Anglo-Chinese School
(Independent)

FINAL EXAMINATIONS 2020

YEAR 3 INTEGRATED PROGRAMME
ADVANCED MATHEMATICS
PAPER 1

Monday **5 October 2020** **1 hour and 30 minutes**

Additional Materials:
Writing paper (6 sheets)

INSTRUCTIONS TO STUDENTS

Do not open this examination paper until instructed to do so.
A calculator is required for this paper.
Answer all questions on the answer sheets provided.
Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

INFORMATION FOR STUDENTS
The maximum mark for this paper is 80.

This question paper consists of 4 printed pages.

[Turn over]

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. *[Maximum marks: 8]*

It is given that $f(x) = 2x^3 + Ax^2 + Bx + 1$ is exactly divisible by $(x-1)$ and leaves a remainder of 9 when divided by $(x-2)$.

(a) Find the value of *A* and of *B*. [4 marks]

(b) Hence, prove that the equation $f(x) = 1 - 4x$ has only one real root. [4 marks]

2. *[Maximum marks: 10]*

(a) Prove that $(x-2)$ is a factor of the polynomial, $g(x) = 6x^3 - 5x^2 - 19x + 10$. [2 marks]

(b) Factorise $g(x)$ completely. [4 marks]

(c) Hence, express $\frac{5+3x}{g(x)}$ in partial fractions. [4 marks]

3. *[Maximum marks: 10]*

(a) When the cubic polynomial, $P(x)$, is divided by x, $(x+1)$ and $(x-2)$, the remainders are 5, 6 and -3 respectively. Given that $P(x)$ is divisible by $(x-1)$, find the polynomial, $P(x)$, in the form, $Ax^3 + Bx^2 + Cx + D$. [7 marks]

(b) Hence, find the remainder when *P*(*x*) is divided by $(x^2 + 2)$. [3 marks]

4. *[Maximum marks: 13]*

The function, *h*, is defined as $h: x \mapsto ax + b$.

(a) Given that $h(2) = 5$ and $h^{-1}(-3) = -2$, find the value of *a* and of *b*. [4 marks]

Another function, *k*, is defined as $k: x \mapsto 1 - e^{x-4}$.

(b) Solve $k(x) = 0$. [2 marks]

(c) Sketch the graph of $k(x)$, stating the equation of the asymptote and the axes intercepts. [3 marks]

(d) Explain why the inverse of $k(x)$ exists. [1 mark]

(e) Solve the equation $h^{-1}k^{-1}(x) = 4$. [3 marks]

5. *[Maximum marks: 8]*

The functions, *f* and *g*, are defined as follow.

$$f: x \mapsto \frac{1}{x+2}, x \neq -2$$

$$g: x \mapsto 2x + 3$$

(a) Express $fg(x)$ and $f^2(x)$ in similar form. [5 marks]

(b) Solve $2f^2(x) - fg(x) = 0$. [3 marks]

6. *[Maximum mark: 8]*

The function, *f*, is defined as $f: x \mapsto ax^2 - 6x + b$.

(a) Given that $f(0) = f(2) = 8$, find the value of *a* and of *b*. [3 marks]

(b) State the largest possible domain for $f^{-1}(x)$ to exist. [3 marks]

(c) Hence, find $f^{-1}(x)$. [2 marks]

7. *[Maximum mark: 15]*

(a) Find all the angles x, where $0° < x < 360°$, such that

(i) $2\sin\left(\frac{1}{2}x\right) = \frac{1}{4}$ [3 marks]

(ii) $3\sin^2 x = 2\cos x \sin x$ [5 marks]

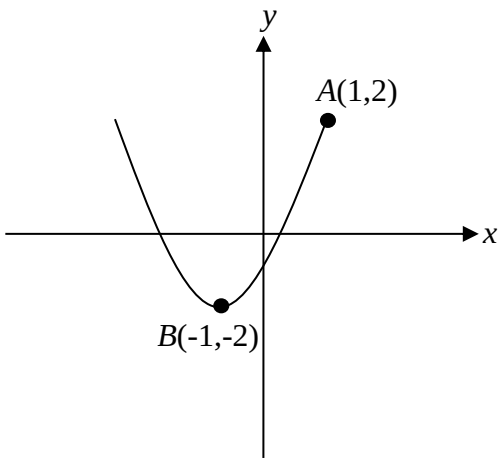
(b) Solve the following equations for $0 < x < 2\pi$, leaving your answers in terms of π.

(i) $\sec^2 x = \frac{4}{3}$ [3 marks]

(ii) $2\cos^2 x = 5\cos x - 2$ [4 marks]

8. *[Maximum mark: 8]*

The diagram below shows the graph of $y = f(x)$ which passes through $A(1,2)$ and $B(-1,-2)$. The graph of $y = f(x)$ is mapped onto the graph of $y_1 = -2f(x-2)+1$ after undergoing a series of transformations.



(a) Describe fully the transformations which map the graph of $y = f(x)$ onto the graph of $y_1 = -2f(x-2)+1$. [4 marks]

(b) Write down the co-ordinates of A and B after the transformations. [2 marks]

(c) Hence, sketch the graph of $y_1 = -2f(x-2)+1$. [2 marks]

End of Paper

Answers

1. (a) $\therefore A=-1,\ \therefore B=-2$

2. (b) $(x-2)(2x-1)(3x+5)$ (c) $\dfrac{1}{3(x-2)}-\dfrac{2}{3(2x-1)}$

3. (a) $\therefore A=1,\ \therefore B=-2,\ \therefore C=-4,\ \therefore D=5$ (b) $(x-2)(x^2+2)+(9-6x)$

4. (a) $\therefore a=2,\ \therefore b=1$ (b) $\therefore x=4$ (e) $\therefore x=-147.4$

5. (a) $\dfrac{x+2}{2x+5}, x\neq -\dfrac{5}{2}$ (b) $\therefore x=-\dfrac{3}{2}$

6. (a) $\therefore a=3,\ \therefore b=8$ (b) $x\geq 1$ or $x\leq 1$

   (c) For $x\geq 1$, $\therefore f^{-1}(x)=1+\sqrt{\dfrac{x-5}{3}}$ OR For $x\leq 1$, $\therefore f^{-1}(x)=1-\sqrt{\dfrac{x-5}{3}}$

7. (a) (i) $\therefore x=14.4°, 345.6°$ (ii) $\therefore x=33.7°, 180°, 213.7°$

   (b) (i) $\therefore x=\dfrac{\pi}{6},\dfrac{5\pi}{6},\dfrac{7\pi}{6},\dfrac{11\pi}{6}$ (ii) $\therefore x=\dfrac{\pi}{3},\dfrac{5\pi}{3}$

8. (a) $T_1=f(x-2)$: Horizontal translation of 2 units along the positive *x*-axis.
   $T_2=-f(x-2)$: Reflection on the *x*-axis.
   $T_3=-2f(x-2)$: Stretch, factor 2, along the *y*-axis.
   $T_4=-2f(x-2)+1$: Vertical translation of 1 unit along the positive *y*-axis.

   (b) $A_1(3,-3),\ B_1(1,5)$