1. $\frac{4^x}{4} = \sqrt{2}\left(2^x\right)$

$4^{x-1} = 2^{\frac{1}{2}}\left(2^x\right)$

$2^{2x-2} = 2^{\frac{1}{2}}\left(2^x\right)$

$2x - 2 = \frac{1}{2} + x$

$x = 2\frac{1}{2}$

2. (i) $x^2 = (k-3)x - 2$

$x^2 - (k-3)x + 2 = 0$

Sum of roots = $\alpha + 1 + \alpha + 4$

$\Rightarrow 2\alpha + 5 = k - 3$

$\alpha = \frac{k-8}{2}$ -----------------(1)

Product of roots = $(\alpha + 1)(\alpha + 4)$

$\Rightarrow \alpha^2 + 5\alpha + 4 = 2$ --------(2)

$\left(\frac{k-8}{2}\right)^2 + 5\left(\frac{k-8}{2}\right) + 4 = 2$

$k^2 - 6k - 8 = 0$

$k = 3 + \sqrt{17}$

4. (i) $200 = 22 + 265e^{-k(5)}$

$178 = 265e^{-5k}$

$e^{-k(5)} = \frac{178}{265}$

$-5k = \ln\frac{178}{265}$

$k = 0.0796$ **[A1]**

(ii) $T = 22 + 265e^{-0.07959(20)}$

$= 75.9^{o}\text{C}$ **[A1]**

(iii) $T = 22$

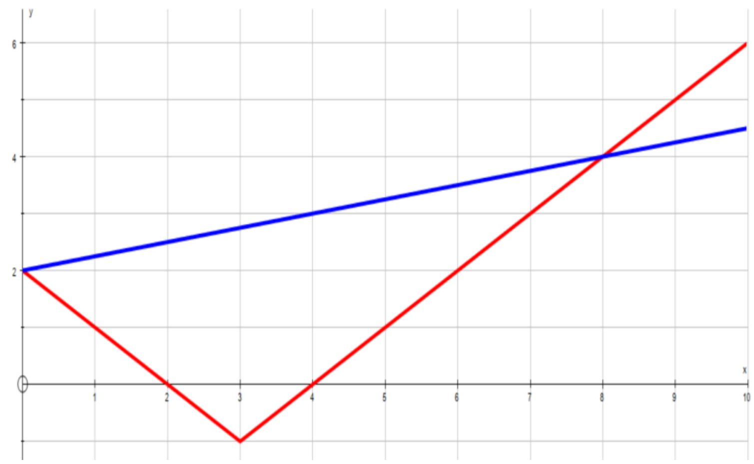
6(i) $|3-x| - 1 = \frac{1}{4}x + 2$

$|3-x| = \frac{1}{4}x + 3$

$3 - x = \frac{1}{4}x + 3$ **or** $3 - x = -\frac{1}{4}x - 3$

$x = 0$ $x = 8$

6(ii)



Graph of $y = |3-x| - 1$ (Red line)

Graph of $y = \frac{1}{4}x + 2$ (Blue line)

6(iii) $4|3-x| \le x + 12$

$|3-x| \le \frac{x}{4} + 3$

$|3-x| - 1 \le \frac{x}{4} + 2$

From the graph, $0 \le x \le 8$.

7(i) Gradient of *SR* = $\frac{10-4}{4-0} = \frac{3}{2}$

Equation of *SR*: $y = \frac{3}{2}x + 4$

7(ii) Gradient of SP = $-\frac{2}{3}$

Equation of SP: $y=-\frac{2}{3}x+4$

When y = 0, x = 6 $\Rightarrow P(6,0)$

7**(iii)** $41 = \frac{1}{2}\begin{vmatrix} 0 & 6 & q & 4 & 0 \\ 4 & 0 & 0 & 10 & 4 \end{vmatrix}$

$41=\frac{10q+16-24}{2}$

$q=9$

Q (9, 0)

7(iv) $\frac{x-4}{9-x}=\frac{2}{3}$

$3x-12=18-2x$

$5x=30$

$x=6$

$\frac{10-y}{y-0}=\frac{2}{3}$

$30-3y=2y$

$y=6$

The coordinates of U is $(6,6)$.

7(v) Gradient of SR = Gradient of $\perp$ bisector = $\frac{3}{2}$.

Mid-point = $\left(\frac{0+6}{2},\frac{4+0}{2}\right)=(3,\ 2)$

Equation of perpendicular bisector

$y-2=\frac{3}{2}(x-3)$

$y=\frac{3}{2}x-\frac{5}{2}$

When x = 6, $y=\frac{3}{2}(6)-\frac{5}{2}=6\frac{1}{2}$

Hence, the point U does not lie on the $\perp$ bisector.

[Alternatively, students can compare distance of U from S and P. If they are equal, Then U is on the perpendicular bisector.]

8(a) $\tan^2 x = \dfrac{3}{\cos x} - 3$

$\sec^2 x - 1 = 3\sec x - 3$

$\sec^2 x - 3\sec x + 2 = 0$

$(\sec x - 1)(\sec x - 2) = 0$

$\cos x = 1$ **or** $\cos x = \dfrac{1}{2}$

$x = 0^o, 60^o, 300^o \; and \; 360^o$

8(bi) LHS = $\dfrac{\sin x}{\sec x - 1} + \dfrac{\sin x}{\sec x + 1}$

= $\dfrac{\sin x(\sec x + 1) + \sin x(\sec x - 1)}{\sec^2 x - 1}$

= $\dfrac{2\sin x \sec x}{\tan^2 x}$

= $\dfrac{2\tan x}{\tan^2 x}$

= 2 Cot x

8(bii) $\dfrac{\sin 2x(\sec 2x + 1) + \sin 2x(\sec 2x - 1)}{2\sqrt{3}} = -1$

$\sin 2x(\sec 2x + 1) + \sin 2x(\sec 2x - 1) = -2\sqrt{3}$

$\sin 2x(\sec 2x + 1) + \sin 2x(\sec 2x - 1) = 2\cot 2x(\sec^2 2x - 1)$

$= 2\cot 2x(\tan^2 2x)$

$= 2\tan 2x$

$2\tan 2x = -2\sqrt{3}$

$\tan 2x = -\sqrt{3}$

$ref \angle of \; 2x = \dfrac{\pi}{3}$

$2x = -\dfrac{\pi}{3}, -\dfrac{4\pi}{3}, \dfrac{2\pi}{3}$ and $\dfrac{5\pi}{3}$

$x = -\dfrac{\pi}{6}, -\dfrac{2\pi}{3}, \dfrac{\pi}{3}$ and $\dfrac{5\pi}{6}$

Otherwise,

$$\frac{\sin 2x(\sec 2x+1)+\sin 2x(\sec 2x-1)}{2\sqrt{3}}=-1$$

$$\sin 2x(\sec 2x+1)+\sin 2x(\sec 2x-1)=-2\sqrt{3}$$

$$\tan 2x+\sin 2x+\tan 2x-\sin 2x=-2\sqrt{3}$$

$$2\tan 2x=-2\sqrt{3}$$

$$\tan 2x=-\sqrt{3}$$

$$ref\ \angle of\ 2x=\frac{\pi}{3}$$

$$2x = -\frac{\pi}{3}, -\frac{4\pi}{3}, \frac{2\pi}{3} \text{ and } \frac{5\pi}{3}$$

$$x = -\frac{\pi}{6}, -\frac{2\pi}{3}, \frac{\pi}{3} \text{ and } \frac{5\pi}{6}$$

Solutions:

9(i) Drawing of graph:
Creation of Table
Correct point**s**
Line of best fit

9(iia) From the graph,

$$a=\frac{12.40-2.00}{1.50-3.60} = -4.95$$

$b = 20.00$

9(iib) Abnormal reading is $y = 9.2$ when $x = 1.4$

From the graph, $xy = 10.00$

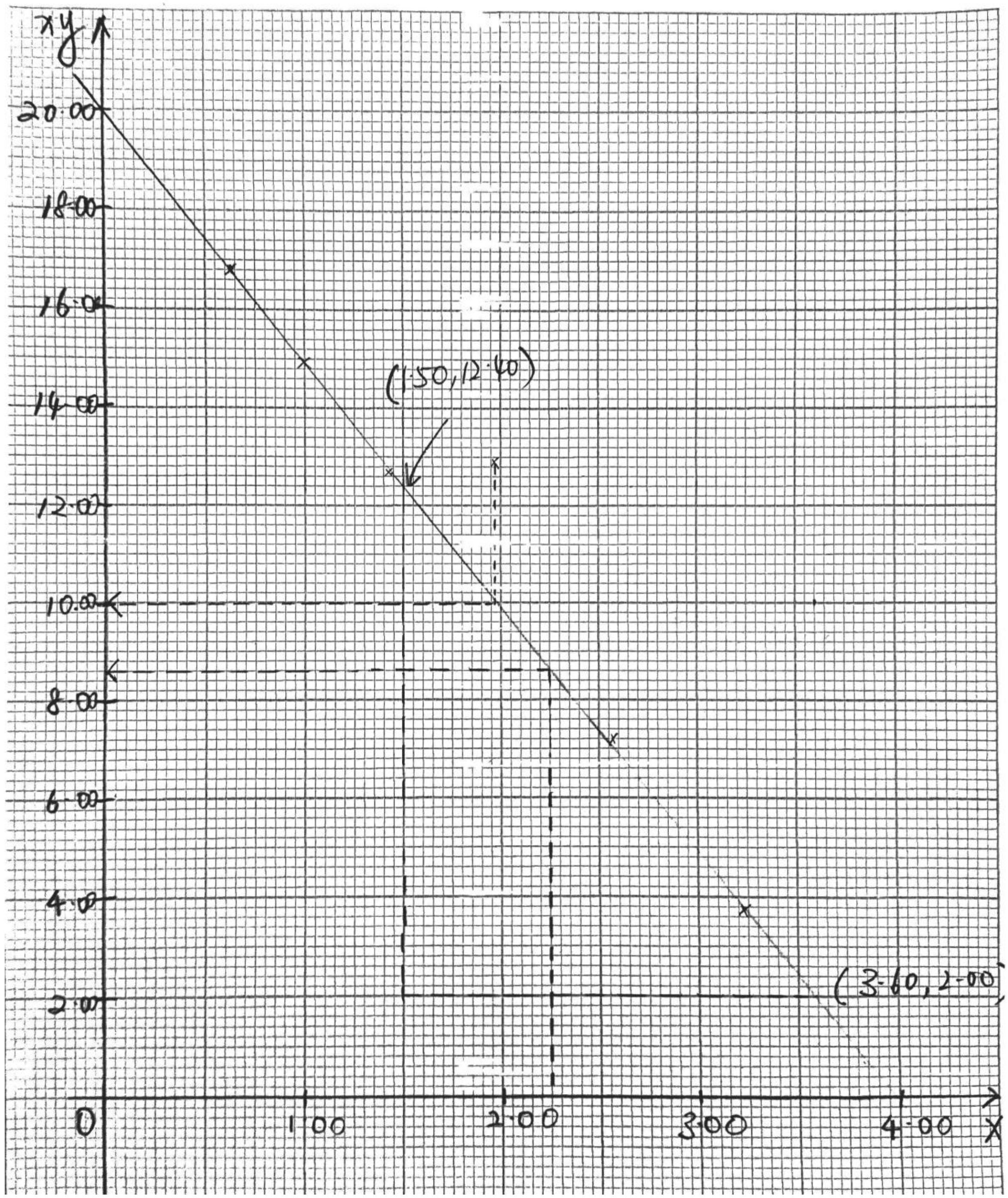
$1.4y = 10.00$

$y = 7.14$

9(iic) When $x = 1.5$, $x^2 = 2.25$,

From the graph, $xy = 8.60$

$$y=\frac{8.60}{1.50} = 5.73$$


xy
20.00
18.00
16.00
14.00
12.00
10.00
8.00
6.00
4.00
2.00
0
1.00
2.00
3.00
4.00
x
(1.50, 12.40)
(3.60, 2.00)