H2 Topic 12b – Superposition

A Water Spouting Bowl is a dramatic demonstration of standing waves. When damp hands run over the handles under the right conditions, resonance occurs and standing waves form around the circular bronze bowl, resulting in water ejecting at antinodes around the rim.

Content (H211 Waves)
- Determination of frequency and wavelength of sound waves

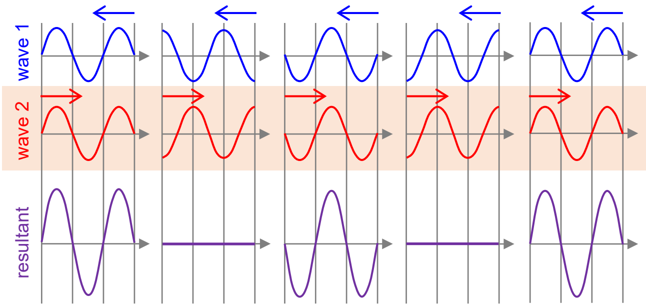
Learning Objectives:
Candidates should be able to:
(k) determine the wavelength of sound using stationary waves

Content (H212 Superposition)
- Stationary waves

Learning Outcomes
Candidates should be able to:
(c) show an understanding of experiments which demonstrate stationary waves using microwaves, stretch strings and air columns
(d) explain the formation of a stationary wave using a graphical method, and identify nodes and antinodes

12.4 Stationary Waves

In H211 Waves, we learnt that a progressive wave transports energy. Here, we shall see how the superposition of two waves can cause interference such that energy is localised (confined within a region) despite wave motion. The waves formed are called *stationary* waves or *standing* waves.

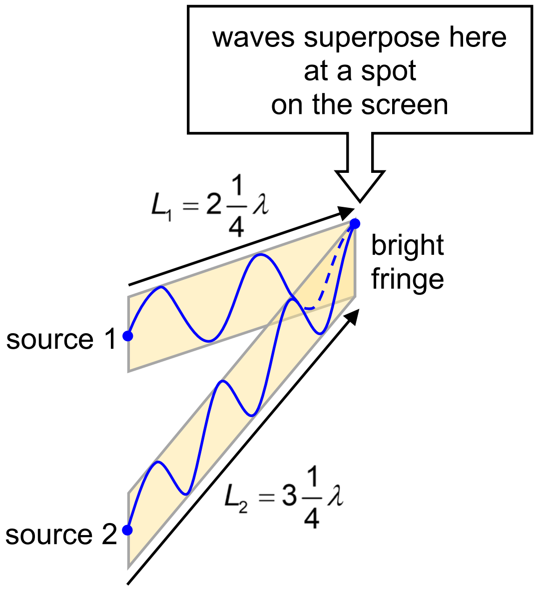


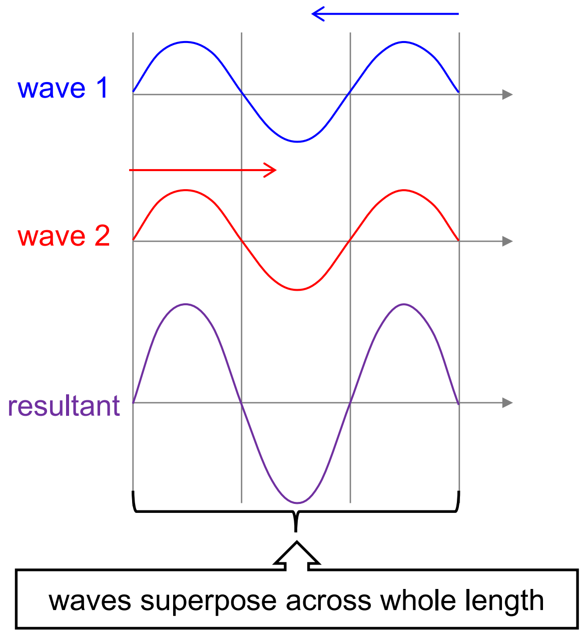
The resulting standing wave has a wave profile that visually moves up and down. We can also think of it as two waves transporting energy in opposite directions, so "net-net" the energy no longer is transported and hence is "stationary".

The wave is stationary because the wave profile does not propagate although the particles still oscillate. Because the wave profile does not move, energy is not transmitted along the wave.

12.4.1 Region of Overlap

For the phenomena that we have seen earlier in H212a, we apply the Principle of Superposition at a *particular location* on the screen. For standing waves, we apply the principle over the *entire length* over which two waves meet and overlap.



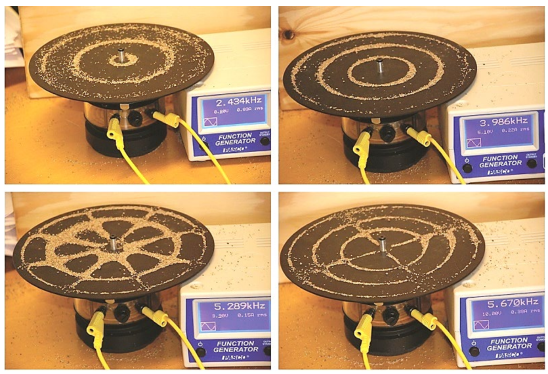


12.4.2 Formation of Stationary Waves

> A stationary wave is formed when
>
> two waves of the same type, same amplitude, same frequency, wavelength and speed,
> travelling in opposite directions towards each other,
> meet and overlap to superpose across a length.

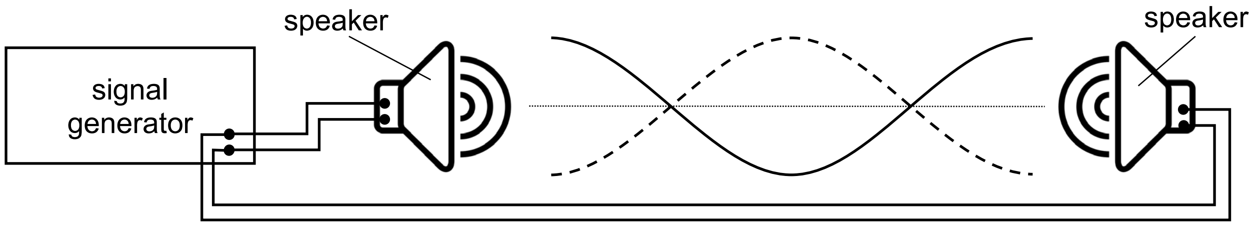
When asked to explain the formation of stationary waves in a particular set up, with the necessary features mentioned above in mind, be sure to describe:

- *how* 2 progressive waves of the same type are generated such that they are the same amplitude, frequency, wavelength and speed
- *how* the 2 waves are travelling in opposite directions towards each other
- *where* the 2 waves meet and overlap



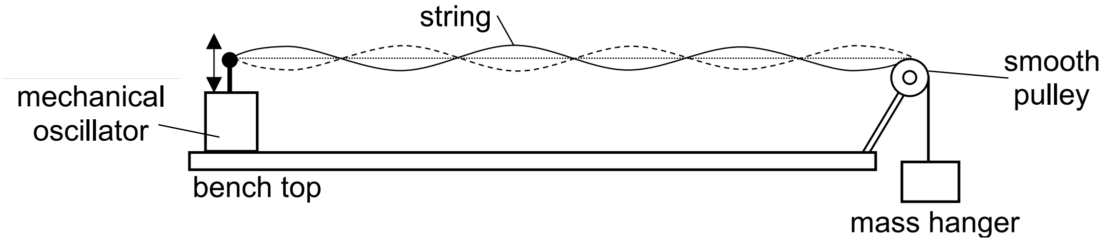
For A-Level H2 Physics, standing waves are almost only 1D such as on a string or along a column. In real life, there are stationary waves on 2D surfaces (e.g. drum skins) and in 3D structures.

Example 20
Two speakers connected in parallel to the same signal generator are directed facing each other to investigate stationary waves. Explain how a stationary wave is formed.



Solution
Both speakers output progressive longitudinal sound waves in opposite directions
that have the same speed as they travel in the same medium of air
and have the same wavelength and frequency as they are connected to same signal generator.
Both waves meet and overlap to superposes along the line joining the two speakers

Example 21
In the set up below, explain how the stationary wave is formed in the string.



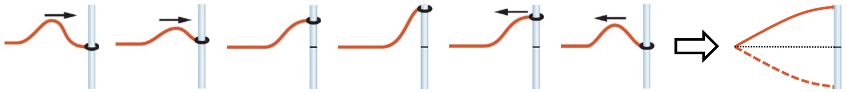
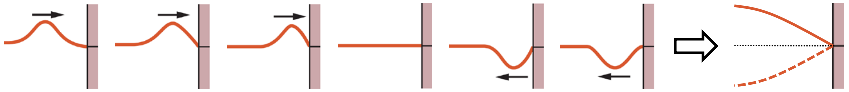
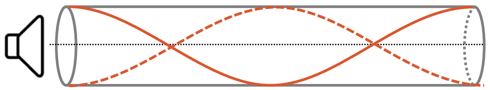
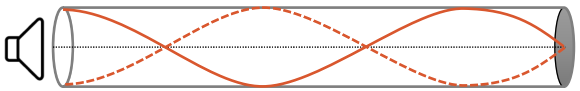
Solution
oscillator sends out progressive transverse wave in string towards pulley
wave reflects at the end of the string where pulley is, back towards oscillator
reflected wave has same speed, frequency and wavelength but travels in opposite direction
meets and overlap with wave from oscillator to superpose along the string between the oscillator and the smooth pulley

Note: for both E.g. 20 and 21, check against the features mentioned at bottom of page 2.

12.4.3 Reflection of Progressive Waves at Boundary Conditions

Stationary waves are often (but not always as per Example 20) formed when an *original* wave meets and overlaps with a *reflected* wave travelling in the opposite direction because the reflection preserves the type, amplitude, speed, frequency and wavelength.

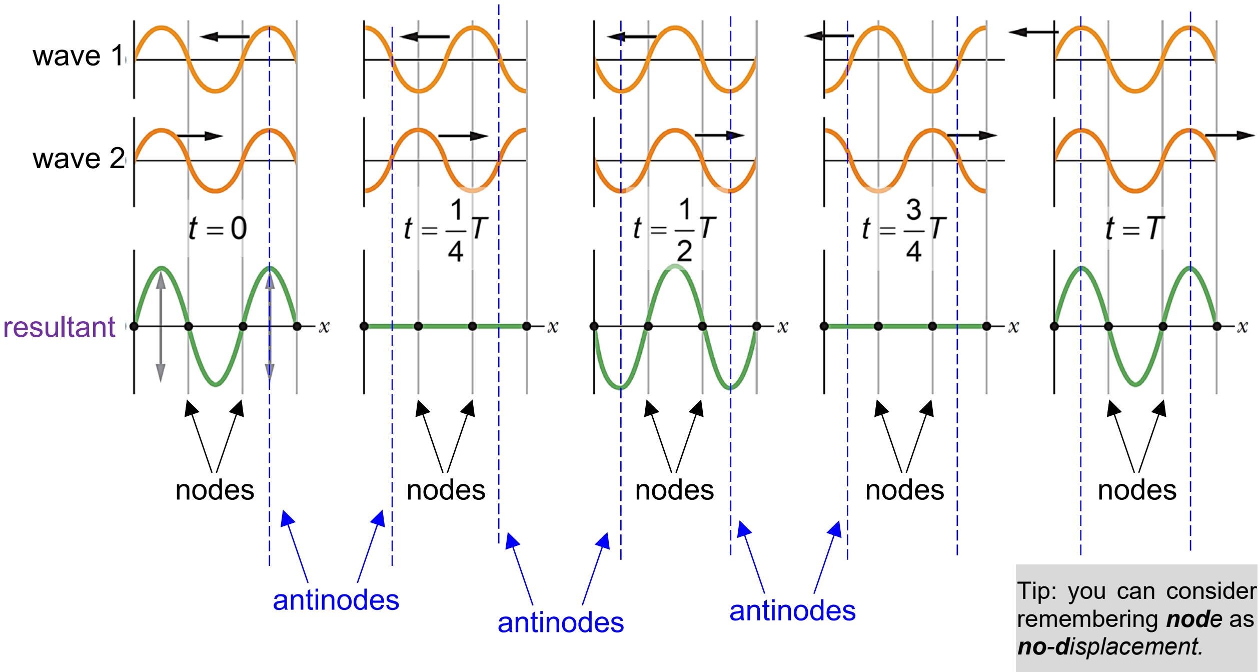
Boundaries are where the reflections take place. Different *boundary conditions* result in different resultant amplitudes (from the superposition of original and reflected waves) at the boundary. Boundaries can be either *fixed ends* or *free ends*.

type of wave	physical condition at boundary	boundary type	resultant displacement at boundary	phase change of reflected wave
transverse waves on stretched string	loop that slides without friction 	free end	maximum amplitude original and reflected waves meet in-phase	$\Delta\phi = 0$
	tied to fixed end 	fixed end	no displacement original and reflected waves meet in anti-phase	$\Delta\phi = \pi$ or 180°
transverse microwaves in air or vacuum	microwaves reflect off a sheet of metal 	fixed end	no displacement original and reflected waves meet in anti-phase	$\Delta\phi = \pi$ or 180°
longitudinal sound waves along a tube	sound waves travel down a tube and reflect at *open* end 	free end	maximum amplitude original and reflected waves meet in-phase	$\Delta\phi = 0$
	sound waves travel down a tube and reflect at *closed* end *sound is longitudinal so it is physically impossible to oscillate into closed wall* 	fixed end	no displacement original and reflected waves meet in anti-phase	$\Delta\phi = \pi$ or 180°

For Example 20, the 2 waves meet and overlap into each other in free space. There are no boundary conditions in free space so a standing wave will always be formed. For Example 21, the boundary conditions are such that the two ends must be fixed ends along the string. Therefore, standing waves can *only* be formed when the wavelength of the waves can fit into the boundaries and satisfy the boundary conditions.

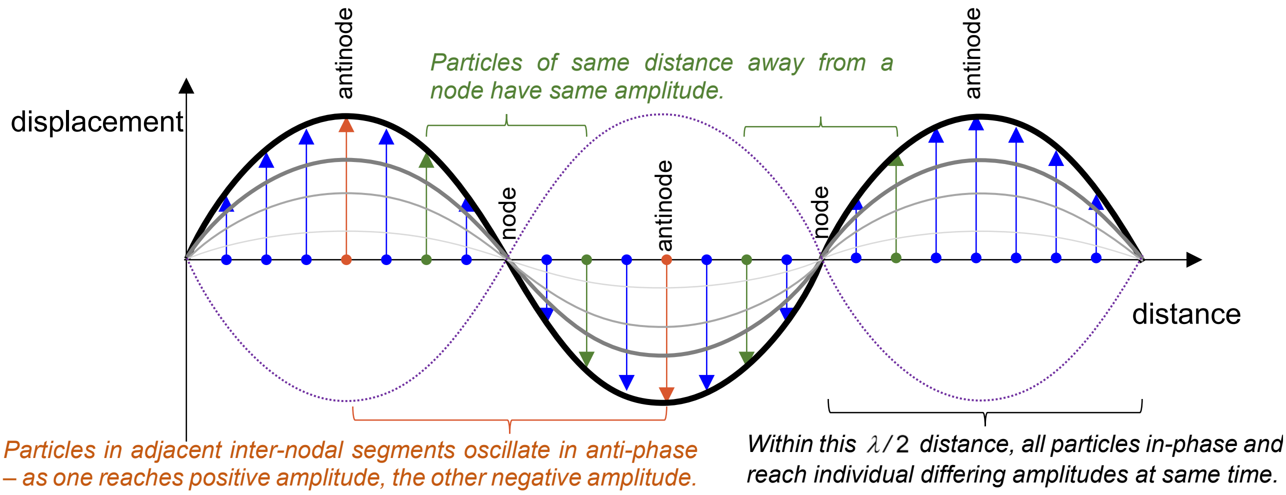
12.4.4 Properties of Stationary Waves

When the two identical progressive waves meet and overlap with each other while travelling in opposite directions, some points will *never* move and some others will move the most.



The resultant waveform has the following features:

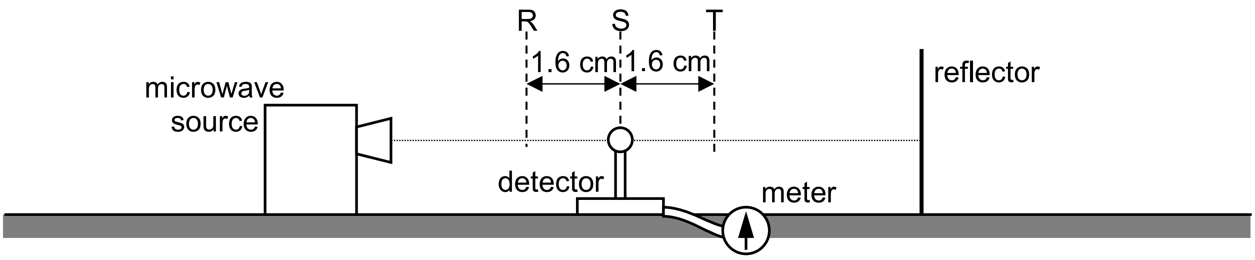
1. Wave profile does not propagate.
2. Wave profile has a characteristic pattern of nodes and antinodes
   - node: a point where the amplitude is constantly zero
   - antinode: a point of maximum amplitude
3. Every particle of the wave, except nodes, oscillates with same frequency (same frequency as the incident or reflected progressive waves).
4. Distance between adjacent nodes (or adjacent antinodes) is $\frac{1}{2}\lambda$.
5. Between two adjacent nodes, every particle oscillates in-phase.
6. Particles in adjacent inter-nodal segments oscillate in anti-phase.
7. Particles that are of same distance away from a node have the same amplitude.
8. The speed in $v = f\lambda$ is the speed of the *incident or reflected progressive wave*.



By convention, we use dotted lines (---) to denote the wave profile half a period later.

Example 22

A detector is placed between microwave source and a metal reflector. At three adjacent points, R, S and T, the meter shows zero intensity. Find the frequency of the microwaves.



Solution

R, S, and T are nodes.

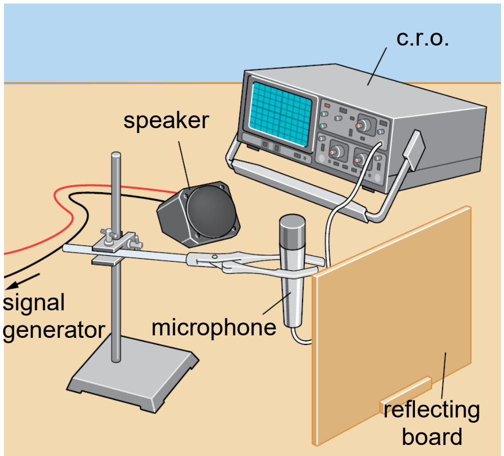
Distance between adjacent nodes is $\frac{1}{2}\lambda$ so $\lambda = 3.2$ cm

$$v = f\lambda$$

$$f = \frac{c}{\lambda} = \frac{3\times10^{8}}{3.2\times10^{-2}} = 9.38\times10^{9}\ \text{Hz}$$

Note: this is a bounded system –there is a standing wave formed when the source and the reflector are at some particular distances apart only.

A similar set up can be used to find the speed of sound. Replace the microwave source with a speaker that is connected to a signal generator, and replace the microwave detector with a microphone connected to a cathode ray oscilloscope (c.r.o.).



1. Output a constant power voltage signal from signal generator to the speaker of known frequency *f*.
2. Measure intensity of sound using a microphone connected to a cathode ray oscilloscope
   - Place microphone along a line perpendicular to the reflecting board joining up with speaker
   - Take preliminary measurements at different locations along the line to adjust for an input sensitivity on the c.r.o that can display the highest possible intensity that is the highest vertical trace on the c.r.o. display.
3. Move microphone along line joining board and speaker to detect positions of maxima and minima.
   - Keep distance between board and speaker constant.
4. Use metre ruler to measure the distance between several nodes.
   - Find average length of separation between adjacent nodes.
   - Wavelength λ is twice the separation between adjacent nodes.
5. Find speed of sound $v = f\lambda$.
   - Vary the frequency *f* and repeat Steps 2, 3, 4 for 6 other frequencies to find average speed.

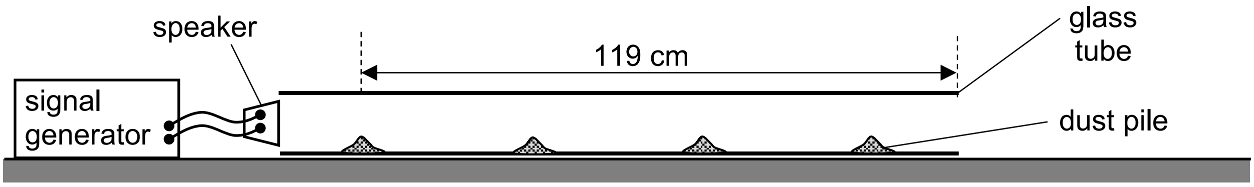
Example 23

A small amount of dust is scattered along the tube. When the frequency of sound is set to 512 Hz, the dust collects in small piles as shown below.

(a) Find the wavelength of sound waves at 512 Hz in air.

(b) Calculate the speed of sound in air in the tube.

(c) Mark two points P and Q where the movement of air particles are π out of phase with the same amplitude.

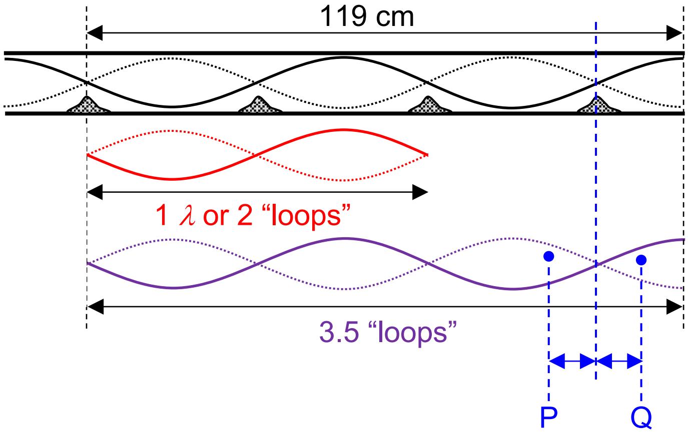


Solution

(a) $\lambda = \frac{2}{3.5}(119) = 68.0$ cm

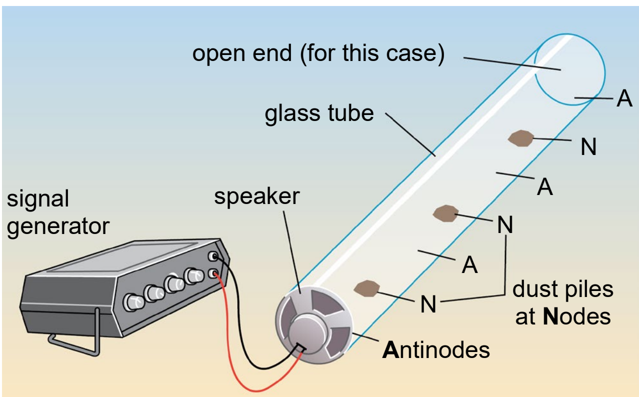
(b) $v = f\lambda = 512\left(68 \times 10^{-2}\right) = 348$ m s^{-1}

(c) two points same distance from either side of a node e.g.:



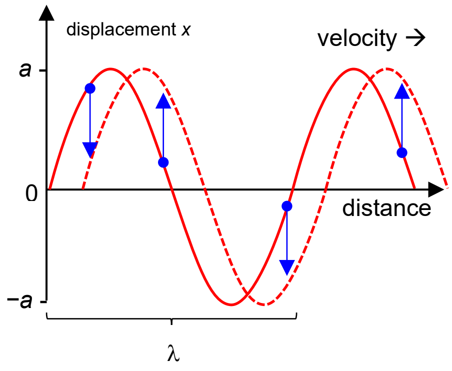
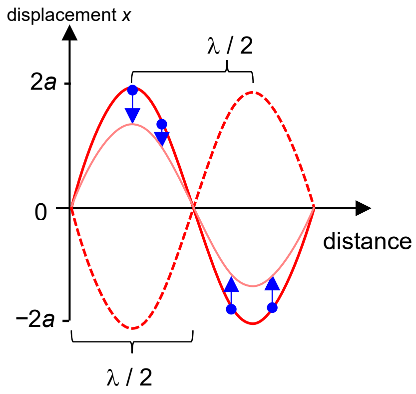
Example 23 is known as the Kundt's tube. A speaker sends sound waves along the inside of a tube. The sound is reflected at the other end (which may be open or closed - results in different modes). Note that the air column, which acts as the medium for sound waves to travel in, is *bounded* by the tube: therefore stationary waves can only be set up at *certain* wavelengths/frequencies.

When the standing wave is established, dust or fine powder that are originally at displacement antinodes get pushed away and accumulate at nodes where movement of air is zero. Therefore, the positions of nodes and antinodes can be clearly seen.



August Kundt (1839 – 1894) was the doctorate advisor to Heinrich Lepold Rubens (1865 – 1922).
While the Kundt's tube worked with powder, Ruben's tube worked with a more dramatic medium - that of fire:

12.4.5 Comparing Stationary Waves and Progressive Waves

wave type	progressive	stationary
variation of displacement with distance		
wave profile	advances in the direction of energy transfer of the wave	does not advance
wavelength	distance between adjacent points on the wave having same phase	twice the distance between 2 adjacent nodes/ antinodes
energy	transferred in the direction of wave propagation	kept within wave as KE and PE of vibrating particles
amplitude of oscillation of individual particles	same for all particles in the wave regardless of position (assuming no energy loss)	varies from zero at nodes to maximum at antinodes
frequency	all points oscillate at frequency of wave	except at nodes, all points oscillate with at same frequency as the incident or reflected progressive wave
phase of wave particles	all particles within one wavelength have different phases ranging from 0 to 2π	all particles within 2 adjacent nodes oscillate in phase; particles on either sides of a node oscillate in anti-phase

In unbounded systems (Example 20), stationary waves can form under all conditions so long as two waves of the same type, frequency, wavelength and speed, travel in opposite directions towards each other, meet and overlap.

More often, we work with systems that have boundary conditions: these systems can only support some fixed frequencies, which we alluded to as *natural frequencies* in H210 Oscillations.

We come full circle and discuss situations when resonance occurs, when the external periodic driving force (waves) matches the natural frequency of the system (a length of string or air).

12.4.6 Some Vocabulary Concerning Standing Waves in Bounded Systems

The rows in grey are less often seen at A-Levels H2 Physics:

mode of oscillation	a particular pattern of nodes and antinodes
fundamental mode fundamental frequency	the mode of oscillation that has the lowest possible frequency of the standing wave (or the longest possible wavelength of the incident/reflected wave)
overtone	the *next* possible mode of higher frequency from the fundamental mode
harmonics	integer relation of the frequency to the fundamental frequency e.g. the fundamental frequency of a system is 3 Hz if the system supports a 9 Hz mode, the mode is the 3rd harmonic therefore, the fundamental mode is also the first harmonic

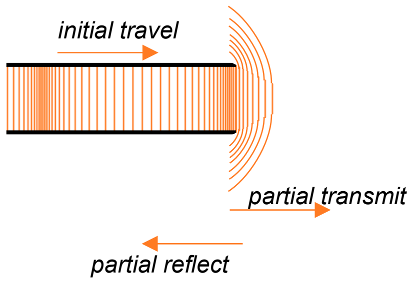
12.4.7 Problem Solving Strategies

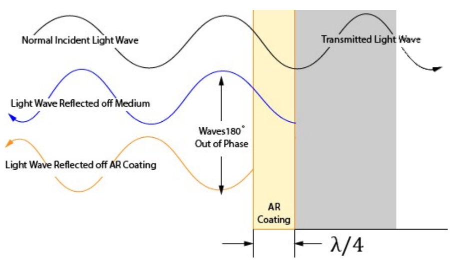
Most questions involving stationary waves require some or all of the following steps:

1) Identify the boundary conditions (2 free ends, 2 fixed ends or 1-fixed-1-free)
2) Decide which mode of oscillation is required (fundamental mode or higher)
3) Sketch out the wave form needed based on mode and/or relationship between wavelength and system length
4) Generalise the relationship between wavelength and system length alongside $v = f\lambda$

You are expected to be able to demonstrate the process of determining the general relationship by first sketching the mode of oscillation. Therefore, *do not* memorise the various relationships. Instead, practise the process of getting to them.

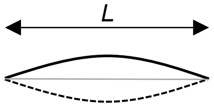
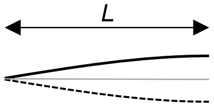
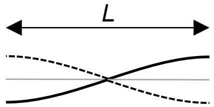
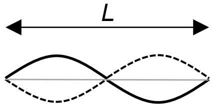
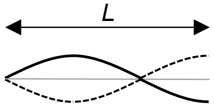
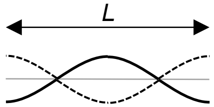
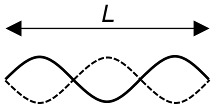
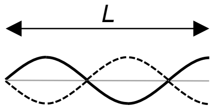
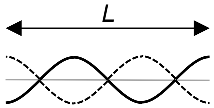
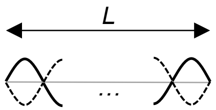
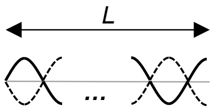
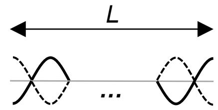
It may seem counter-intuitive that longitudinal sound waves can reflect at an open end of a tube, since the direction of oscillations is parallel to the direction of energy transfer. The idea is that of impedance – how resistant a medium is to the transfer of a particular type of wave. When a pulse of compression travels down the tube and reaches the open end, it suddenly has the ability to spread out into the geometric shadow (diffract). The difference in impedance causes a partial reflection, some energy continues moving in the original direction out of the tube and the remnant is reflected back along the tube without a phase change. A continuous external periodic input of energy into the tube causes wavelengths of fixed lengths to grow in amplitude along the wave.





This is in fact how anti-reflection coating works. When light goes from a medium of lower refractive index to higher refractive index, the partial reflection has a π phase change (no phase change when going from higher to lower refractive index analogous to sound waves above). A thin coating of thickness λ/4 is applied to lens to generate a reflected wave than destructively interferes with the partial reflectance; $n_{air} < n_{coat} < n_{lens}$.

12.4.8 Oscillation Modes under Different Boundary Conditions

	2 fixed ends			1 fixed, 1 free			2 free ends		
mode	wave profile	wavelength	frequency	wave profile	wavelength	frequency	wave profile	wavelength	frequency
fundamental		$L=\frac{\lambda_0}{2}$	$f_0=\frac{v}{\lambda_0}=\frac{v}{2L}$		$L=\frac{\lambda_0}{4}$	$f_0=\frac{v}{\lambda_0}=\frac{v}{4L}$		$L=\frac{\lambda_0}{2}$	$f_0=\frac{v}{\lambda_0}=\frac{v}{2L}$
1st overtone		$L=\lambda_1$	$f_1=\frac{v}{\lambda_1}$ $=2\left(\frac{v}{2L}\right)$ $f_1=2f_0$		$L=\frac{3\lambda_1}{4}$	$f_1=\frac{v}{\lambda_1}$ $=3\left(\frac{v}{4L}\right)$ $f_1=3f_0$		$L=\lambda_1$	$f_1=\frac{v}{\lambda_1}$ $=2\left(\frac{v}{2L}\right)$ $f_1=2f_0$
2nd overtone		$L=\frac{3}{2}\lambda_2$	$f_2=\frac{v}{\lambda_2}$ $=3\left(\frac{v}{2L}\right)$ $f_2=3f_0$		$L=\frac{5\lambda_2}{4}$	$f_2=\frac{v}{\lambda_2}$ $=5\left(\frac{v}{4L}\right)$ $f_2=5f_0$		$L=\frac{3}{2}\lambda_2$	$f_2=\frac{v}{\lambda_2}$ $=3\left(\frac{v}{2L}\right)$ $f_2=3f_0$
in general:	 *n* loops	$L=n\left(\frac{\lambda}{2}\right)$	$f=n\left(\frac{v}{2L}\right)$ $f=nf_0$	 *n* loops + ½ loop	L $=n\frac{\lambda}{2}+\frac{\lambda}{4}$ $=(2n+1)\frac{\lambda}{4}$	$f=$ $(2n+1)\left(\frac{v}{4L}\right)$ $f=(2n+1)f_0$	 *n*-1 loops + 2 ½'s = *n* loops	$L=n\left(\frac{\lambda}{2}\right)$	$f=n\left(\frac{v}{2L}\right)$ $f=nf_0$
			all harmonics $(f=nf_0)$ possible			odd harmonics $f=(2n+1)f_0$ possible			all harmonics $(f=nf_0)$ possible

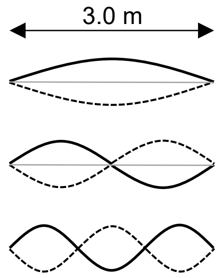
Example 24

A taut wire is clamped at two points 3.0 m apart. It is plucked near one end.

(a) State the three longest wavelengths present in the oscillating wire.

(b) When the wire is plucked, many different modes are present in the wire at the same time. Suggest a method to make the 2nd longest wavelength the mode of oscillation.

Solution



(a) wavelengths (decreasing order): 6.0 m, 3.0 m, 2.0 m

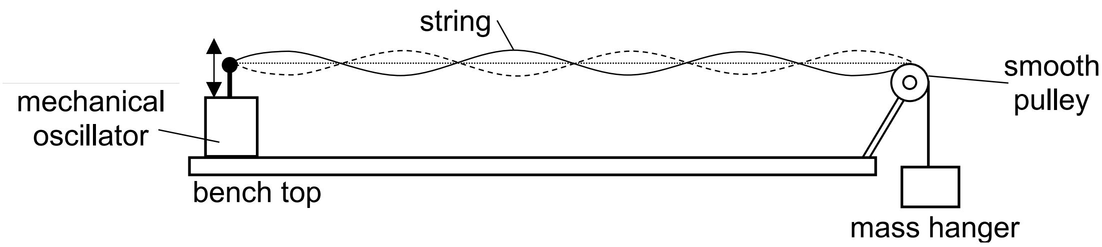
(b) after plucking, make a node in the centre of wire by placing finger lightly at the 1.5 m mark of wire

Example 25

Stationary waves are investigated via the set up below. It is known that the speed of waves in a stretched string v is directly proportional to square-root of tension in string $v \propto \sqrt{T}$. The frequency of the oscillator is initially set to 0 Hz (no oscillation).

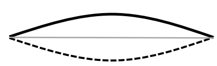
(a) Explain how to obtain standing waves of lowest possible frequency in the string.

(b) The distance from the oscillator to the pulley 2.4 m. At a frequency of 40 Hz, a stationary wave of the profile shown below is formed. Find the lowest frequency as in (a).



Solution

(a) For a fixed mass providing constant tension, and a fixed distance between oscillator and pulley, slowly increase the frequency of oscillations until a standing wave with nodes at the oscillator and pulley is formed.
Fine-tune the frequency such that the antinode in the middle of the string is maximum.



(b) wavelength $\lambda = \dfrac{2.4}{3} = 0.8$ m

Fixed mass so speed is constant $v = f\lambda = 40\left(0.8\right) = 32.0 \text{ m s}^{-1}$

wavelength at lowest frequency = 4.8 m

lowest frequency $f_0 = \dfrac{v}{\lambda} = \dfrac{32}{4.8} = 6.67$ Hz (40 Hz is the 6th harmonic)

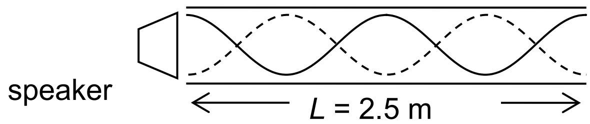
Note: oscillator end can be considered a node because the amplitude is relatively small when compared to amplitude at antinodes. Work with information given in the question.

Example 26

A loud sound is heard when a speaker producing frequency of 280 Hz is placed near the end of a tube with both ends open, as shown in the figure below.

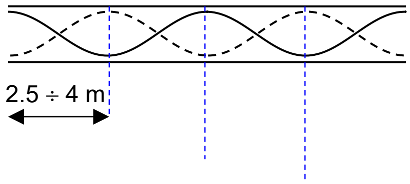
(a) State a possible length of another tube that can result in a loud sound with the same speaker.

(b) Find the lowest possible frequency that can result in a loud sound with the original tube.



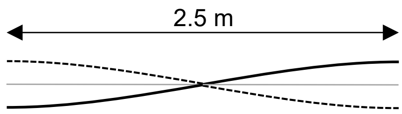
Solution

(a) integer multiples of $\frac{2.5}{4}$

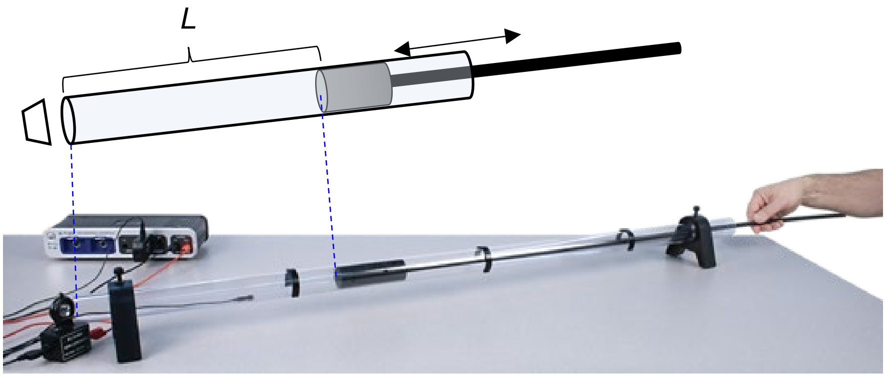


(b) $v = f\lambda = 280\left(\frac{2.5}{2}\right) = 350 \text{ m s}^{-1}$

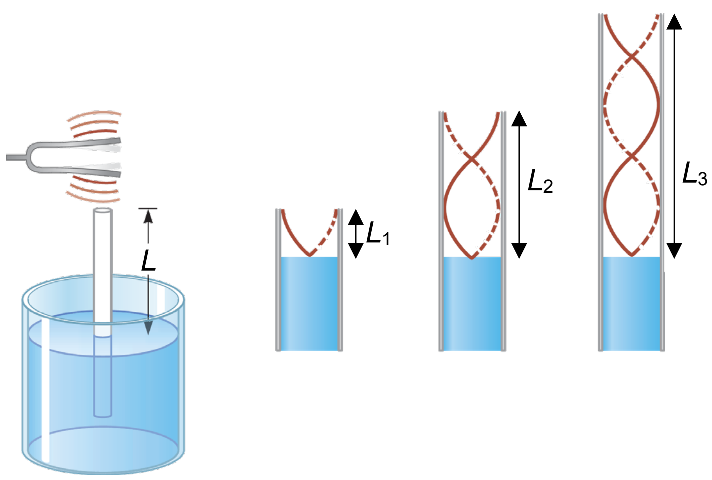
$f_0 = \frac{v}{\lambda} = \frac{350}{5} = 70.0 \text{ Hz}$

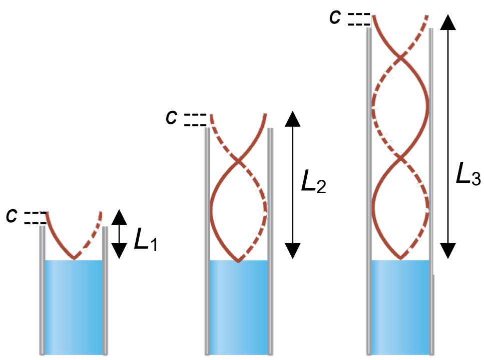


The length of an open-pipe (1 fixed and 1 free end) can be varied via a piston in hollow pipe. The sound waves reflect off the piston as a fixed end:



Another way is do vary the depth to which a hollow tube is submerged into water. The sound waves reflect off the water surface as a fixed end:





in practice, the antinodes are slightly outside the tube and 'c' is known as the "open end correction". we can perform
$L_2 - L_1 = L_3 - L_2 = \lambda / 2$

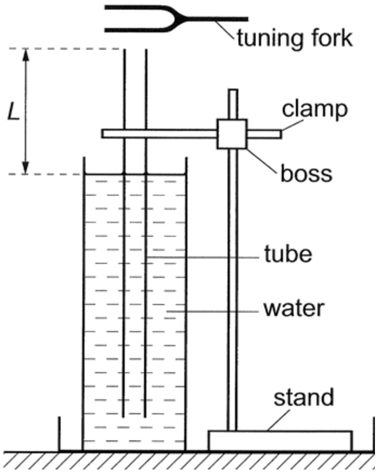
Example 27

A tuning fork is sounded off and placed near the top of a partially-submerged tube as shown. Length L is varied from 0 until a loud sound is heard.

(a) By considering the speed of sound in air v, the frequency of the tuning fork used f, show that L is inversely proportional to f.

(b) Several values of L are measured using tuning forks of different natural frequencies.

f / Hz	L / cm
480	16.8
384	21.6
320	25.7
288	28.8



(i) Find the speed of sound in air v.

(ii) Suggest why the expected relationship in (a) was not obtained.

Solution

(a) $L = \dfrac{\lambda}{4}$

$v = f\lambda = f(4L)$

$L = \left(\dfrac{v}{4}\right)\left(\dfrac{1}{f}\right)$

(b)

f / Hz	L / cm	$\dfrac{1}{f}$ / 10^{-3} Hz^{-1}
480	16.8	2.08
384	21.6	2.60
320	25.7	3.13
288	28.8	3.47

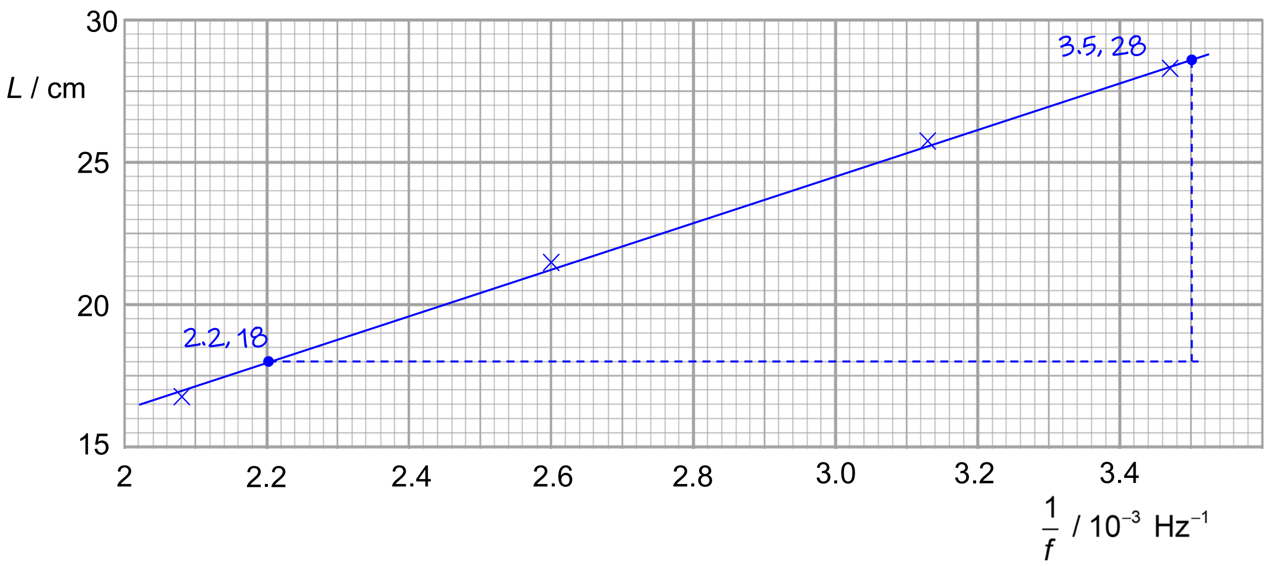
$v = 4(\text{gradient})$

$= 4\left(\dfrac{(28-18)\times 10^{-2}}{(3.5-2.2)\times 10^{-3}}\right)$

$= 308 \text{ m s}^{-1}$

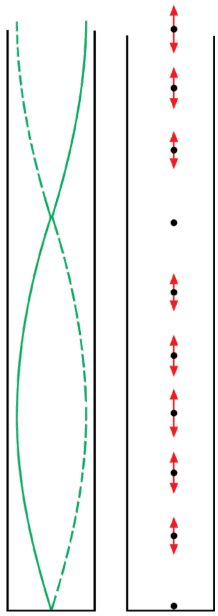
$\text{gradient} = \dfrac{28-c}{3.5-0}$

$c = 1.09 \text{ cm}$

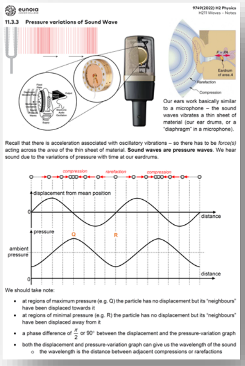


Note: the y-intercept is the "open end correction". Sound waves are longitudinal and experiments show that oscillations go slightly beyond open end. When dealing with multiple different frequencies in the same set up, we typically assume constant c (systematic error).

12.4.9 Sound Waves are Pressure Waves

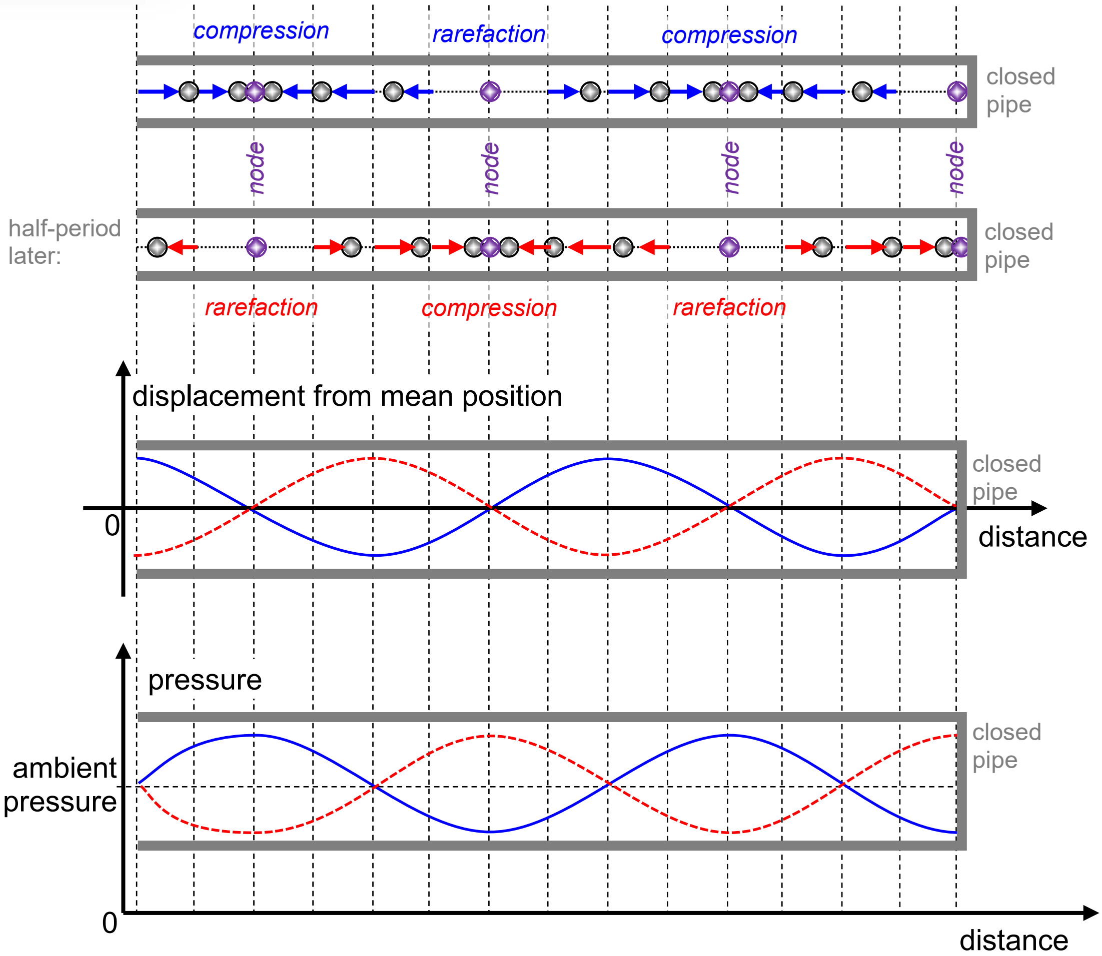


The wave profiles on page 10 depict the variation of displacement of particles. It is directly applicable for transverse waves. But for longitudinal waves e.g. sound, be aware that the wave profiles are merely *representations* of the displacements.



This is not new information – it has been introduced in H211 Waves. The following section uses graphical relationships to show this in the context of stationary wave.

a displacement node is a
pressure (variation) antinode
and vice versa



At the displacement node, the neighbouring air particles will be alternately crowding and separating away from it after half a period. The air particles at the displacement nodes are fixed in position.

Note, when a stationary wave is formed in a pipe, a loud sound is heard outside the tube – the whole air column is in resonance and acts as a long-ish source of sound. This is how wind musical instruments work. However, if we move a small-enough microphone down the length of the pipe, it will detect variations in audio volume as it will detect the *pressure* (sound waves are pressure waves) nodes and antinodes.

12.5 Summary

only for waves

principle of superposition
when 2 or more waves meet & overlap, resultant displacement is vector sum of displacement of each individual wave

basis

interference
(based on principle of superposition) pattern of maxima and minima

waves meet in-phase

constructive
resultant displacement larger than largest individual displacement

waves meet in anti-phase

destructive
resultant displacement smaller than largest individual displacement

diffraction
spreading of a wave into geometric shadow when it passes through a slit or past an edge of an obstacle

significant when

size of aperture or obstacle same order of magnitude as wavelength $\lambda \sim b$

coherent waves of same type meet and overlap (and if transverse: unpolarised or polarised in same plane)

conditions for pattern

constant phase difference

if 2 source

compare path difference in λs & check if sources in-phase

superpose across a length

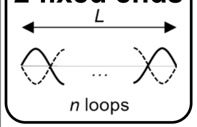
stationary waves
2 waves of same amplitude, type, f, λ & v, travel in oppo dxn towards each other, superpose along length

node
point of constant **0** amplitude

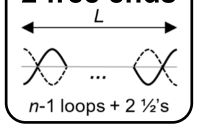
antinode
point of maximum amplitude

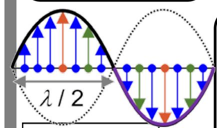
2nd wave can be reflection

2 fixed ends

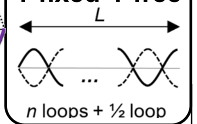


2 free ends





1-fixed-1-free



compare inter-nodal segment	**same**	**different**
within same	phase	amplitude
adjacent	amplitude (if same distance from node)	π out of phase

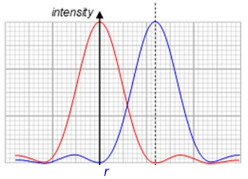
explained

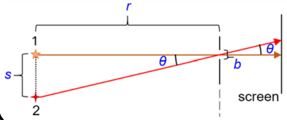
screen very far: $D \gg a$

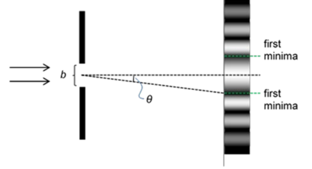
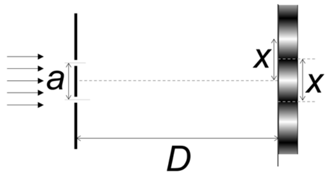
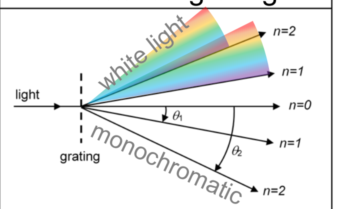
basis for

Rayleigh Criterion
limit for which 2 sources of light can be just distinguished is when 1st minima of diffraction pattern of 1 source coincides with central maxima of diffraction pattern of the other source

$$\theta \approx \frac{\lambda}{b}\left(\approx \frac{s}{r}\right)$$

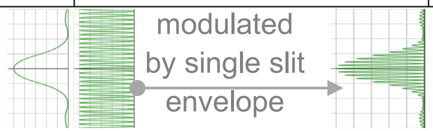




	single slit	(Young's) double slit	diffraction grating
geometry			
equation	$\sin\theta = \frac{\lambda}{b}$	$x = \frac{\lambda D}{a}$	$d\ \sin\theta = n\lambda$
equation describes	position of 1st minimum	separation of adjacent bright/dark fringes	position of n^{th} order maxima
separation	not constant	constant	not constant

when conditions change

check extent of spreading



modulated by single slit envelope

find max order

indirect

$$d = \frac{L_{unit}}{N}$$

$$\sin\theta \to 1\ ;\ n_{max} = \frac{d}{\lambda}$$

Separation
Amplitude
Overlap

if individual amplitudes unequal

bright fringes intensity ↓, dark fringes not totally dark, contrast ↓