HWA CHONG INSTITUTION
2012 PRELIMINARY EXAMINATION
Higher 2

MATHEMATICS **9740/01**
Paper 1 **12 September 2012**
3 hours

Additional Materials: Answer Paper
List of Formulae (MF15)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number, name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use a graphic calculator.
Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, place the completed cover page on top of your answer scripts and fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

1. The function f is defined for $x \neq 2$ by

$$\mathrm{f} : x \to \frac{kx^2 - 5x + 3}{x - 2},$$

where k is a real constant. Find the range of values of k such that the function f is an increasing function for all real values of x, $x \neq 2$. [5]

2. The point A represents a fixed complex number a such that $-\frac{\pi}{2} < \arg a < 0$. The complex numbers $\mathrm{i}a$ and $a + \mathrm{i}a$ are represented by the points B and C respectively. On a clearly labelled Argand diagram, show the points A, B, C and the set of points representing the complex numbers z satisfying the following:

(i) $|z - \mathrm{i}a| = |a|$,

(ii) $|z| \geq |z - (a + \mathrm{i}a)|$.

Write down the complex number z that gives the greatest value of $|z|$, giving your answer in terms of a. [5]

3. It is given that $y = x^{xy}$, where $x > 0$, $y > 0$. Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y. Hence find the equation of the tangent to the curve $y = x^{xy}$ which is parallel to the y-axis. [6]

4. **(a)** Find the complex number z in the form $x + \mathrm{i}y$, where $x, y \in \mathbb{R}$ such that $\frac{3\mathrm{i}z}{z^* - 1 + 2\mathrm{i}} = -\mathrm{i}$. [Note: z^* is the conjugate of z.] [3]

(b) The two roots z_1, z_2 of the equation $z^2 + z + p = 0$, where $p \in \mathbb{R}$, are such that $z_1 = a + \mathrm{i}b$ where $a, b \in \mathbb{R}$, $b \neq 0$ and $|z_1 - z_2| = \sqrt{3}$. Find the value of p. [3]

5. **(i)** Use the standard series for $\ln(1+x)$ to find the first three terms of the Maclaurin's series for $y=\dfrac{1}{\sqrt{1+\ln(1+2x)}}$. [3]

(ii) Find the range of values of x for the above expansion to be valid. [3]

(iii) Use your answer in part **(i)** to calculate an approximate value of $\int_0^2 y \, \mathrm{d}x$. Explain why the answer obtained is not a good approximation. [2]

6. **(a)** One NETO logistics commander was tasked to deliver goods to an army camp in Abghan during June 2012. The commander delivered 1000 tons of goods on 1^{st} June, 1100 tons on 2^{nd} June, ... etc. Each subsequent day's delivery was 100 tons more than the previous day's delivery. After several days of delivery, due to more road attacks from Bl Paeda, the commander had to change the plan so that each subsequent day's delivery was 100 tons less than the previous day's delivery. By 15^{th} June, the commander managed to deliver a total of 21300 tons of goods for the past 15 days. On which day did the commander deliver the most goods to the camp? How many tons of goods did he deliver on this particular day? [5]

(b) Given that all terms in a geometric progression $\{u_n\}$, $n=1, 2, 3, \ldots$ are positive with first term *a* and common ratio *r*, where $r \neq 1$, the sums *H* and *C* are defined as follows:

$$H = u_1 + u_2 + \ldots + u_n , \qquad C = \frac{1}{u_1} + \frac{1}{u_2} + \ldots + \frac{1}{u_n} .$$

(i) By expressing *H* and *C* in terms of *a* and *r*, prove that $\dfrac{H}{C} = u_1 u_n$. [3]

(ii) Express the product $u_1 u_2 u_3 \ldots u_n$ in the form of $\left(\dfrac{H}{C}\right)^{\mathrm{f}(n)}$, where f(*n*) is a function of *n*. [2]

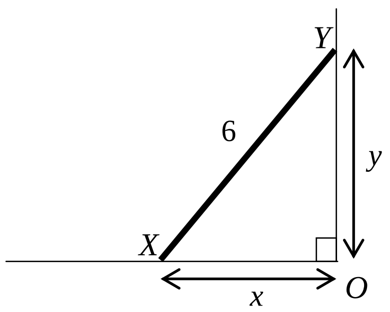
7. **(a)** A sequence u_1, u_2, u_3, … is such that $u_1 = -1$ and $u_{n+1} = u_n + 2\left(3^{-n}\right) + a$, where $n \in \mathbb{Z}^+$ and a is a constant. By considering $u_{n+1} - u_n$, find u_n in terms of n and a. [4]

(b) Prove by induction that, for every positive integer n,

$$(1^3 + 3\cdot 1^5) + (2^3 + 3\cdot 2^5) + (3^3 + 3\cdot 3^5) + \ldots + (n^3 + 3n^5) = \frac{1}{2}n^3(n+1)^3.$$ [4]

Given that $\sum_{r=1}^{n} r^3 = \frac{n^2}{4}(n+1)^2$, find $\sum_{r=1}^{2n} r^5$ in terms of n. [2]

8. Jesse leans a 6 metre rod XY in the position as shown below. At time t seconds, the two ends X and Y are x and y metres from O respectively.



(i) State an equation relating x and y. [1]

As the rod slips, Jesse proposes that y decreases at a rate proportional to y.

(ii) Based on Jesse's proposal, show that

$$\frac{dx}{dt} = \frac{k(36 - x^2)}{x},$$

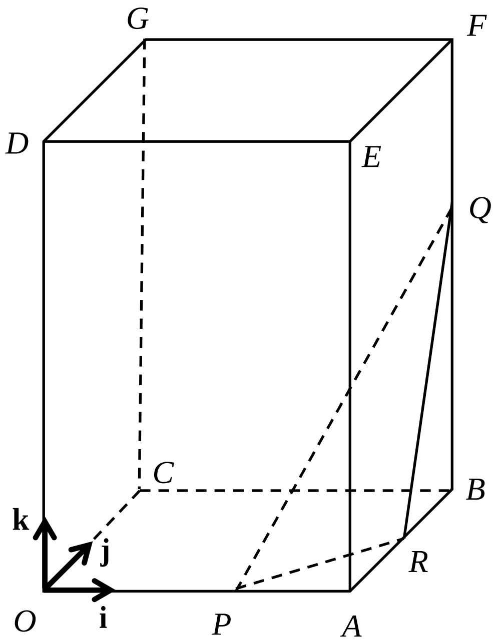
where k is a constant. [3]

Initially, X is 4 metres from O. Take $k = 2$.

(iii) Find an expression for x in terms of t and find the time taken for Y to be 3 metres from O from the instance Jesse lets go of the rod, giving your answer to 1 decimal place. [5]

(iv) Sketch the graph of x against t. Hence, comment on whether Jesse's proposed model is appropriate. Justify your answer. [3]

9.



The diagram above shows a cuboid *OABCDEFG* with horizontal base *OABC* where $OA = 6$ cm and $AB = 4$ cm. *OD*, *AE*, *BF* and *CG* are vertical with height 10 cm. The point *O* is taken as the origin and vectors **i**, **j** and **k** are unit vectors in the directions *OA*, *OC* and *OD* respectively. The point *P* on *OA* and the point *Q* on *BF* are such that $OP : PA = 2:1$ and $BQ : QF = 3:2$. Point *R* is the midpoint of *AB*.

(i) Show that the equation of the plane *PQR* is $3x - 3y + z = 12$. [3]

(ii) Find the distance from the point *C* to the plane *PQR*. [2]

(iii) Find the acute angle between the plane *PQR* and the plane *OABC*. [2]

(iv) Find the position vector of the point of intersection *M* of the line *PQ* and the plane *OCGD*. Hence write down the distance from *M* to the plane *OABC*. [3]

(v) Find a vector equation of the plane, in the form $\mathbf{r} = \mathbf{u} + \lambda\mathbf{v} + \mu\mathbf{w}$, when the plane *PQR* is reflected about the plane *OABC*. [2]

10. **(a)** Find the following integrals:

(i) $\int\left(\cos^4 x-\sin^4 x\right)\,\mathrm{d}x$; [3]

(ii) $\int \log_3(3x-1)\,\mathrm{d}x$. [4]

(b) By using the substitution $x=4\sin\theta$, show that

$$\int\frac{\sqrt{16-x^2}}{x^2}\,\mathrm{d}x=-\frac{\sqrt{16-x^2}}{x}-\sin^{-1}\left(\frac{x}{4}\right)+C,$$

where C is an arbitrary constant. [5]

11. A curve G has parametric equations

$$x=t-\frac{1}{t},\qquad y=2t+\frac{1}{t},$$

where $t>0$.

(a)(i) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t, and deduce that $-1<\frac{\mathrm{d}y}{\mathrm{d}x}<2$. [4]

(ii) Find the x-coordinate of the stationary point of G. [3]

(iii) Denoting the graph of G by $y=\mathrm{f}(x)$, sketch the graph of $y=\mathrm{f}\,'(x)$, stating clearly the equation of any asymptote(s) and any axial-intercept(s). [3]

(b) Let P denote the point (x, y) on the curve G. A vertical line and a horizontal line through P are drawn, intersecting the x-axis and the y-axis at $X=(x,\ 0)$ and $Y=(0,\ y)$ respectively. If A represents the area of the rectangle $OXPY$, find the rate of change of A with respect to x when $t=5$. [4]