Temperature and Ideal Gas

<u>Temperature</u>: A measure of "hotness" of object. It indicates the direction of thermal energy flow – thermal energy flows from a region of higher temperature to a region of lower temperature.

<u>Thermal Equilibrium:</u>

- No net heat flow between the two objects in thermal contact
- Two objects' temperatures are the same

<u>Convert kelvin to degree Celsius:</u> $T / K = T / °C + 273.15$

<u>Gas Laws</u>

Boyle's Law	**Charles's Law**	**Pressure Law**
When temperature is constant, pressure increases as volume decreases. $p \propto \frac{1}{V}$	When pressure is constant, volume increases as temperature increases. $V \propto T$	When volume is constant, pressure increases as temperature increases. $P \propto T$
Ideal Gas Equation		
$pV = nRT$ or $pV = NkT$ • Reflects Boyle's, Charles's and Pressure Laws • An ideal gas is one that obeys the ideal gas equation (pV = nRT) at all values of temperature (T), pressure (p) and volume (V). [n is the number of moles of gas; R is the molar gas constant] • Ideal Gas does not exist in reality. Most gases behave more like an ideal gas at high temperature and low pressure. • n is the number of moles of gas. One mole of gas contains 6.02×10^{23} particles. Avogradro number, $N_A = 6.02 \times 10^{23}$ mol^{-1}. Hence, $n = N/N_A$		

<u>Kinetic Theory of Gases</u>

Assumptions of Kinetic Theory of Gases:

1. A container with volume V contains a very large number of identical atoms/molecules, each with mass m.
2. The volume of the atoms/molecules is negligible as compared to the volume occupied by the gas (i.e. volume V).
3. The atoms/molecules are in constant motion randomly in agreement with Newton's Law of Motion.
4. The atoms/molecules exert no forces on each other or on the walls of the container except during collisions.
5. The collisions among atoms/molecules or with the walls of the container are elastic.

How molecular movement causes pressure:

- When a molecule hits the wall of the container elastically, there is a change in momentum of the molecule over time.
- By Newton's 1st and 2nd law, there is a net force on the molecule.
- By Newton's 3rd law, there is a force on the wall by the molecule.
- Hence there is a pressure exerted by the gas which exert a force over the area of the wall.

Derivation of $pV = \frac{1}{3}Nm\langle c^2 \rangle$

- Consider an ideal gas particle of mass *m* and speed *U*, hence momentum *mU* colliding perpendicularly with a wall of a container. By assumptions 5, the final momentum of the particle is *-mU*. $|\Delta p| = |-mU - mU| = 2mU$
- By assumption 3, there is a change in momentum of the particle resulting in a force by the particle on the wall. By Newton's 3rd law of motion, there is a force on the wall by the particle.
- The particle is hitting on the wall constantly by moving back and forth the container of length *L*. Therefore, the frequency of collision is $\frac{1}{\Delta t} = \frac{U}{2L}$
- By Newton's 2nd law, the magnitude of the force is $F = \frac{dp}{dt} = (\Delta p) \times \frac{1}{\Delta t} = \frac{mU^2}{L}$
- Therefore, pressure by one particle is $p = \frac{F}{A} = \frac{mU^2}{LA} = \frac{mU^2}{V}$
- By assumption 1, due to large numbers of particles with various velocity *U*, summing all mean square values of the velocities in the x-direction, the pressure in x-direction is,
$$p = \frac{m\left(U_1^2 + U_2^2 + ... + U_N^2\right)}{V} = \frac{Nm\langle U^2 \rangle}{V}$$
- In 3-dimensions, by the random motion of particles, the mean square speeds in each of the three directions are the same, the mean square speed in 3D is
$$\langle c^2 \rangle = 3\langle U^2 \rangle$$
$$\langle U^2 \rangle = \frac{1}{3}\langle c^2 \rangle$$
- Therefore,
$$pV = \frac{1}{3}Nm\langle c^2 \rangle$$

Mean Kinetic Energy of a molecule of an Ideal Gas:

$$pV = \frac{1}{3}Nm\langle c^2 \rangle = NKT$$

$$3 \times \frac{1}{2} \times \frac{1}{3}Nm\langle c^2 \rangle = 3 \times \frac{1}{2} \times NKT$$

$$\frac{1}{2}m\langle c^2 \rangle = \frac{3}{2}KT$$

thus the mean kinetic energy of a molecule of an ideal gas is proportional to the thermodynamic temperature.

Pressure in Kinetic Theory of Gas:

- The higher the frequency of collision, the larger the pressure
- The higher the change in momentum of the particles (e.g. higher temperature, hence higher speed), the larger the pressure

Volume in Kinetic Theory of Gas:

- Higher volume suggests larger average spacing between the particles