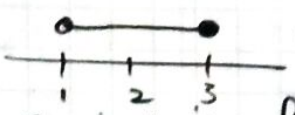
Chapter 1 **AMATHS**

Sum of roots → $\frac{-b}{a}$

product of roots → $\frac{c}{a}$

equation → x^2 - sum$x$ + product = 0

Inequalities:



eg. $1 < x \leq 3$

● → inclusive of (3)

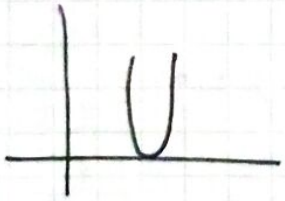
○ → bigger/smaller than (1)

$b^2 - 4ac = 0$

- curve & line intersect at ONE pt.

- line is a tangent to the curve

- curve meets x-axis at only one point



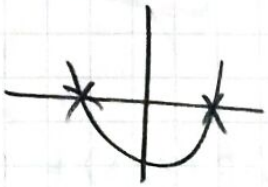
→ 2 equal & real roots

$b^2 - 4ac > 0$

- curve meets line at two distinct points

- curve meets the line at points A and B

- curve cuts x-axis at 2 points



→ 2 real and distinct roots

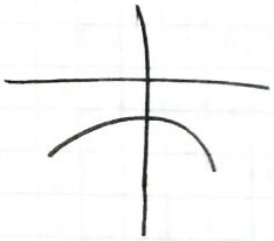
$b^2 - 4ac < 0$

- 2 imaginary roots
- no real roots

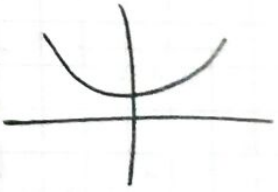
- the line and curve do not intersect each other

⇒ no real roots

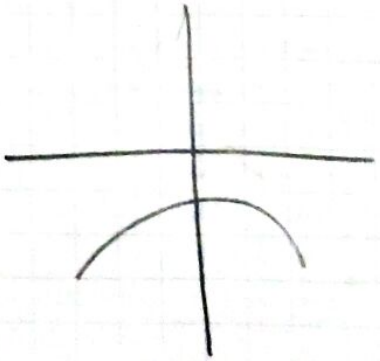
- given that y is entirely below the x-axis



- given that y is positive for all values of x



- given that y is negative for all values of x



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if 2 real roots,

$$b^2 - 4ac \geq 0$$

Chapter 2

zero index → a^0

negative index → $a^{-p} = \frac{1}{a^p}$

fractional index → $(\sqrt[p]{a})^q = a^{\frac{q}{p}}$

→ $\sqrt[p]{a} = a^{\frac{1}{p}}$

(a) $a^m \times a^n = a^{m+n}$

(b) $\frac{a^m}{a^n} = a^{m-n}$

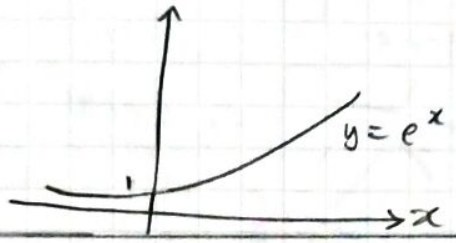
(c) $(a^m)^n = a^{mn}$

(d) $a^n \times b^n = (ab)^n$

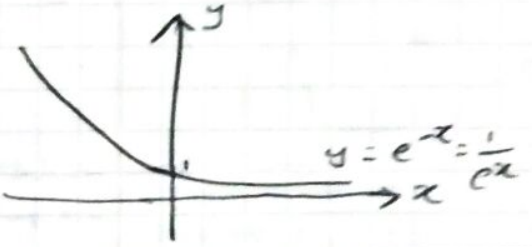
(e) $\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$

(f) $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$

$a > 1$



$0 < a < 1$



Chapter 3

- $a^2 - b^2 = (a-b)(a+b)$
- $(a+b)^2 = a^2 + 2ab + b^2$
- $a^2 + b^2 = (a+b)^2 - 2ab$
- $(a-b)^2 = a^2 - 2ab + b^2$
- $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
- $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
- $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
- $a^4 + b^4 = [(a+b)^2 - 2ab]^2 - 2(ab)^2$

Partial fractions:

$$\frac{A}{(x+y)(x+z)} = \frac{B}{x+y} + \frac{C}{x+z} \Rightarrow B(x+z) + C(x+y) = A$$

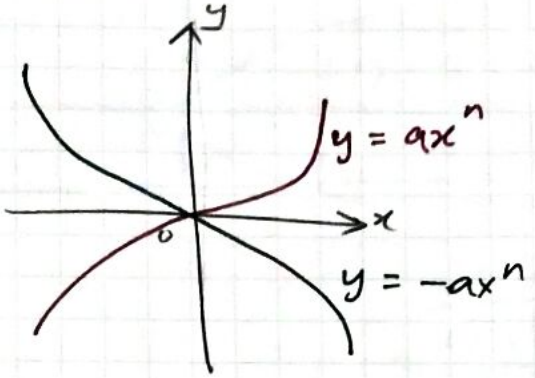
$$\frac{A}{(x+y)(x+z)^2} = \frac{B}{x+y} + \frac{C}{x+z} + \frac{D}{(x+z)^2} \Rightarrow B(x+z)^2 + C(x+y)(x+z) + D(x+y) = A$$

$$\frac{A}{(x+y)(x^2+z)} = \frac{B}{x+y} + \frac{Cx+D}{x^2+z} \Rightarrow B(x^2+z) + (Cx+D)(x+y) = A$$

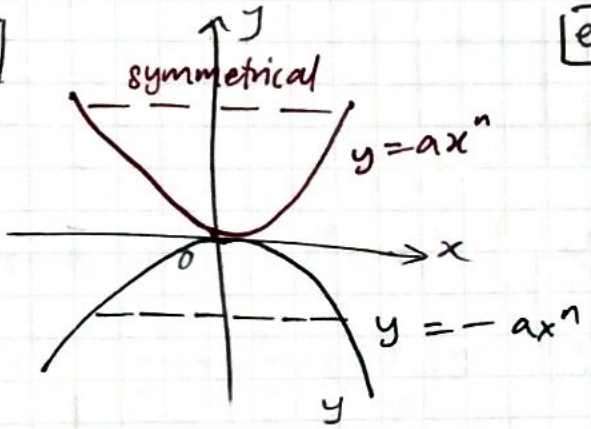
Chapter 4

positive integers : $(n) > 0$

odd :

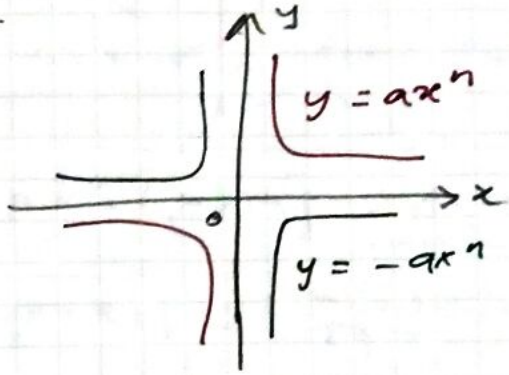


even :

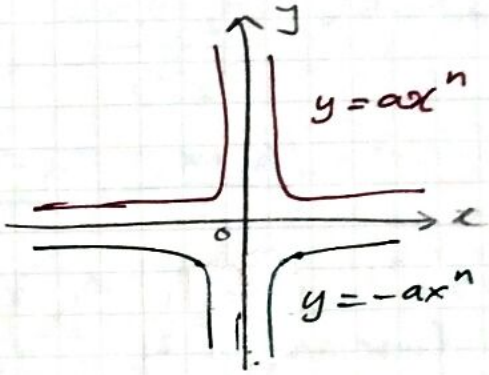


negative integers : $(n) < 0$

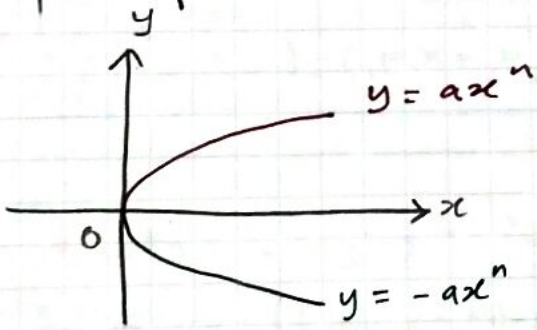
odd :



even :

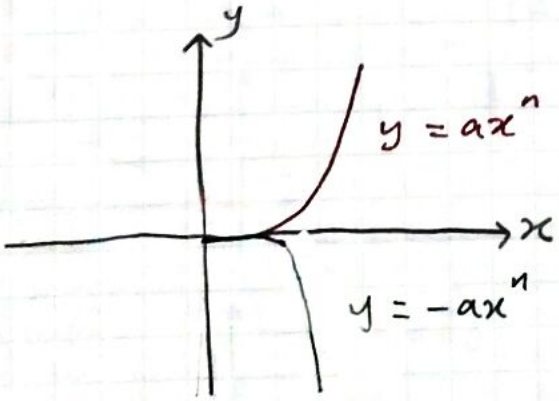


$0 < n < 1$
$x \geq 0$



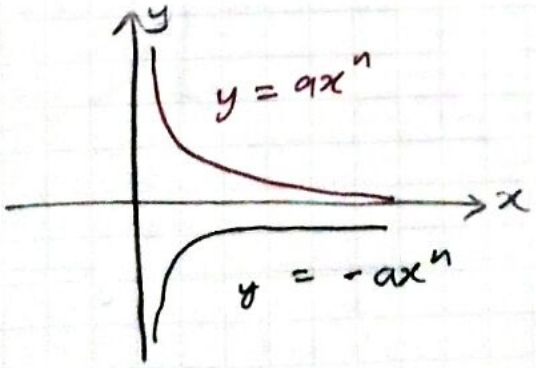
$n = \frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \frac{5}{9}, \cdots$

$n > 1$
$x \geq 0$



$n = \frac{3}{2}, \frac{5}{4}, \frac{6}{5}, \cdots$

$n < 0$
$x > 0$



$n = -\frac{1}{2}, -\frac{1}{3}, -\frac{2}{3}, -\frac{3}{2}, -\frac{5}{9}, \cdots$

Chapter 5

general term $\Rightarrow T_{r+1} = \binom{n}{r} a^{n-r} b^r$

Term independent of x $\Rightarrow$ power $= 0$

Chapter 6 $[A(x_1, y_1)$ and $B(x_2, y_2)]$

distance between two pts $= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

gradient $= \dfrac{y_2 - y_1}{x_2 - x_1}$

midpoint $= \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2} \right)$

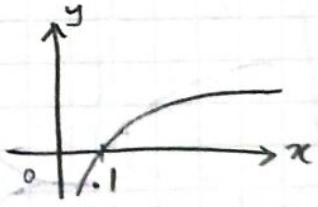
area of triangle (shoelace) $= \frac{1}{2} \left| \begin{matrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{matrix} \right|$

$= \frac{1}{2} \left| (x_1 y_2 + x_2 y_2 + x_3 y_1) - (x_1 y_3 + x_3 y_2 + x_2 y_1) \right|$

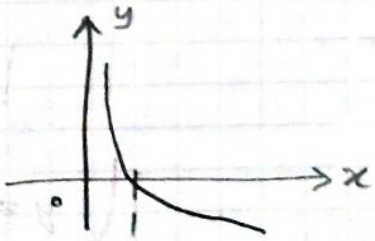
Chapter 7

- $y = a^x \Leftrightarrow \log_a y = x$
- $y = 10^x \Leftrightarrow x = \lg y$
- $y = e^x \Leftrightarrow x = \ln y$
- $\log_a m + \log_a n = \log_a mn$
- $\log_a m - \log_a n = \log_a \left(\frac{m}{n}\right)$
- $\log_a m^n = n \log_a m$
- $\log_a 1 = 0$
- $\log_a a = 1$
- $a^{\log_a x} = x$
- $\log_b m = \dfrac{\log_a m}{\log_a b}$

$\boxed{y = \log_a x}$ $a > 1$:

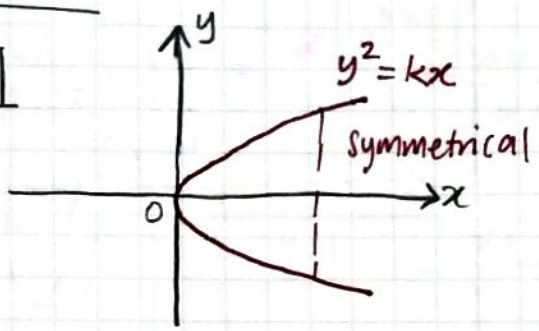


$0 < a < 1$:

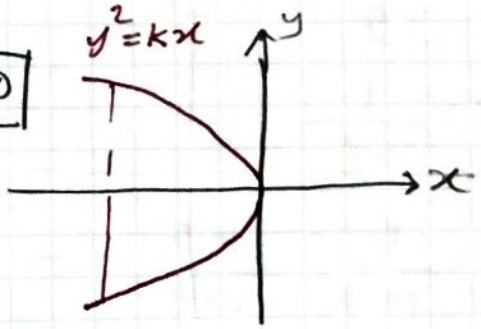


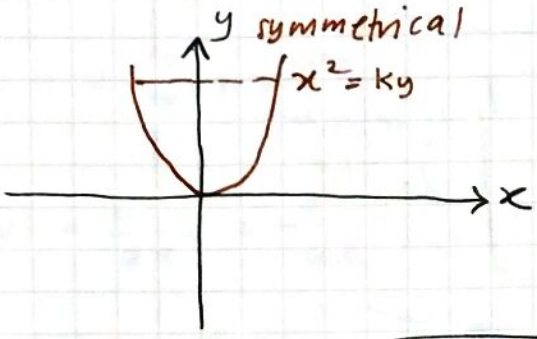
Chapter 9

k > 0



k < 0





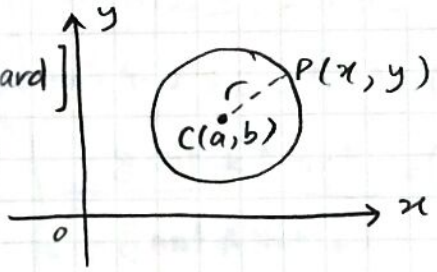
Circles:

$(x-a)^2 + (y-b)^2 = r^2$ [standard]

$x^2 + y^2 + 2gx + 2fy + c = 0$ [general]

centre $(-g, -f)$

radius $\sqrt{f^2 + g^2 - c}$



Chapter 11 & 12

	$30°\left(\frac{\pi}{6}\right)$	$45°\left(\frac{\pi}{4}\right)$	$60°\left(\frac{\pi}{3}\right)$
$\sin\theta$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
$\cos\theta$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
$\tan\theta$	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

- $\sin(90° - \theta) = \cos\theta$
- $\cos(90° - \theta) = \sin\theta$
- $\tan(90° - \theta) = \frac{1}{\tan\theta}$

$\cos(-\theta) = \cos\theta$

$\sin(-\theta) = -\sin\theta$

$\tan(-\theta) = -\tan\theta$

$180° - \alpha$ S	A
$180° + \alpha$ T	$360° - \alpha$ C

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$
$$1 + \tan^2\theta = \sec^2\theta$$
$$1 + \cot^2\theta = \operatorname{cosec}^2\theta$$
} pythagorean identity

(a) $\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$

(b) $\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$

(c) $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$
} addition formulae

(a) $\sin 2A = 2\sin A\cos A$

(b) $\cos 2A = \cos^2 A - \sin^2 A$
$= 2\cos^2 A - 1$
$= 1 - 2\sin^2 A$

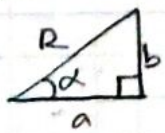
(c) $\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$
} double-angle formulae

$\sin A\cos A = \frac{1}{2}\sin 2A$

$\cos^2 A = \dfrac{1 + \cos 2A}{2}$ $\quad \sin^2 A = \dfrac{1 - \cos 2A}{2}$

$a\sin\theta \pm b\cos\theta = R\sin(\theta \pm \alpha)$

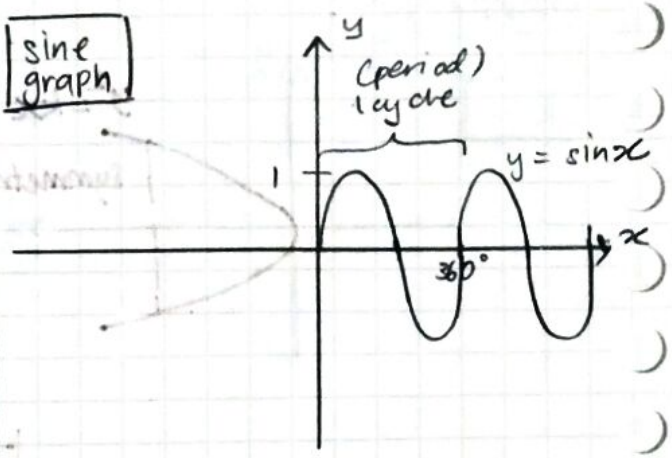
$a\cos\theta \mp b\sin\theta = R\cos(\theta \mp \alpha)$

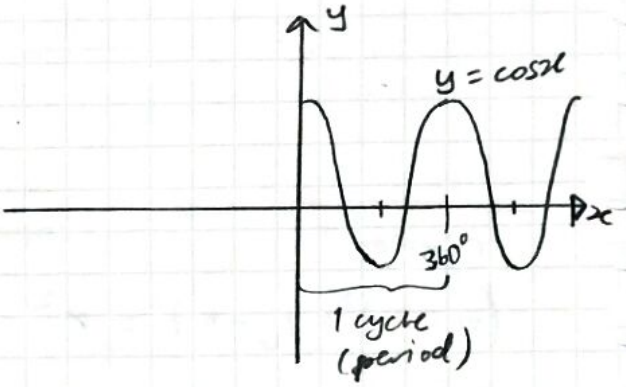


$R = \sqrt{a^2 + b^2}$, $\alpha = \tan^{-1}\frac{b}{a}$

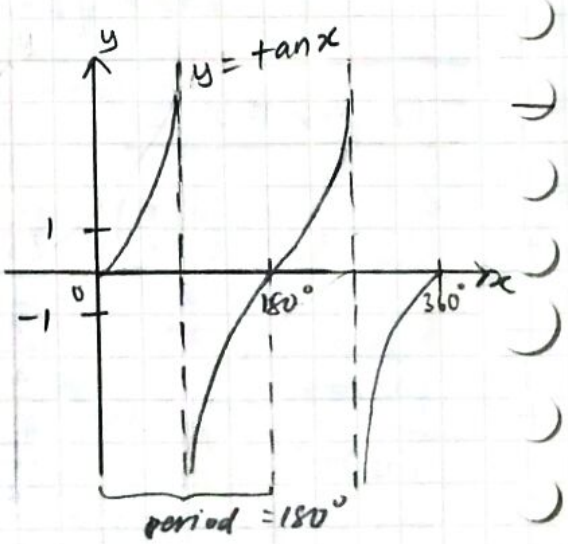
max value $= R$ when $\begin{cases}\sin(\theta \pm \alpha) = 1\\ \cos(\theta \mp \alpha) = 1\end{cases}$ | min value $= -R$ when $\begin{cases}\sin(\theta \pm \alpha) = -1\\ \cos(\theta \mp \alpha) = -1\end{cases}$
} R-formulae

sine graph





$y = a\sin bx + c$ / $a\cos bx + c$ | $y = c$, a = amp., period $= \frac{360}{b}$ / $\frac{2\pi}{b}$



Chapter 14

chain rule → $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

quotient rule → $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

product rule → $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

sum rule → $\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$

difference rule → $\frac{d}{dx}(u-v) = \frac{du}{dx} - \frac{dv}{dx}$

power rule → $\frac{d}{dx}(x^n) = nx^{n-1}$

constant multiple rule → $\frac{d}{dx}[kf(x)] = k \cdot f'(x)$

Chapter 15

1. $\boxed{\frac{dy}{dx} > 0}$ for all x in $(a,b) \Rightarrow y = f(x)$ is **increasing**

2. $\boxed{\frac{dy}{dx} < 0}$ for all x in $(a,b) \Rightarrow y = f(x)$ is **decreasing**

Chapter 16

stationary point → $\frac{dy}{dx} = 0$

maximum point → $\frac{dy}{dx} < 0$

minimum point → $\frac{dy}{dx} > 0$

1st Derivative Test

$y = f(x)$

$\frac{dy}{dx} = 0 \rightarrow$ find x

x	1^-	1	1^+
$\frac{dy}{dx}$	/	0	/

(example)

(point of inflexion, no change)

2nd Derivative Test

$y = f(x)$

$\frac{dy}{dx} = 0 \rightarrow$ find x

$\frac{d^2y}{dx^2} > 0 \rightarrow$ min pt.

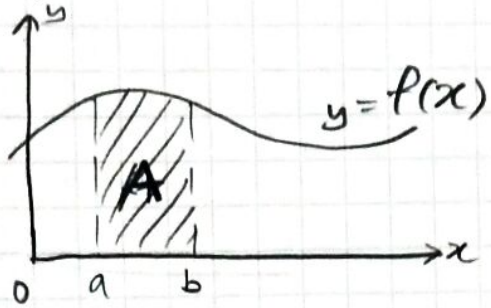
$\frac{d^2y}{dx^2} < 0 \rightarrow$ max pt.

$\frac{d^2y}{dx^2} \rightarrow$ test failed → use 1st DT

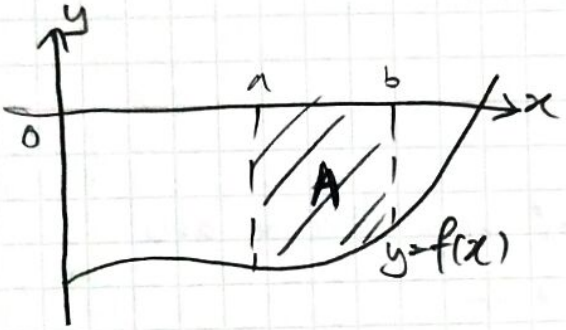
Chapter 17 and 18

$\frac{d}{dx} \sin x = \cos x$	$\int \cos x \, dx = \sin x + C$
$\frac{d}{dx} \cos x = -\sin x$	$\int \sin x \, dx = -\cos x + C$
$\frac{d}{dx} \tan x = \sec^2 x$	$\int \sec^2 x \, dx = \tan x + C$
$\frac{d}{dx} e^x = e^x$	$\int e^x dx = e^x + C$
$\frac{d}{dx} e^u = e^u \cdot \frac{du}{dx}$	$\int e^u du = e^u \div \frac{du}{dx} + C$
$\frac{d}{dx} \ln x = \frac{1}{x}$	$\int \frac{1}{x} dx = \ln x + C$
$\frac{d}{dx} \ln u = \frac{1}{u} \cdot \frac{du}{dx}$	$\int \frac{1}{u} du = \ln u \div \frac{du}{dx} + C$

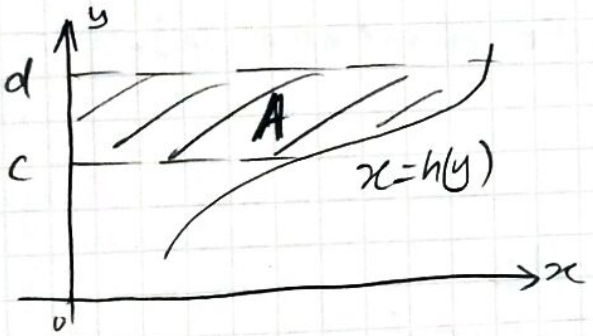
chapter 19



$$A = \int_a^b f(x)\,dx$$

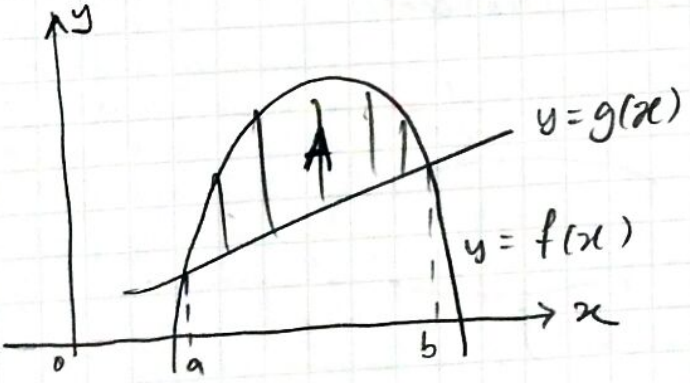


$$A = \left| \int_a^b f(x)\,dx \right|$$

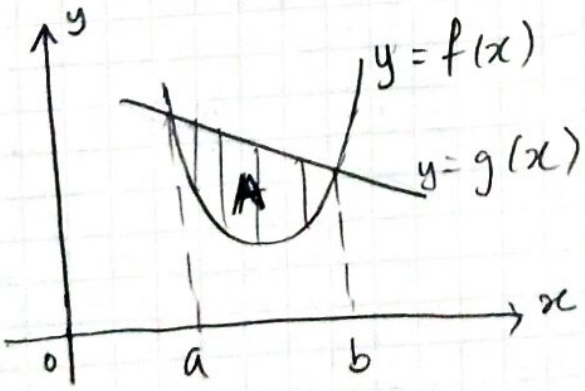


$$A = \int_c^d h(y)\,dy \quad \text{for } h(y) \geq 0$$

$$= \int_c^1 x\,dy$$



$$A = \int_a^b [f(x) - g(x)]\,dx \text{ if } f(x) \geq g(x)$$



$$A = \int_a^b [g(x) - f(x)]\,dx \text{ if } f(x) < g(x)$$

Chapter 20

s = displacement

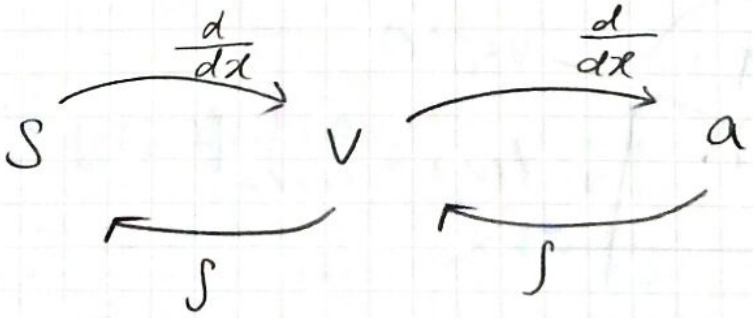
if $s > 0$, the particle is in the positive x-axis
if $s = 0$, the particle is at the fixed point O
if $s < 0$, the particle is in the negative s-axis

v - velocity

if $v > 0$, the particle moves in the direction of the positive x-axis
if $v = 0$, the particle is instantaneously at rest
if $v < 0$, the particle moves in the direction of the negative s-axis

a = acceleration

if $a > 0$, the particle speeds up or accelerates
if $a = 0$, the particle moves at a constant speed
if $a < 0$, the particle slows down or decelerates



$v = \frac{ds}{dt} \Big/ \int a$

$a = \frac{dv}{dt}$

$s = \int v$