rate of change of its parameter θ with respect to time t is given by $\frac{\mathrm{d}\theta}{\mathrm{d}t} = \cos\theta$. Find the exact rate of change of the area of triangle OPN when $\theta = \frac{\pi}{6}$. **[4]**

4 Adam and Gregory signed up for a marathon. In preparation for this marathon, Adam and Gregory each planned a 15-week personalised training programme. Adam runs 2.4 km on the first day of Week 1, and on the first day of each subsequent week, the distance covered is increased by 20% of the previous week. Gregory also runs 2.4 km on the first day of Week 1, but on the first day of each subsequent week, the distance covered is increased by d km, where d is a constant. Assume Adam and Gregory only run on the first day of each week.

(i) Find, in terms of d, the total distance covered by Gregory in these 15 weeks. [2]

(ii) Adam targets to cover a total distance of 170 km in these 15 weeks. Can Adam achieve this target? You must show sufficient working to justify your answer. [2]

(iii) It is given that Adam covers a longer distance than Gregory on the first day of the 15th week. Find the maximum value of d, correct to 2 decimal places.
Using this value of d, show that the difference in the distance covered by Adam and Gregory for their 15th week training is 0.134 km correct to 3 significant figures. [4]

(iv) Due to unforeseen circumstances, Adam has to end his training programme early. In order for Adam to cover a total distance of 170 km by the end of the 13 weeks, the distance covered has to be increased by x % of the previous week on each subsequent week from Week 1. Find x. [3]

Section B: Statistics [60 marks]

5 A bag contains four red and eight blue balls of which two of the red balls and six of the blue balls have the number "0" printed on them. The remaining balls have the number "1" printed on them. Three balls are randomly drawn from the bag without replacement.

(i) Show that the probability that at least one blue ball is drawn is $\frac{54}{55}$. [1]

Find the probability that

(ii) at least one ball of each colour is drawn, [2]

(iii) the sum of the numbers on the balls drawn is at least two. [3]

6 Packets of a particular brand of potato chips are delivered to a supermarket in boxes of 60. On average, 1.8 packets in a box are underweight. The number of underweight packets from a randomly chosen box is the random variable X.

Assume that X has a binomial distribution.

(i) Use a suitable approximation to find the probability that two randomly chosen boxes of potato chips contain more than 6 packets of underweight potato chips. State the parameter(s) of the distribution that you use. [4]

A batch of 50 boxes of potato chips is delivered to the supermarket.

(ii) Use a suitable approximation to find the probability that the mean number of underweight packets per box is more than 2. [3]

7 A group of 9 friends, including Albert and Ben, are having dinner at Albert's house. They sit in two groups: a row of 4 on a couch and a group of 5 at a round dining table with 5 identical seats.

Find the number of ways they can sit if

(i) there are no restrictions, **[2]**

(ii) Albert and Ben sit beside each other, **[3]**

(iii) Albert and Ben both sit on the couch or both sit at the round table, but they do not sit beside each other. **[3]**

8 A factory manufactures a certain product for sale. The following table gives the quantity of product manufactured, x units in thousands, and its corresponding cost of production, y dollars in thousands. The data is recorded during different months of a certain year.

Quantity of product, x	2.0	2.4	3.0	3.8	4.8	6.0	7.2	8.2	9.4
Cost of production, y	10	19	35	47	58	35	78	80	81

(i) Draw a scatter diagram for the data. **[1]**

One of the values of y appears to be incorrect.

(ii) Indicate the corresponding point in your diagram by labelling it P. **[1]**

Remove P from the set of data.

(iii) By using the scatter diagram for the remaining points, explain whether $y = a + bx$ or $y = a + b \ln x$ is the better model for the relationship between x and y. **[1]**

(iv) Using the better model chosen in part **(iii)**, find the product moment correlation coefficient and the equation of a suitable regression line.
Explain what happens to the product moment correlation coefficient and the equation of the regression line if the factory decides to include a fixed cost of M thousand dollars for purchasing a packing machine to the cost of production, y. **[4]**

(v) Use the regression line found in part **(iv)** to estimate the cost of production when the quantity produced is 6000 units and comment on its reliability. **[2]**

9 The finishing times in a 10km race with a large number of runners follow a normal distribution. After 40 minutes, 10% of the runners have completed the race. After one hour, 35% of the runners have yet to complete the race. The first 20% of runners who finish the race receive a medal.

(i) Show that, correct to 1 decimal place, the runners have running times with mean 55.4 minutes and standard deviation 12.0 minutes. **[2]**

(ii) Find the maximum time a runner can take to finish the race in order to receive a medal. **[2]**

A random sample of 12 runners is selected.

(iii) Find the probability that more than four runners receive a medal. **[2]**

(iv) Given that none of the runners receives a medal, find the probability that the slowest runner completes the race in under one hour. **[3]**

10 (a) A Physical Education teacher wants to plan a volleyball training programme for all students in a secondary school, where each student has exactly one CCA. In order to check on the current fitness level of students in the school, he selects a sample of students by choosing the Captains of every sports team and the Presidents of every Club and Society in the school.

(i) Explain briefly why this may not provide a representative random sample of the student population. **[1]**

(ii) Name a more appropriate sampling method which would provide a representative random sample and explain how it can be carried out in this context. **[3]**

(b) The vertical jump heights of players from a volleyball team are normally distributed with mean 40 cm. The coach claims that a particular training regime is effective in improving the players' jump heights. After the regime is implemented for a period of time, a random sample of 7 players is taken and their jump heights are recorded.

The sample mean is 42.1 cm and the sample standard deviation is k cm.
A test is to be carried out at the 10% level of significance to determine whether the training regime has been effective.

(i) State appropriate hypotheses for the test. **[1]**

(ii) Find the set of values of k for which the result of the test would be to reject the null hypothesis. **[3]**

(iii) State the conclusion of the test in the case where the sample variance is 15. **[2]**

11 The average number of calls per hour received by telephone operators at the Call Centre of bank ECBC is being reviewed.

(i) State, in context, two assumptions that need to be made for the number of calls received by a telephone operator to be well modelled by a Poisson distribution. **[2]**

The Call Centre has only three telephone operators at any point in time. One handles calls pertaining to credit card queries, another handles calls pertaining to business banking queries while the last operator handles calls pertaining to personal banking queries, with the numbers of calls received in one hour assumed to have the independent distributions $\text{Po}(\mu)$, $\text{Po}(6)$ and $\text{Po}(7)$ respectively.

(ii) It is given that the probability of receiving two calls pertaining to credit card queries within an hour is eight times that of receiving two calls pertaining to credit card queries within four hours.

Find the exact value of μ, expressing your answer in the form $\frac{a}{b}\ln 2$ where a and b are two positive integers to be found. **[3]**

(iii) On a certain day, the Call Centre receives more than 50 calls from 1200 to 1400 hours. Find the probability that there are no calls pertaining to credit card queries during this period. **[3]**

(iv) Using suitable approximations, find the probability that there are more calls pertaining to business banking queries than personal banking queries within a two-hour period. **[3]**

— THE END —