SWISS COTTAGE SECONDARY SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATION

NA

Name: ________________________________( ) Class: __________

ADDITIONAL MATHEMATICS
Paper 1

4051/01
Tuesday 1 August 2023
1 hour 45 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 70.

Questions	1	2	3	4	5	6	7	8	9	10	11
Marks	0	4	4	3	1	3	2	7	2	9	6

Total: 41/70

This document consists of **14** printed pages.

Setter: Mdm Zoe Pow
Vetter: Mr Ang Hanping

[Turn over

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

take out common factor!

Answer **all** the questions.

(a^2-ab+b^2)

1 Factorise $54x^3+128$ completely. [3]

$(3x+4)(18x^2-bx+32)$

By comparing x term:

$96x - 4bx = 0$

$4b = 96$

$b = 24$

$\therefore 2(3x+4)(9x^2-12x+16)$

$\therefore (3x+4)(18x^2-24x+32)$

2 **(i)** Given that $x^2+4kx+1-3k=0$ has two real and distinct roots, show that k satisfies $4k^2+3k-1>0$. [2]

$b^2-4ac>0$

$(4k)^2-4(1)(1-3k)>0$

$16k^2-4+12k>0$

$4k^2+3k-1>0$ $)\div 4$ (shown)

(ii) Solve the inequality $4k^2+3k-1>0$. [2]

$(4k-1)(k+1)>0$

-1 $\frac{1}{4}$

$k>-1$ or $k<\frac{1}{4}$

4

3 **(i)** Write down the period and amplitude of $y = 4\cos\left(\frac{x}{2}\right) - 1$. [2]

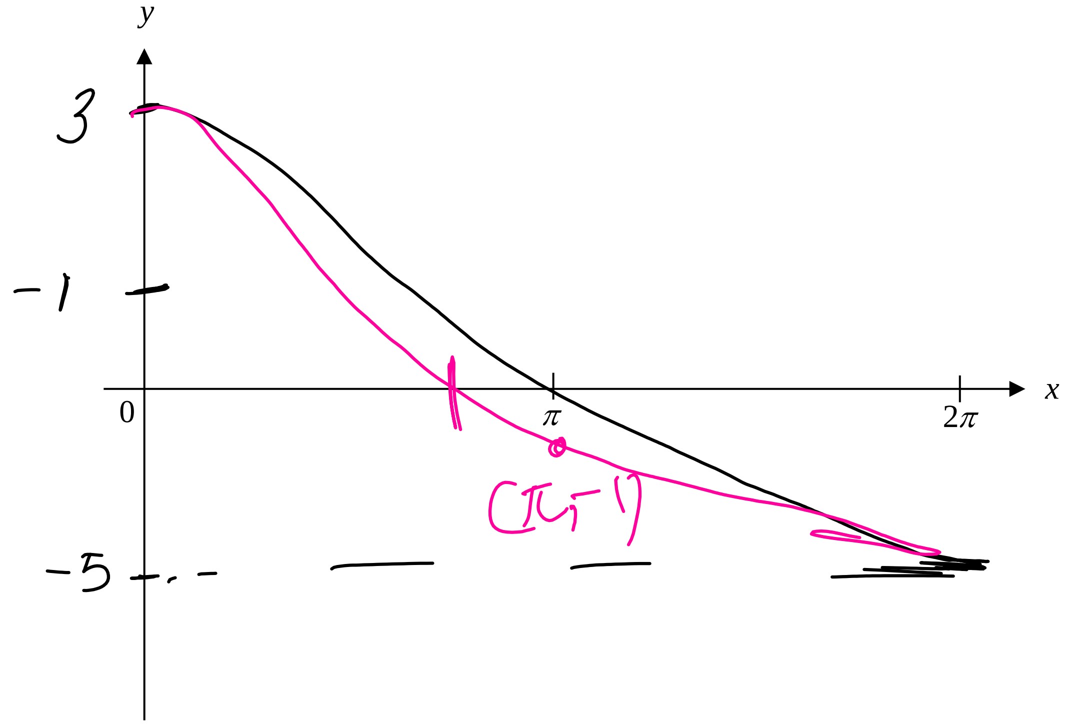
amplitude $= 4$

period $= 360 \div \frac{1}{2}$

$= 720° \,/\, 4\pi$

(ii) Sketch, on the axes below, the graph of $y = 4\cos\left(\frac{x}{2}\right) - 1$ for $0 \le x \le 2\pi$ radians. [3]

2



4

$\frac{dy}{dx}$ (tangent)

4 A curve has gradient $2(x^2+3)$ at every point on the curve. It is given that the curve passes through the point (1, 12).

(i) Find the equation of the curve. ~~[4]~~ 2

$$\int 2(x^2+3)\,dx = \int 2x^3 + 6\,dx$$

M1

$$= \frac{2x^4}{4_2} + 6x + C \qquad \frac{2x^3}{3}$$

$$= \frac{x^4}{2} + 6x + C$$

$$\therefore y = \frac{x^4}{2} + 6x + \frac{11}{2}$$

Sub (1, 12)

$$12 = \frac{1^4}{2} + 6(1) + C \quad \text{M1}$$

$$C = \frac{11}{2}$$

(ii) Explain why this curve has no stationary point. ~~[1]~~

$$2(x^2+3) = 0$$

$$2x^2+6 = 0$$

$$x^2 = -3$$

$\therefore$ Since, there is no solution. the curve have no stationary point

$x^2 \geq 0$ since

$2x^2 \geq 0$ $\frac{dy}{dx} > 0$

$2x^2+6 > 0$

(iii) Given that the equation of the curve can be written in the form $y = P(x)$, explain why $P(x)$ is an increasing function. [1]

$$2(x^2+3)$$

$$2x^2+6$$

since $x^2 \geq 0$, $2x^2+6 \geq 6$

$\therefore$ $2x^2+6 > 0$, $P(x)$ is a increasing function.

$\frac{dy}{dx} > 0$

3

5 **(a)** **Without using a calculator, and showing all your working**, find the exact value of $\sin 30° \cos 45° + \cos 30° \sin 45°$. [2]

$$\left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right)+\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$$

$$=\frac{1}{\sqrt{8}}+\frac{\sqrt{3}}{\sqrt{8}}$$

$$=\frac{1+\sqrt{3}}{\sqrt{8}}\times\frac{\sqrt{8}}{\sqrt{8}}$$

$$=\frac{\sqrt{8}(1+\sqrt{3})}{8}$$

$$\frac{\sqrt{8}+\sqrt{24}}{8}$$

$$=\frac{2\sqrt{2}+\cancel{4}\,2\sqrt{6}}{8}$$

$$=\frac{\cancel{2}(\sqrt{2}+\cancel{2}\sqrt{6})}{\cancel{8}\,4}$$

$$=\frac{\sqrt{2}+\sqrt{6}}{4}$$

$$\tan = \frac{\sin}{\cos}$$

→ find x.

(b) Solve $\cos\left(2x+\frac{\pi}{3}\right)=\frac{\sqrt{2}}{2}$ for $0 \le x \le \pi$ radians giving your answers in terms of π.

[4]

$$\cos 2x \cos\frac{\pi}{3} - \sin 2x \sin\frac{\pi}{3} = \frac{\sqrt{2}}{2}$$

$$2\cos 2x \cos\frac{\pi}{3} - 2\sin 2x \sin\frac{\pi}{3} = \sqrt{2}$$

$$\cancel{2}\cos 2x\left(\frac{1}{\cancel{2}}\right) - \cancel{2}\sin 2x\left(\frac{\sqrt{3}}{\cancel{2}}\right) = \sqrt{2}$$

$$\cos 2x - \sqrt{3}\sin 2x = \sqrt{2}$$

$$1-2\sin^2 2x-\sqrt{3}\sin 2x=\sqrt{2}$$

$$-2\sin^2 2x-\sqrt{3}\sin 2x-\sqrt{2}+1=0$$

1

6 **(i)** Differentiate $\frac{1}{\left(1-x^2\right)^3}$. [2]

$$\frac{d}{dx}\left((1-x^2)^{-3}\right) = -3(1-x^2)^{-4}(-2x)$$

$$= 6x(1-x^2)^{-4}$$

$$= \frac{6x}{(1-x^2)^4} \quad //$$

(ii) Hence evaluate $\int_0^2 \frac{x}{\left(1-x^2\right)^4}\, dx$. ~~[4]~~

$$\int \frac{6x}{(1-x^2)^4}\, dx = \frac{1}{(1-x^2)^3}$$

1

$$\int_0^2 \frac{x}{(1-x^2)^4}\, dx = \left[\frac{1}{6(1-x^2)^3}\right]_0^2$$

$$= -\frac{1}{162} \quad //$$

3

7 The following trigonometric function models the depth of water motion along Kallang River.

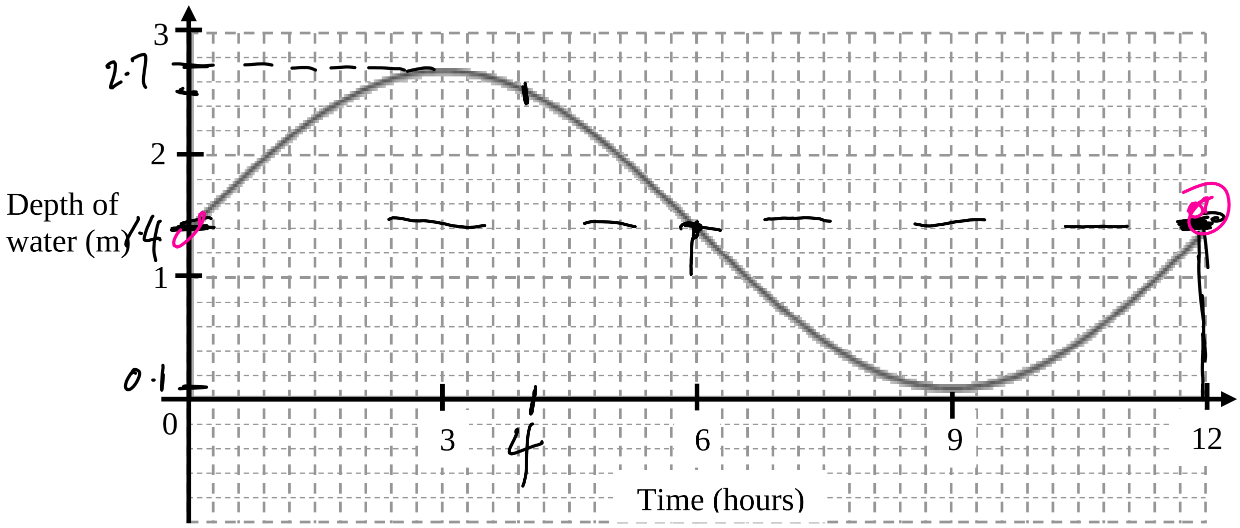
depth hour

$$y = a\sin(bt) + c$$

y is the depth of water, t is time in hours and a, b and c are constants.

At high tide, the depth of water is 2.7 m. Six hours later, at low tide, the depth of water is 0.1 m.

This diagram shows part of the above trigonometric function.



(i) Find the values of a, b and c. [4]

$a = 2.7 - 1.4$
$= 1.3$

$c = 1.4$

$b = 12$

$\frac{2\pi}{12} = \frac{\pi}{6}$

2

2

16 − 12 = 4

(ii) Find the depth of water after 16 hours. [2]

depth = 2.5m

(iii) Kayaking is permitted when the depth of water is at least 1.4 m.

State the time interval(s) between 0 hour and 12 hours where kayaking is permitted. [1]

0

8 **(a)** Show that $\frac{\sqrt{3}+2}{5\sqrt{3}-1}$ can be written in the form of $\frac{a+b\sqrt{3}}{c}$ where a, b and c are integers. [4]

$$\frac{\sqrt{3}+2}{5\sqrt{3}-1} \times \frac{5\sqrt{3}+1}{5\sqrt{3}+1}$$

$$\frac{5(3)+\sqrt{3}+10\sqrt{3}+2}{75-1}$$

$$\frac{17+11\sqrt{3}}{74}$$

$\therefore\ a=17,\ b=11,\ c=74$

(b) It is given that $\sqrt{7}(x+2)=1-x$. Find x in the form $p+q\sqrt{7}$, where p and q are rational numbers. [4]

$$x\sqrt{7}+2\sqrt{7}=1-x$$

$$x\sqrt{7}+x=1-2\sqrt{7}$$

$$x(\sqrt{7}+1)=1-2\sqrt{7}$$

$$x=\frac{1-2\sqrt{7}}{\sqrt{7}+1} \times \frac{\sqrt{7}-1}{\sqrt{7}-1}$$

$$=\frac{\sqrt{7}-1-2(7)+2\sqrt{7}}{7-1}$$

$$\frac{-15+3\sqrt{7}}{6}$$

3

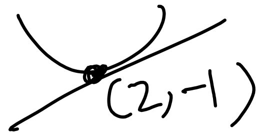
$p=\frac{-15}{6}$ //

$q=\frac{3\sqrt{7}}{6}$ //

7

$= -\frac{5}{2}+\frac{1}{2}\sqrt{7}$

M1



9 A curve is such that $\frac{d^2y}{dx^2} = -3$. The point $(2, -1)$ lies on the curve and the gradient of the curve at this point is 2.

(i) Find the equation of the curve. ~~[6]~~ 2

$$\int -3 \, dx = -3x + c \quad \checkmark \quad \frac{dy}{dx}$$

M1

$$-3(2) + c = 2$$

$$c = 8 \quad \checkmark \quad \text{M1}$$

$$\int -3x + 8 \, dx = \frac{-3x^2}{2} + 8x + d$$

$$\therefore y = \frac{-3x^2}{2} + 8x$$

$$-1 = \frac{-3(2)^2}{2} + 8(2) + d$$

$$d = -11$$

$$\boxed{\therefore y = -\frac{3}{2}x^2 + 8x - 11}$$

<0

(ii) Show that the curve does not intersect the x-axis. ~~[2]~~ 0

$$b^2 - 4ac < 0$$

$$(8)^2 - 4\left(-\frac{3}{2}\right)(0) =$$

$$8^2 - 4\left(-\frac{3}{2}\right)(-11)$$

$$= -2 < 0$$

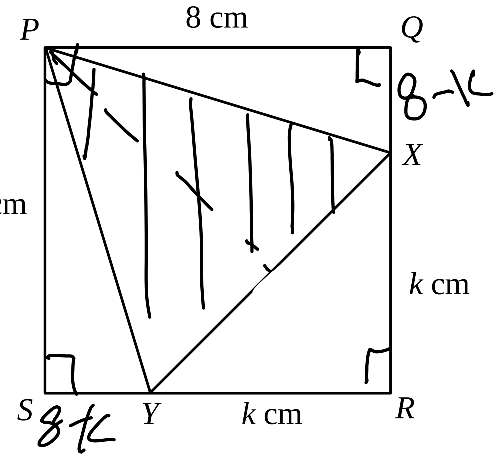
2

Since discriminant < 0,

it has no real roots.

$\therefore$ it does not intersect the curve.

10



The diagram shows a square $PQRS$ of sides 8 cm each and a triangle PXY.
X and Y lie on QR and SR respectively, and $XR = YR = k$ cm.

(i) Show that the area, A cm^2, of triangle PXY is given by

$$A = 8k - \frac{1}{2}k^2.$$

[3]

$$A = 8^2 - \cancel{2}\left(\tfrac{1}{2} \times 8-k \times 8\right) - \left(\tfrac{1}{2} \times k \times k\right)$$

$$= 64 - 8(8-k) - \frac{k^2}{2}$$

$$= \cancel{64} - \cancel{64} + 8k - \frac{k^2}{2}$$

$$= 8k - \frac{1}{2}k^2 \text{ (shown)} //$$

(ii) Given that k can vary, find the value of k for which there is a stationary value of A and calculate this value of A.

[5]

$$\frac{dA}{dk} = 8 - k = 0$$

$$k = 8$$

$$A = 8(8) - \frac{1}{2}(8)^2$$

$$= 32 //$$

8

(iii) Determine whether this stationary value of A is a maximum or minimum value. [1]

$$\frac{d^2A}{dk^2} = -1$$

Since, $\frac{d^2A}{dk^2} < 0$.

by 2^{nd} deverative test, it is a maximum value.

1

11 The equation of a circle is $x^2 + y^2 - 6x + 8y = 39$.

2

(i) Find the coordinates of the centre of the circle and find the radius of the circle. [4]

$2g\cancel{x} = -6\cancel{x}$ centre $(3, -4)$ //

$g = -3$

$2f\cancel{y} = 8\cancel{y}$ radius $= \sqrt{(-3)^2 + 4^2 + 39}$

$f = 4$ $= 5$ units //

8 units

(ii) Show that the points $A(-5, -4)$ and $B(11, -4)$ lie on the circle and that AB is a diameter of the circle. [4]

Midpoint $= \left(\frac{-5+11}{2}, \frac{-4-4}{2} \right)$

$= (3, -4)$

A:

LSH $= (-5)^2 + (-4)^2 - 6(-5) + 8(-4) = 39$ = RHS

6

B:

LSH $= (11)^2 + (-4)^2 - 6(11) + 8(-4) = 39$ = RHS

∴ Since midpoint of AB is the centre of

End of Paper

the circle, it is a diameter. point A & B satisfy the eq^n, point A & B lies on the circumference. (Shown) //