AMath Prelim 2019 Paper 2 Answer Scheme

Qn	Answer	Mark Allocation
1(i)	When $t=0; P=50000$ $t=2; 100000=50000e^{2k}$ $e^{2k}=2$ $2k=\ln 2$ $k=\frac{1}{2}\ln 2$	M1 M1
1(ii)	When $t=5; P=50000e^{5(\frac{1}{2}\ln 2)}$ $P=282842$ ≈ 280000	M1 A1
1(iii)	$50000e^{kt}=450000$ $e^{\frac{1}{2}\ln 2t}=9$ $\frac{1}{2}\ln 2t=\ln 9$ $t=6.34$ $\text{Year}=2016$	 M1 M1 A1
2(a)	$\frac{dy}{dx}=e^x\cos x+e^x\sin x$ $\frac{d^2y}{dx^2}=-e^x\sin x+e^x\cos x+e^x\cos x+e^x\sin x$ $=2e^x\cos x$ $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+2y$ $=2e^x\cos x-2(e^x\cos x+e^x\sin x)+2e^x\sin x$ $=2e^x\cos x-2e^x\cos x-2e^x\sin x+2e^x\sin x$ $=0(\text{shown})$	M1 M1 M1 M1

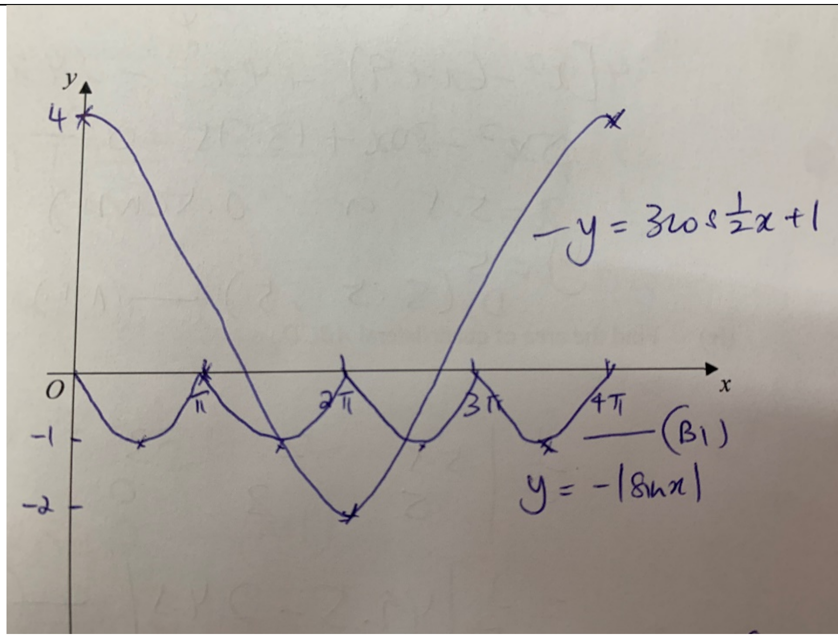
2(b)	$\frac{dy}{dx}=\frac{2(x-1)-(2x+16)}{(x-1)^2}$ $=\frac{-18}{(x-1)^2}$ $\frac{dy}{dx}=-2\frac{dx}{dt}$ $-2\frac{dx}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}$ $-2\frac{dx}{dt}=\frac{-18}{(x-1)^2}\times\frac{dx}{dt}$ $(x-1)^2=9$ $x-1=3$ or $x-1=-3$ $x=4$ or $x=-2$(NA)	M1 M1 M1 M1 A1
3(a)	$\lg x^2-2\left(\frac{\lg 10}{\lg x}\right)=3$ $\lg x^2-\frac{2}{\lg x}=3$ $2\lg x-\frac{2}{\lg x}=3$ $2(\lg x)^2-2=3\lg x$ Let $\lg x=y$ $2y^2-3y-2=0$ $(2y+1)(y-2)=0$ $y=-\frac{1}{2}$ or $y=2$ $\lg x=-\frac{1}{2}$ or $\lg x=2$ $x=10^{-\frac{1}{2}}$ or $x=10^2$ $x=0.316$ or $x=100$	M1 M1 M1 M1 A1

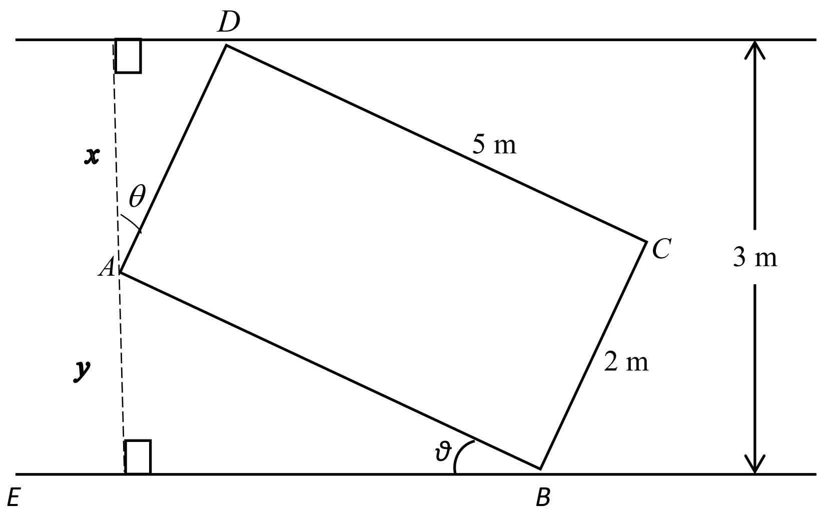
3(b)	$\log_m x + \log_m y^3 = a$ $\log_m x + 3\log_m y = a \quad -(1)$ $\log_m x^2 = b$ $2\log_m x = b$ $\log_m x = \frac{b}{2} - (2)$ $\frac{b}{2} + 3\log_m y = a$ $b + 6\log_m y = 2a$ $\log_m y = \frac{2a-b}{6}$ $\log_m x - \log_m y$ $= \frac{b}{2} - \frac{2a-b}{6}$ $= \frac{3b-2a+b}{6}$ $= \frac{4b-2a}{6}$ $= \frac{2b-a}{3}$	M1 M1 M1 M1 A1
3(c)	$8 \times 4^a = 2^{2b-1}$ $2^3 \times 2^{2a} = 2^{2b-1}$ $3 + 2a = 2b - 1$ $2a - 2b = -4$ $a - b = -2$ $3^a \sqrt{3^b} = 81$ $3^a \times 3^{\frac{b}{2}} = 3^4$ $a + \frac{b}{2} = 4$ $b - 2 + \frac{b}{2} = 4$ $b = 4, a = 2$	M1 M1 M1 A1

4(a)	$\frac{dy}{dx}=\frac{-10}{kx^3}+30x^2$ $\frac{-10}{kx^3}+30x^2=0$ $-10+30kx^5=0$ $10=30(k)(2^5)$ $k=\frac{1}{96}$	M1 M1 A1
4(b)(i)	$3x+2>0$ $x>-\frac{2}{3}$	B1
4(b)(ii)	$\frac{dy}{dx}=\frac{1}{2}\left(\frac{3}{3x+2}\right)$ $=\frac{3}{2(3x+2)}$	M1 A1
4(b)(iii)	$y=0$ $\ln\sqrt{3x+2}=0$ $\frac{1}{2}\ln(3x+2)=0$ $\ln(3x+2)=0$ $3x+2=e^0$ $3x+2=1$ $x=-\frac{1}{3}$ $\frac{dy}{dx}=\frac{3}{2[3\times-\frac{1}{3}+2]}$ $=\frac{3}{2}$ Equation of normal: $y=-\frac{2}{3}x+c$ $\text{At}(-\frac{1}{3},0), c=-\frac{2}{9}$ $y=-\frac{2}{3}x-\frac{2}{9}$	M1 A1 M1 M1 A1

5(i)	$\frac{dy}{dx}=2-\frac{8}{(2x-1)^2}$ $2-\frac{8}{(2x-1)^2}=0$ $\frac{8}{(2x-1)^2}=2$ $(2x-1)^2=4$ At(p,q); $(2p-1)^2=4$ $2p-1=2$ or $2p-1=-2$ $p=1\frac{1}{2}$ or $p=-\frac{1}{2}$(NA) $q=5$	M1 M1 M1 A1 A1
5(ii)	$\frac{d^2y}{dx^2}=\frac{32}{(2x-1)^3}$ At $x=-\frac{1}{2}$; $y=-3$ $\left(-\frac{1}{2},-3\right)$ At $x=-\frac{1}{2}$; $\frac{d^2y}{dx^2}=\frac{32}{(2\times-\frac{1}{2}-1)^3}<0$ $\left(-\frac{1}{2},-3\right)$ is max point	M1 A1 M1 A1
6(i)	$1-6px+15p^2x^2$	A1, A1 for both terms
6(ii)	$(1-4x+4x^2)(1-6px+15p^2x^2)$ $=15p^2x^2+24px^2+4x^2$ Coefficient of $x^2=15p^2+24p+4$	M1 A1
6(iii)	$-6px-4x$ $=(-6p-4)x$ $15p^2+24p+4=2(-6p-4)$ $15p^2+24p+8p+8=0$ $15p^2+36p+12=0$ $(5p+2)(3p+6)=0$ $p=-\frac{2}{5}$ or $p=-2$	M1 M1 M1 A1

7(i)	Midpoint of AC $\left(\frac{2+6}{2},\frac{3+2}{2}\right)$ $=(4,2)$ Gradient of $AC=-\frac{1}{2}$ Gradient of $BD=2$ $y=2x+c$ At$(4,2);2=(2)(4)+c$ $c=-6$ $y=2x-6$	M1 M1 M1 A1
7(ii)	$y=0$ $2x-6=0$ $x=3$	B1
7(iii)	$2\sqrt{(x-3)^2+y^2}=5\sqrt{(4-3)^2+2^2}$ $4[(x-3)^2+y^2]=25(5)$ $(x-3)^2+(2x-6)^2=\frac{125}{4}$ $4[(x^2-6x+9)+4x^2-24x+36]=125$ $5x^2-30x+13.75=0$ $x=5.5$ or 0.5(NA) $y=5$ $D(5.5,\ 5)$	M1 M1 A1
7(iv)	Area $=\frac{1}{2}\begin{vmatrix}5.5 & 2 & 3 & 6 & 5.5\\ 5 & 3 & 0 & 1 & 5\end{vmatrix}$ $=\frac{1}{2}\lvert 49.5-24.5\rvert$ $=12.5\text{units}^2$	M1 A1
8(i)	$a=3$ $b=0.5$ $c=1$	B1 B1 B1
8(ii)	Correct shape, amplitude 3 Correct turning points, end points $y=-\lvert\sin x\rvert$	M1 M1 B1

		
8(iii)	$k=4$	B1
9(i)	$x(-2\sin 2x)+\cos 2x$ $=-2x\sin 2x+\cos 2x$	M1, M1 A1
9(ii)	$\int_0^{\frac{\pi}{2}} (-2x\sin 2x+\cos 2x)dx=\left[x\cos 2x\right]_0^{\frac{\pi}{2}}$ $\int_0^{\frac{\pi}{2}} (-2x\sin 2x)dx+\left[\frac{\sin 2x}{2}\right]_0^{\frac{\pi}{2}}=\frac{\pi}{2}\cos\pi$ $\int_0^{\frac{\pi}{2}} (-2x\sin 2x)dx=-\frac{\pi}{2}-\frac{\sin\pi}{2}$ $\int_0^{\frac{\pi}{2}} (-2x\sin 2x+3)dx=-\frac{\pi}{2}+\left[3x\right]_0^{\frac{\pi}{2}}$ $=-\frac{\pi}{2}+\frac{3\pi}{2}$ $=\pi$	M1 M1 M1 M1 A1
10(a)(i)	$x^2-px+2p-3=0$ $b^2-4ac>0$ $p^2-4(2p-3)>0$ $p^2-8p+12>0$ $(p-2)(p-6)>0$ $p<2$ or $p>6$	M1 M1 M1 A1
10(a)(ii)	$y=2(x-2)$ $p=2$ therefore, $y=2x-4$ is tangent to curve	M1 A1

10(b)	$b^2-4ac=(k-2)^2-4(1)(-2k)$ $=k^2-4k+4+8k$ $=k^2+4k+4$ $=(k+2)^2$ ≥ 0 Since $(k+2)^2\geq 0$, $b^2-4ac\geq 0$ roots are real for all values of k.	M1 M1 A1
11(i)	 $\sin\theta=\frac{y}{5}$ $y=5\sin\theta$ $\cos\theta=\frac{x}{2}$ $x=2\cos\theta$ $x+y=3$ $2\cos\theta+5\sin\theta=3$	 M1 M1
11(ii)	$R\sin\alpha=5$ $R\cos\alpha=2$ $R=\sqrt{2^2+5^2}$ $R=\sqrt{29}$ $\tan\alpha=\frac{5}{2}$ $\alpha=68.1°$ $\sqrt{29}\cos(\theta-68.2°)$	M1 M1 M1 A1

11(iii)	$\sqrt{29}\cos(\theta - 68.2°) = 3$ $Acute\angle = 56.14°$ $\theta - 68.2° = -56.14°$ $\theta = 12.1°$	M1 A1