HCI 2024 H2 Mathematics Preliminary Examinations Paper 2 Section A (Pure Maths)

Question	**1**	**2**	**3**	**4**	**5**	**6**
Marks	6	5	7	5	7	10

Question 1 [6]

Part (a) [3]

By considering $tan\,(A - B)$, show that

$$tan^{-1}\,(\tfrac{1}{x}) - tan^{-1}(\tfrac{1}{1+x}) = tan^{-1}(\tfrac{1}{x^2+x+1}).$$

Part (b) [3]

Hence show that

$$\sum_{r=1}^{1} tan^{-1}(\tfrac{1}{r^2+r+1}) = k - f(n),$$

Where $f(n)$ is an inverse trigonometric function and k is an exact constant to be found.

Question 2 [5]

A sequence is defined by the recurrence relation

$$u_{n+1} = \frac{1 + u_n}{1 - u_n}, \qquad \text{for } n \geq 1.$$

Part (a) [1]

State what happens to the sequence when $u_1 = 0$.

It is now given that $u_1 = 2$.

Part (b) [2]

Find u_2, u_3, u_4, u_5, and u_6.

Part (c) [2]

By observing the pattern in part **(b)**, find $\sum_{r=1}^{4n} u_r$ in terms of n.

Question 3 [7]

Part (a) [5]

A curve C has equation $2y^3 - y^2 = xe^x$.

Find the equations of the tangents which are parallel to the y-axis.

Part (b) [2]

It is given that the tangents found in part **(a)** make an acute angle of $\frac{\pi}{6}$ radians with the line $y = mx + 1$. Find the values of m.

Question 4 [5]

Part (a) [2]

For any non-parallel and non-zero vectors **m** and **n**, **explain clearly** and show that

$$(\mathbf{m}\cdot\mathbf{n})^2 + |\mathbf{m}\times\mathbf{n}|^2 = |\mathbf{m}|^2\,|\mathbf{n}|^2$$

P and Q are two distinct points, where $\vec{OP} = \mathbf{p}$ and $\vec{OQ} = \mathbf{q}$. It is also known that **p** and **q** are non-zero vectors.

Two parallel lines l_1 and l_2 have vector equations

$$\mathbf{r} = \mathbf{p} + s\mathbf{u} \text{ and } \mathbf{r} = \mathbf{q} + t\mathbf{v} \text{ respectively, where } s, t \in \mathbf{R}.$$

Part (b) [3]

If $\mathbf{v} \times (\mathbf{p} - \mathbf{q}) = \mathbf{0}$, what can be said about the relationship between the two lines?
Justify your answer.

Question 5 [7]

Part (a) [4]

Using standard series from the **List of Formulae (MF26)**, find the Maclaurin expansion of $\frac{1}{(1+cos x)^2}$ in ascending powers of x up to and including the term in x^4.

Part (b) [3]

Find the set of values of x for which $\frac{1}{(1+cos x)^2}$ is within ±0.5 of the polynomial found in part **(a)**, where $0 \le x \le \pi$.

Question 6 [10]

Part (a) [1]

Given that $u = x + iy$, where x and y are real numbers, show that $|u|^2 = u\,u^*$.

Two complex numbers z and w with non-zero real and imaginary parts satisfy

$$|z + w| = |z - w|, \text{ where } z \neq w.$$

Part (b) [3]

By considering part (a), show that $zw^* + z^*w = 0$.

Part (c) [2]

Hence show that zw^* is purely imaginary.

It is now given that $w = -1 + i\sqrt{3}$, and the argument of z is θ, where $-\pi < \theta \leq \pi$.

Part (d) [4]

Using the result in part (c), find the possible exact values of θ.

HCI 2024 H2 Mathematics Preliminary Examinations Paper 2 Section B (Statistics)

Question	**7**	**8**	**9**	**10**	**11**	**12**
Marks	6	7	9	12	12	14

Question 7 [6]

A factory produces a large number of monitor screens. It is known that, on average, $100p\%$ of the monitor screens are faulty. The number of faulty monitor screens produced each day is independent of that on other days. Each day, the quality control manager will produce a check on n randomly chosen monitor screens produced on that day.

Let M be the number of faulty monitor screens found. You may assume that M can be modelled by a binomial distribution.

Part (a) [2]

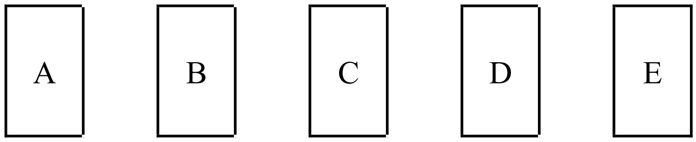
State the probability that on a particular day, there are at least 2 but no more than 3 faulty monitor screens found, giving your answer in terms of n and p.

Part (b) [4]

Each day, the quality control manager will perform a check on 10 randomly chosen monitor screens produced. Find the possible values of p such that there is a 25% chance that on a randomly chosen week with 5 working days, there are exactly 3 days with at least 2 but no more than 3 faulty monitor screens found.

Question 8 [7]

A conference hall has five doors, labelled A, B, C, D and E, which are located side by side as shown below. The doors are to be painted using four distinct colours, and each door will be painted with a single colour.



Part (a) [1]

By considering the number of colours available for each door, find the number of ways to paint the five doors such that there is no restriction to the colour of each door.

Part (b) [3]

Find the number of ways to paint the doors such that there are no consecutive doors which are of the same colour.

Part (c) [3]

Find the number of ways to paint the doors if all four colours are to be used.

Question 9 [9]

In this question, you should state the parameters of any distribution you use.

A ceramic shop sells handmade ceramic cups. The mouths of the cups are assumed to be circular in shape. The diameter of the outer circumferences of the top rim of the cups, S, are assumed to follow a normal distribution with mean μ mm and standard deviation σ mm.

Part (a) [3]

It is given that $P(S < 80.5) = P(S > 84.5)$ and that the probability of the diameter of the outer circumference of the top rim of a randomly chosen cup being more than 85mm is 1.15%. Find the value of μ, and show that $\sigma = 1.10$, when corrected to 3 significant figures.

Part (b) [3]

The shop also makes covers of circular shape that can be fitted over the mouths of the cups. The diameter of any randomly chosen cover, C, in mm, follows a normal distribution with mean 83mm and standard deviation 1.5mm. A cover would be considered to be well-fitted over the mouth of the cup if the diameter of the cover is not larger than that of the outer circumference of the top rim of the cup by not more than 2mm. Find the probability that a randomly chosen cover is well-fitted over the mouth of a randomly-chosen cup.

Part (c) [3]

A cover and a cup are randomly chosen. If the cover is well-fitted over the mouth of the cup, find the probability that the diameter of the cover is larger than that of the cup by more than 1.5mm.

Question 10 [12]

An online website, Star-Salary, which shares information on the salaries for fresh graduates in Singapore, claimed that the mean monthly salary of a fresh graduate with a Bachelor of Science (B.Sc) degree was \$3600.

However, another website, First-Pay, stated a higher mean monthly salary for a fresh graduate with the same degree. A random sample of 80 fresh graduates with a B.Sc degree is surveyed and their monthly salaries, \$$x$, are summarised by

$$\Sigma(x - 3600) = 1000, \qquad \Sigma(x - 3600)^2 = 205000.$$

Part (a) [1]

Give a reason why it is challenging to obtain a random sample in this context.

Part (b) [2]

Calculate **exact** unbiased estimates of the population mean and variance for the monthly salaries of fresh graduates with a B.Sc degree.

Part (c) [4]

Test, at the 5% level of significance, whether First-Pay's claim is justified. You should state your hypotheses and define any parameters that you use.

Part (d) [1]

Explain, with justification, whether any assumption about the population is needed for the test in part **(c)** to be valid.

Part (e) [1]

State, in the context of the question, the meaning of “5% level of significance”.

A second sample of 60 randomly chosen fresh graduates with B.Sc degree is surveyed and the sample mean and standard deviation of their monthly salaries are found to be $\$\bar{y}$ and \$355 respectively.

Part (f) [3]

Find the largest value of $\bar{y}$ such that this second sample would conclude the test in favour of Star-Salary’s claim at 5% level of significance, giving your answer correct to the nearest dollar.

Question 11 [12]

An experiment was carried out to investigate the growth rate of a particular species of plant. The following table gives the height of the plant specimen, h centimetres, at the start of the nth month.

n	1	2	4	6	8	10
h	6.22	9.06	13.62	16.62	18.46	19.72

A possible model for the growth rate is given to be

$$h = \frac{an}{b+n}, \text{ where } a \text{ and } b \text{ are constants.}$$

Part (a) [4]

By writing the above equation in a form that is linear in $\frac{1}{h}$ and $\frac{1}{n}$, calculate the equation of the least squares regression line of $\frac{1}{h}$ on $\frac{1}{n}$. Hence, find estimates for the values of a and b, correct to 3 decimal places.

Part (b) [1]

Sketch a scatter diagram for $\frac{1}{h}$ on $\frac{1}{n}$ and include the least squares regression line found in part **(a)**.

Part (c) [1]

Explain, in the context of the question, the significance of the value of a.

Part (d) [3]

Use the least squares regression line in part **(a)** to find the least integer value of n required for the plant to reach a height of 18 centimetres. Explain whether you would expect this estimate to be reliable.

For a line of best fit $y = f(x)$, the residual for a point (p, q) plotted on the scatter diagram is the vertical distance between $(p, f(p))$ and (p, q).

Part (e) [1]

Mark the residual for each point on the scatter diagram in part **(b)**.

Part (f) [1]

Find the sum of squares of the residuals for the least squares regression line of $\frac{1}{h}$ on $\frac{1}{n}$, giving your answer correct to 5 significant figures.

Question 12 [14]

The probability distribution function of a discrete random variable, X, is given as follows:

$$
P(X = x) \quad \begin{cases} = \frac{1}{2}P(X = |x| + 1) & , \quad \text{if } x = -2, -1 \\ = a & , \quad \text{if } x = 0, 1 \\ = b & , \quad \text{if } x = 2, 3 \\ = 0 & , \quad \text{otherwise} \end{cases}
$$

Part (a) [3]

Show that $b = \frac{1-2a}{3}$.

Part (b)(i) [3]

Show that $E(X) = \frac{7}{6} - \frac{4a}{3}$.

Part (b)(ii) [3]

Find $Var\ (X)$ in terms of a.

Part (b)(iii) [2]

Find the range of values of a for which $Var\ (X)$ exists.

Let $a = \frac{7}{20}$.

Part (c) [3]

A random sample of 50 observations of X is taken. Find the probability that the sum of these observations differs from 36 by less than 5.