1 Solve the inequality $\dfrac{x-5}{x-1} \le -1$. [3]

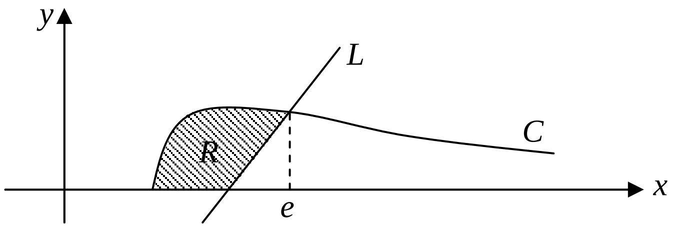
Hence solve $\dfrac{\cos x - 4}{\cos x} \le -1$ for $0 \le x \le \pi$. [2]

2 A circular cake of radius r is cut into 22 sectors. The areas of the sectors form an increasing arithmetic progression. The area of the eighth sector is twice the area of the smallest sector. Find, in terms of π, the angle of the largest sector. [4]

[Area of Sector = $\dfrac{1}{2}r^2\theta$ where θ is the angle of the sector]

3(a) Find $\displaystyle\int \frac{6+2x}{\sqrt{1-4x-x^2}}\,\mathrm{d}x$. [5]

3(b)



The diagram above shows the region R bounded by the curve C with equation $y = \dfrac{\sqrt{\ln x}}{x}$, $x \ge 1$, the x-axis and the line L with equation $y = \dfrac{1}{e\,(e-2)}(x-2)$.

Find the exact volume of the solid of revolution when R is rotated completely about the x-axis. [5]

4 It is given that $\mathrm{f}(x) = \dfrac{a - x^2}{1 + x^2}$ where a is a real constant and $a \ne -1$.

Find the range of values of a such that the curve of $y = \mathrm{f}(x)$ has a maximum point. [4]

Given that $a > 1$.

(i) Sketch $y = \mathrm{f}(x)$, showing clearly the coordinates of the turning point, any intersections with the axes and the equation(s) of any asymptote(s). [2]

(ii) By drawing a sketch of another suitable curve in the same diagram, find the number of roots of the equation

$$\pi\left(a - x^2\right) = \left(1 + x^2\right)\tan^{-1} x \;.$$ [2]

5 The function f is defined by $\mathrm{f}: x \to \dfrac{1}{1-x^2}$, $-1 < x \le 0$.

(i) Define f^{-1}. [3]

(ii) Sketch the graphs of $y = \mathrm{f}\mathrm{f}^{-1}(x)$ and $y = \mathrm{f}^{-1}\mathrm{f}(x)$ on the same diagram. Hence, solve the equation $\mathrm{f}\mathrm{f}^{-1}(x) = \mathrm{f}^{-1}\mathrm{f}(x)$. [3]

Figure 1 shows the graph $y = \dfrac{1}{2} + e^{-x}$, $x \ge 0$, which undergoes a sequence of two geometrical transformations as shown below. P_1 and P_2 are points corresponding to the point P after each transformation.

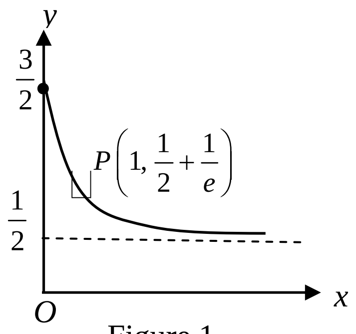


Figure 1

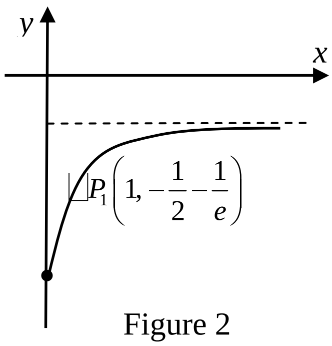


Figure 2

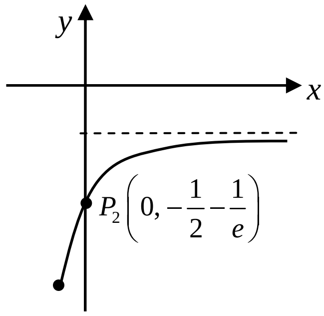


Figure 3

The resulting graph in Figure 3 shows the graph of the function h.

(iii) Find h(x). [2]

(iv) Explain clearly why the composite function fh does not exist. [2]

(v) Find the maximal domain of h for fh to exist and hence find the range of fh. [4]

6 The points (x, y) on a curve satisfy the equations

$$x = \frac{1}{2}t^2 - 2t \quad \text{and} \quad \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2t-1} \qquad \text{where } t \text{ is a parameter,} \quad t > 2.$$

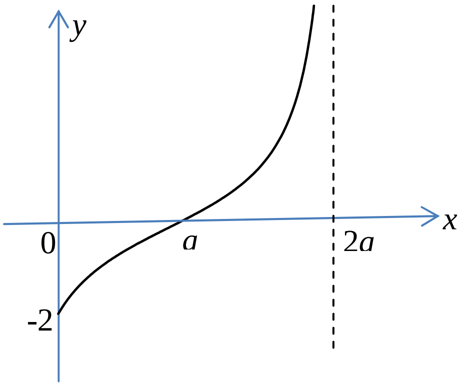
It is given that $y = 3 - \dfrac{3}{4}\ln 7$ when $t = 4$.

(i) Find y in terms of t. [4]

(ii) Show that $\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2} = \dfrac{-2}{(2t-1)^2(t-2)}$. [2]

(iii) Find the Maclaurin's series for y in terms of x up to and including the term in x^2. [3]

7 The figure below shows a sketch of part of the graph of $y = \mathrm{f}(x)$ for $0 \le x < 2a$. The vertical asymptote $x = 2a$ is also a line of symmetry of the graph.



Sketch, on separate diagrams, the graph of

(i) $y = \mathrm{f}(x)$ for $0 \le x \le 4a$; [2]

(ii) $y = \dfrac{1}{\mathrm{f}(x)}$ for $0 \le x \le 4a$; [3]

(iii) $y = \mathrm{f}\left(\dfrac{x}{2} + a\right)$ for $-2a \le x \le 6a$. [3]

indicating clearly the asymptote(s) and the axial intercept(s).

8 The equations of planes P_1, P_2 are

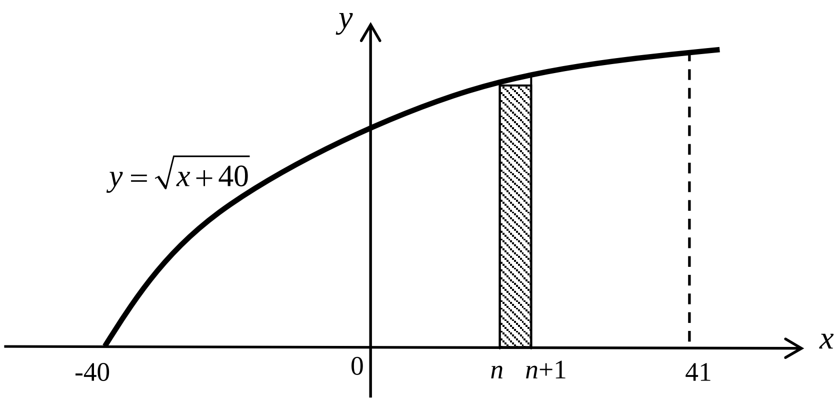
P_1: $\mathbf{r} = \begin{pmatrix} -3 \\ 10 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 12 \\ 3 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, $\lambda, \beta \in \mathbb{R}$ P_2: $\mathbf{r} \cdot \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} = 1$

Find the coordinates of the foot of perpendicular from the point $A(-3, 10, 3)$ to the plane P_2 and show that the point $B(2, 10, -2)$ is the reflection of point A in P_2. [5]

The planes P_1 and P_2 meet in a line L. Find a vector equation of line L. [3]

Plane P_3 is the reflection of P_1 in P_2. Using the results above, find a vector perpendicular to P_3. Hence find, in scalar product form, the equation of P_3. [3]

9 The graph of $y=\sqrt{x+40}$ is shown in the diagram below.



By considering the shaded rectangle, show that

$$\sqrt{n+40} < \int_{n}^{n+1} \sqrt{x+40}\ \mathrm{d}x .$$ [1]

Deduce that

$$\sqrt{1} + \sqrt{2} + \sqrt{3} + \ldots + \sqrt{80} < \int_{-40}^{41} \sqrt{x+40}\ \mathrm{d}x .$$ [2]

Show also that $$\sqrt{n+41} > \int_{n}^{n+1} \sqrt{x+40}\ \mathrm{d}x .$$ [2]

Deduce that

$$\sqrt{1} + \sqrt{2} + \sqrt{3} + \ldots + \sqrt{81} > \int_{-40}^{41} \sqrt{x+40}\ \mathrm{d}x .$$ [1]

Hence, deduce the value a, where $a \in \mathbb{Z}$, that satisfies the following inequality,

$$9a < \sqrt{1} + \sqrt{2} + \sqrt{3} + \ldots + \sqrt{80} < 9(a+1).$$ [2]

10 Solve the equation $z^5 + 16 = 16\sqrt{3}i$, giving the roots in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \le \pi$. Show the roots on an Argand diagram. [5]

The points A and B represent the two roots with the two smallest positive arguments. Point P, the mid-point of AB, represents the complex number w.

(i) Find, in exact form, the modulus and argument of w. [2]

(ii) w is also an n^{th} root of k, where n is a positive integer and k is a real number. Find the least possible value of n and find the corresponding value of k, leaving your answer in trigonometric form. [3]

11(a) A sequence u_1, u_2, u_3, … is such that $u_1 = \frac{e}{10}$ and

$$u_n - u_{n-1} = \frac{e}{2^{n-1}}, \text{ for all } n \geq 2 .$$

Use the method of difference to show that

$$u_n = \frac{e}{10}\left[11 - 10\left(\frac{1}{2}\right)^{n-1}\right].$$ [4]

State, with a reason, whether u_n is a convergent sequence. [1]

11(b) The sequence a_1, a_2, a_3 ……..... is defined by

$$a_r = \frac{1}{1+2+3+.....+r}, \quad \text{where } r \text{ is a positive integer.}$$

Another sequence $b_1, b_2, b_3 \ldots$ is defined by

$$b_n = \frac{1}{4}\left(\sum_{r=1}^{n} a_r\right), \quad \text{where } n \text{ is a positive integer.}$$

(i) Find the terms b_1, b_2 and b_3. Hence, make a conjecture for b_n and express your answer in the form of $\frac{n}{\mathrm{f}(n)}$. [3]

(ii) Prove your conjecture using mathematical induction. [5]

END OF PAPER