| Name: | | Index Number: | | Class: | |
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**CATHOLIC HIGH SCHOOL**
**Preliminary Examination**
**Secondary 4 (O-Level Programme)**

# Additional Mathematics

**4049/01**

Paper 1

**12 September 2022**
**2 hours 15 minutes**

Candidates answer on the Question Paper.
Additional Materials: Answer booklets A, B and C.

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**READ THESE INSTRUCTIONS FIRST**

Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.

Given non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total of the marks for this paper is **90**.

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This paper consists of **22** printed pages.

*Mathematical Formulae*

## 1. ALGEBRA

*Quadratic Equation*

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

*Binomial Expansion*

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where $n$ is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

## 2. TRIGONOMETRY

*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

*Formulae for $\Delta ABC$*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \tfrac{1}{2}ab\sin C$$

1. (i) The side of an equilateral triangle is $2\left(\sqrt{3}+1\right)$ cm.
**Without using a calculator**, find the exact value of the area of the equilateral triangle in the form $a+b\sqrt{c}$ where $a$, $b$ and $c$ are integers. [2]

(ii) A triangular prism has the equilateral triangle in (i) as its base.
The volume of the prism is $9\left(\sqrt{3}+1\right)$ cm$^3$.
Find the height of the prism in the form $m+n\sqrt{l}$ where $m$, $n$ and $l$ are rational numbers. [3]

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2. (a) Given that $x^{\frac{1}{2}}+x^{-\frac{1}{2}}=3$, find the value of $x^{\frac{3}{2}}+x^{-\frac{3}{2}}$. [3]

(b) Express $2-4x-3x^2$ in the form $-a(x-b)^2+c$ and hence find, in **exact** form, the coordinates of the turning point on the curve. [2]

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3. Find the coordinates of the points of intersection of the curve $x^2-xy+y^2=16$ and the line $2x-3y=4$. [5]

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4. (a) Integrate $e^{-\frac{1}{2}x}+\sin 3x$ with respect to $x$. [2]

(b) (i) Express $\dfrac{x^2+3x+12}{x(x+2)^2}$ in partial fractions. [3]

(ii) Hence evaluate $\displaystyle\int_1^4 \frac{x^2+3x+12}{x(x+2)^2}\,dx$. [4]

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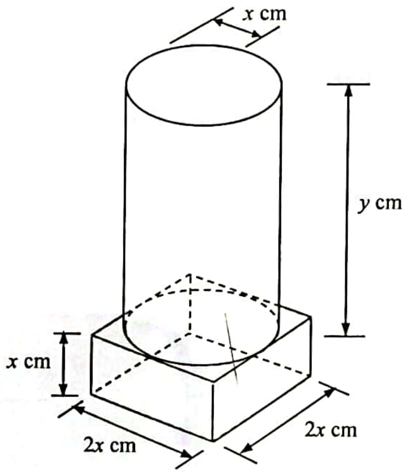
5. A polynomial, $P(x)$, is $3x^3-cx^2-6x+8$, where $c$ is a constant.

(i) Given that $(x-2)$ is a factor of $P(x)$, show that $c=5$. [1]

(ii) Solve $P(x)=0$. [3]

(iii) Hence, solve $3(y-1)^3-5(y-1)^2=6y-14$. [2]

6.




The diagram shows a solid body which consists of a right circular cylinder fixed, with no overlap, to a rectangular block. The block has a square base of side $2x$ cm and a height of $x$ cm. The cylinder has a radius of $x$ cm and a height of $y$ cm.
The total volume of the solid is 27 $cm^3$.

(a) Express $y$ in terms of $x$. [2]

(b) Hence show that the total surface area, $A$ $cm^2$, of the solid is given by [2]

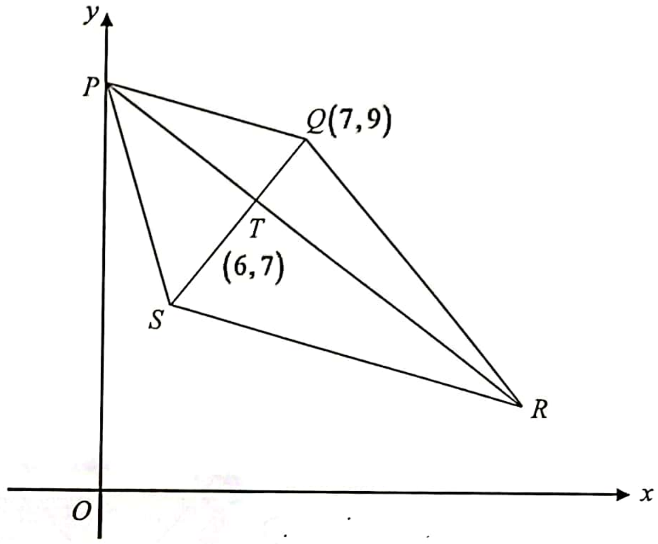
$$A = \frac{54}{x} + 8x^2$$

(c) Find the value of $x$ for which $A$ has a stationary value. [3]

(d) Find the value of $A$ and of $y$ corresponding to this value of $x$. [2]

(e) Determine whether the stationary value of $A$ is a maximum or a minimum. [2]

7.




The diagram, which is not drawn to scale, shows a trapezium $PQRS$ in which $PQ$ is parallel to $SR$. The point $P$ lies on the $y$-axis and the point $Q$ is $(7,9)$. The point $T(6,7)$ lies on $PR$ such that $QT$ is perpendicular to $PR$ and $QS = 3QT$.

(i) Find the equation $PR$. [2]

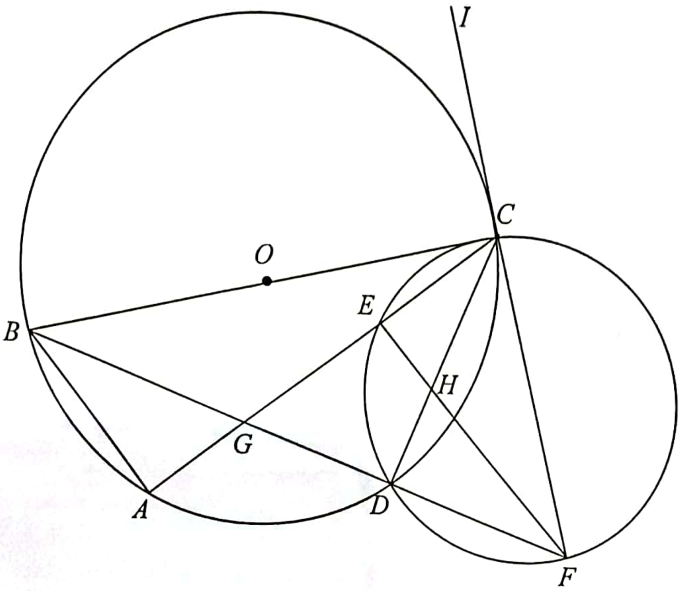
(ii) Hence, write down the coordinates of $P$. [1]

(iii) Find the coordinates of $V$ if $PQRV$ is a kite. [2]

(iv) Show that the coordinates of $S$ are $(4, 3)$. [2]

(v) Find the coordinates of $R$ if the area of $PQRS$ is 67.5 square units. [3]

8.




In the figure, two circles intersect at $C$ and $D$. $BC$ is the diameter of circle with centre $O$ and is a tangent to the other circle at $C$. $FCI$ is tangent to the circle with centre $O$ at $C$. $AGEC$ and $BGDF$ are straight lines.

(i) Prove that $AB \parallel FE$. [2]

(ii) Hence, show that $AB \times FG = BG \times EF$. [3]

(iii) Explain why $CF$ is the diameter of the smaller circle. [2]

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9. (i) On the same diagram, sketch the graphs of

(a) $y = 3\sin 2x + 1$ for $0 \le x \le 2\pi$, and [2]

(b) $y^2 = 4$. [1]

(ii) Hence, state the number of solutions of $4 = (3\sin 2x + 1)^2$ in the interval $0 \le x \le 2\pi$. [1]

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10. (a) Prove the identity $\dfrac{\cot x - \tan x}{\cot x + \tan x} = \cos 2x$. [3]

(b) Hence solve the equation $\dfrac{2\cot x - 2\tan x}{\cot x + \tan x} + 1 = 0$ for $0 \le x \le 2\pi$. [3]

11. Solve the equations

(a) $18^{\log_5 x} = 7$, [3]

(b) $\log_5 x = 4\log_x 5$. [4]

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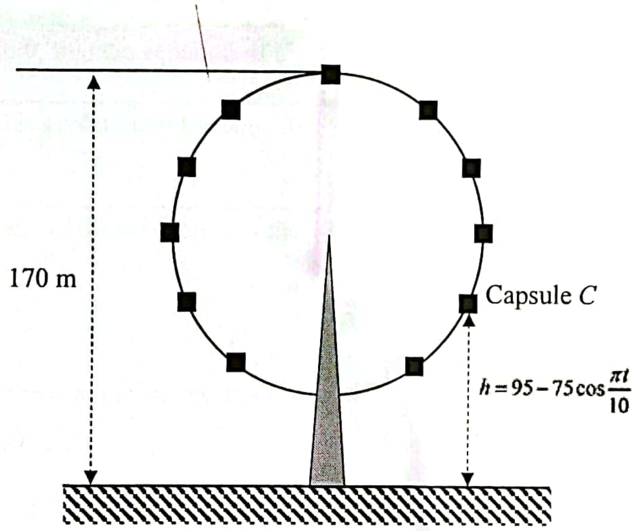
12. (a) A particular Ferris wheel stands at 170 m and has a wheel with diameter of 150 m. The height of a capsule, $C$, above ground level, $h$ m, can be modelled by $h = 95 - 75\cos\frac{\pi t}{10}$, where $t$ is the time in minutes after the ride begins from the lowest point of the flyer.




(i) Find the time taken for the Ferris wheel to make one revolution. [1]

(ii) Find the duration of time within one revolution that a particular capsule is at least 120 m above the ground level. [4]

(b) The function f is defined by $y = \frac{7x-2}{2x+3}$, where $x \neq -1.5$.

It is known that $x$ and $y$ vary with time $t$, and that $x$ decreases at a rate of 0.2 units per second.

Find the rate at which $y$ is changing at the instant when $x = 3$ units. [4]

13. The value, $V$, of a piece of rare jewel at the beginning of 2022 was \$32 000. This value increased continuously such that, after a period of 15 years, the value of the jewel increased exponentially to \$160 000.

Given that $V = A(1.25)^{kt}$, where $A$ and $k$ are constants and $t$ is the time in years from the beginning of 2022, find

(a) the value of $A$ and $k$, [3]

(b) the year in which the value of the stone first reached \$200 000. [3]

# CATHOLIC HIGH SCHOOL
## 2022 Preliminary Examination
## Secondary 4 (O-Level Programme)
## Additional Paper 2

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**1** Solve the equation $2e^{2x} = 7 + 15e^{-2x}$ giving your answer correct to 1 significant figure. [4]

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**2** The points $P(-4, -1)$ and $Q(-4, 9)$ lie on the circumference of a circle $C_1$. The line $y = 17$ is a tangent to $C_1$.

**(a)** Given further that the $x$-coordinate of the centre of $C_1$ is positive, find the equation of $C_1$. [5]

**(b)** Find the equations of the tangents of $C_1$ which are parallel to the $y$-axis. [2]

**(c)** A second circle, $C_2$, has its centre at $P$. Given that the ratio of the area of $C_2$ to the area of $C_1$ is 1 : 9, find the equation of $C_2$. [3]

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**3** Show that $x = 2$ is a solution of the equation $6x^3 + x = 13x^2 - 2$ and hence solve the equation completely. [5]

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**4** **(a)** Find the ratio of the coefficient of the third term to that of the fifth term in the expansion of $\left(2x^2 - \frac{1}{x}\right)^{10}$. [4]

**(b)** The first three terms in the expansion of $(1 + px)^n$ in ascending powers of $x$ are $1 - 45x + 900x^2$.

**(i)** Find the value of $p$ and of $n$. [5]

**(ii)** Hence find the term independent of $x$ in the expansion of $\left(2 - \frac{1}{x}\right)^2 (1 + px)^n$. [2]

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**5** Two points $A$ and $B$ are on a straight line where $AB = 6$ m. A particle $P$ moves along the line so that its velocity, $v$ ms$^{-1}$, is given by $v = 2t^2 - 7t - 4$, where $t$ is the time in seconds after leaving $B$. Initially $P$ is at $B$, moving towards $A$.

**(a)** Find an expression, in terms of $t$, for the acceleration of $P$. [1]

**(b)** Find the minimum velocity of $P$. [3]

**(c)** Find an expression, in terms of $t$, the distance of $P$ from $A$. [2]

**(d)** Find the distance from $A$ of the point where $P$ comes instantaneously to rest. [3]

**(e)** Find the total distance travelled by $P$ in the first six seconds. [3]

[TURN OVER

6 The equation of a curve is $y = ax^2 + bx - 8$, where $a$ and $b$ are integers, $a > 0$.

(a) If the set of values of $x$ for which the curve lies below the $x$-axis is $-4 < x < \frac{2}{3}$, find the value of $a$ and of $b$. [3]

(b) In the case when $b = -11$, find the value of $a$ for which the line $y + 3x + 16 = 0$ is a tangent to the curve. [3]

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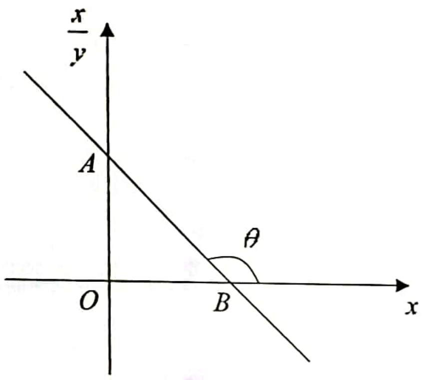
7 (a) Two variables $x$ and $y$ are related by the equation $2xy = 3y - x$. The diagram shows part of a straight line graph obtained by plotting $\frac{x}{y}$ against $x$. This graph cuts the $x$-axis and $y$-axis at $B$ and $A$ respectively; and it makes an angle $\theta$ with the $x$-axis as shown.




(i) Find the coordinates of $A$ and of $B$, [3]

(ii) Find the value of $\theta$ in degrees. [2]

(b) A particle moves in a certain medium with speed $v$ ms$^{-1}$ and experiences a resistance to motion of force $F$ newtons. It is believed that $F$ and $v$ are related by the equation $F = mA^v$, where $A$ and $m$ are constants. The table below shows the experimental values of the variables $F$ and $v$.

| $v$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $F$ | 12 | 18 | 27 | 40 | 61 | 91 |

(i) Express the equation in a form suitable for drawing a straight line graph. [1]

(ii) On the grids in the next page, draw this straight line graph and use it to estimate the value of $A$ and $m$. [4]

(iii) Determine the equation of the straight line which must be drawn on your graph to obtain the solution to the equation $mA^v = 3^v$. Use your graph to estimate the speed, $v$, that satisfies the equation $mA^v = 3^v$. [2]

**8** Given that $y = 3x\sqrt{3x^2+2}$ ,

**(a)** express $\dfrac{dy}{dx}$ in the form $\dfrac{ax^2+b}{\sqrt{3x^2+2}}$, where $a$ and $b$ are constants. [4]

Hence,

**(b)** show that $y$ increases as $x$ increases for all values of $x$, [2]

**(c)** find the exact value of $\int_0^2 \dfrac{3x^2+1}{\sqrt{3x^2+2}}\,dx$. [3]

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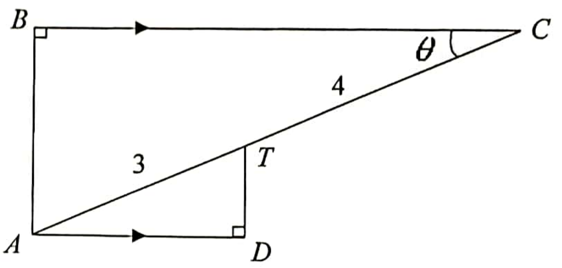
**9** A structure $ABCTD$ is such that $ATC$ is a straight line such that $AT = 3$ cm and $TC = 4$ cm.
$BC$ is parallel to $AD$, $\measuredangle ABC = \measuredangle ADT = 90°$ and $\measuredangle ACB = \theta$, where $\theta$ is a variable angle such that $0° < \theta < 90°$.




**(a)** Show that $BA + AD = 7\sin\theta + 3\cos\theta$. [1]

**(b)** Express $BA + AD$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0° \le \alpha \le 90°$. [3]

**(c)** Find the maximum value of $BA + AD$ and the corresponding value of $\theta$. [2]

**(d)** State the minimum value of $\dfrac{1}{(BA+AD)^2 - 6}$. [1]

**(e)** Solve the equation $7\sin\theta + 3\cos\theta = 4$. [2]

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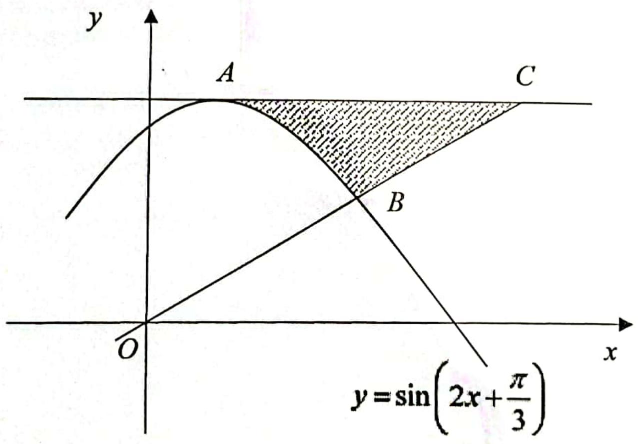
**10**




The diagram shows part of the curve $y = \sin\left(2x + \dfrac{\pi}{3}\right)$ for $0 \le x \le \dfrac{\pi}{2}$, where $A$ is the maximum point of the curve. $OC$ is a straight line that cuts the curve at $B$ and $AC$ is parallel to the $x$-axis. Given that the $x$-coordinate of $B$ is $\dfrac{\pi}{4}$, show that the area of the shaded region is $\left(\dfrac{11\pi}{48} - \dfrac{\sqrt{3}}{4}\right)$ units$^2$. [12]