Anglo-Chinese School
(Independent)

THE BEST IS YET TO BE

FINAL EXAMINATION 2021

YEAR 3 INTEGRATED PROGRAMME

ADVANCED MATHEMATICS
PAPER 1

Tuesday **5 October 2021** **1 hour 30 minutes**

Additional Materials:
Writing paper (6 sheets)

INSTRUCTIONS TO STUDENTS

Do not open this examination paper until instructed to dos o.
A calculator is required for this paper.
Answer all questions on the answer sheets provided.
Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

INFORMATION FOR STUDENTS
The maximum mark for this paper is 80.

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This question paper consists of **5** printed pages.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

*Answer **all** the questions on the answer sheets provided.*

1. *[Maximum mark: 10]*

(a) Given that $2x^2+4x-7=A(x-2)^2+B(x-2)+C$ for all real values of *x*, find the value of *A*, of *B* and of *C*. [4 marks]

(b) (i) Find the value of k, given that $f(x)=2x^3+kx+14$ is exactly divisible by $x-2$. [1 mark]

(ii) Hence, solve $f(x)=0$. [5 marks]

2. *[Maximum mark: 8]*

(a) The function $h(x)=3x^3-8x^2-5x+6$ is exactly divisible by x^2-2x-3.

Find

(i) the other factor, [1 mark]

(ii) the remainder when $h(x)$ is divided by $x-1$. [2 marks]

(b) When $P(x)=2x^2+x+5$ is divided by $(x-a)$, the remainder is twice the remainder when divided by $(x+a)$. Find the possible values of *a*. [5 marks]

3. *[Maximum mark: 10]*

The functions *f* and *fg* are defined for real values of *x* by $f:x\mapsto\dfrac{x-1}{2x+4}$, $x\neq-2$, and $fg:x\mapsto\dfrac{x-3}{2x}$, $x\neq0$.

(a) Find $f^{-1}(x)$. [4 marks]

(b) Find an expression for $g(x)$. [3 marks]

(c) Hence, evaluate $gf(3)$. [3 marks]

4. *[Maximum mark: 9]*

A certain species of fish was introduced into a pond and its number was tracked daily. It was observed that t days later, its population, P, was given by $P = 5000 - 2000e^{kt}$, where k is a constant. Ten days later, there were 3800 fish in the pond.

(a) Find the initial number of fish in the pond. [1 mark]

(b) Determine the value of k. [3 marks]

(c) Find the number of days needed for the population of the fish to increase by 40%. [3 marks]

(d) Find the number of fish in the pond after 12 days. [1 mark]

(e) State the value which P approaches as t becomes increasingly large. [1 mark]

5. *[Maximum mark: 11]*

Express the following in partial fractions.

(a) $\dfrac{5x^2+3x+7}{(x+2)(x^2+3)}$, [5 marks]

(b) $\dfrac{3x^2+2x-5}{x^2-4x+4}$. [6 marks]

6. *[Maximum mark: 8]*

(a) y_2 is obtained after the graph of $y_1 = x^2 + 2x - 8$ undergoes the following transformations, T_1 and T_2 :

T_1 : Translation of 3 units along the negative *x*-direction.

T_2 : Stretch of scale factor 2 along the *y*-axis.

(i) Write down the equation of y_2. [2 marks]

(ii) The point $A(1,-5)$ lies on the graph of $y_1 = x^2 + 2x - 8$. State the coordinates of the image point of *A* after the transformations. [2 marks]

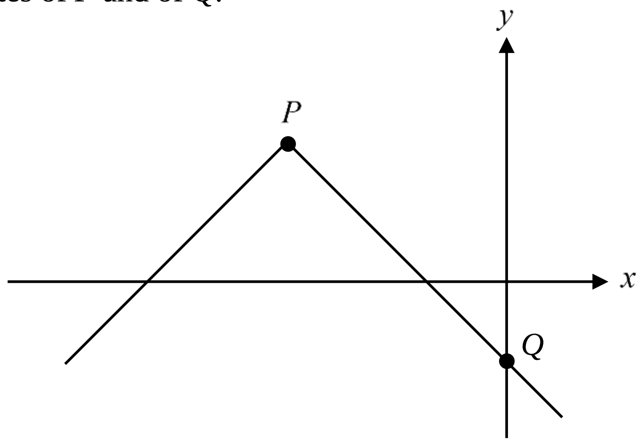
(b) The point $B(2,2)$ lies on the graph of $y = f(x)$ and the function $f(x)$ undergoes some transformations. Write down the new coordinates of *B* on the graph of each of the following functions:

(i) $y = 2 - 3f(x)$, [2 marks]

(ii) $y = 1 + f(1-2x)$. [2 marks]

7. *[Maximum mark: 10]*

(a) The diagram below shows part of the graph of $y = 3 - |2x+5|$, where P is the maximum point of the graph and the graph cuts the y-axis at Q. Write down the coordinates of P and of Q. [2 marks]



(b) Solve

(i) $3 - |2x+5| > 1$, [4 marks]

(ii) $2|2x+5| = 3 + |-2x-5|$. [4 marks]

8. *[Maximum mark: 14]*

(a) Given that θ is acute and that $\sin\theta = \frac{1}{\sqrt{3}}$, prove, without the use of a calculator, that $\frac{1}{\cos\theta - \sin\theta} = \sqrt{3} + \sqrt{6}.$ [4 marks]

(b) In an alternating current circuit, the current is given by the formula, $I = A\sin(Bt) + C$, where I is the current measured in Amperes and t is the time measured in seconds. The current has an amplitude of 20 and a period of 2π.

(i) If the maximum current is 35 Amperes, find the value of A, of B and of C. [3 marks]

(ii) Sketch the graph of $I = A\sin(Bt) + C$ for $0 \le t \le 2\pi$. [3 marks]

(iii) Hence, state the time when the current reaches the maximum. [1 mark]

(iv) Find the time when the current of the circuit first reaches 20 Amperes. [3 marks]

End of Paper 1

Answers

1(a) $A=2, B=12, C=9$

1(b) (i) $k=-15$

(ii) $x=2, x=-1\pm\frac{3\sqrt{2}}{2}$

2(a) (i) $3x-2$

(ii) -4

2(b) No possible solutions of a

3(a) $f^{-1}(x)=\frac{4x+1}{1-2x}, x\neq\frac{1}{2}$

3(b) $g(x)=x-2$

3(c) $-\frac{9}{5}$

4(a) 3000

4(b) –0.0511

4(c) 17.9 or 18 days

4(d) 3917 or 3920

4(e) 5000

5(a) $\frac{5x^2+3x+7}{(x+2)(x^2+3)}=\frac{3}{x+2}+\frac{2x-1}{(x^2+3)}$

5(b) $\frac{3x^2+2x-5}{x^2-4x+4}=3+\frac{14}{x-2}+\frac{11}{(x-2)^2}$

6(a) (i) $y_2=2x^2+16x+14$

(ii) $A(-2,-10)$

6(b) (i) $B(2,-4)$

(ii) $B(-0.5,3)$

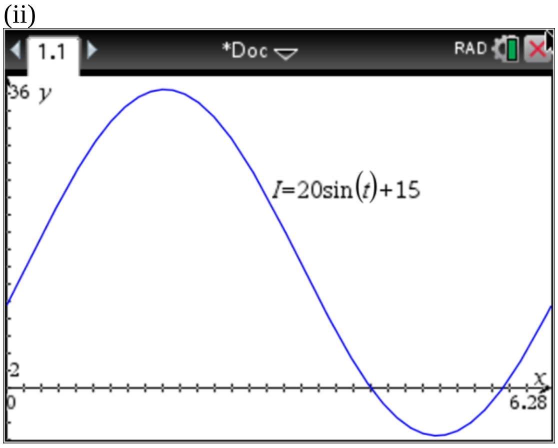
7(a) $P(-2.5,3), Q(0,-2)$

7(b) (i) $-\frac{7}{2}<x<-\frac{3}{2}$

(ii) $x=-1$ or $x=-4$

8(b) (i) $A=20, B=1, C=15$

(ii)



(iii) $\frac{\pi}{2}$ seconds or 1.57 seconds

(iv) 0.253 seconds