Name:		Index Number:		Class:	

DUNMAN HIGH SCHOOL
Promotional Examination Practice Paper 1
Year 5

MATHEMATICS (Higher 2) **9758/01**

3 hours

Additional Materials: List of Formulae (MF27)

READ THESE INSTRUCTIONS FIRST

Write your Name, Index Number and Class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

1 A café sells three types of cake: banana, chocolate and walnut. The café gives 12% discount on total cost to any customer who buys more than 12 cakes. The number of each type of cake bought by Alice, Betty and Charlotte, together with the total amount paid, are shown in the following table.

	Banana	Chocolate	Walnut	Amount paid ($)
Alice	5	3	2	52
Betty	3	4	5	68
Charlotte	4	8	2	66

Determine the original selling price of each type of cake. [3]

2 A curve C undergoes, in succession, the following transformations.

A: Translation of 4 units in the negative *y*-direction.

B: Stretching parallel to the *x*-axis by a factor of 2.

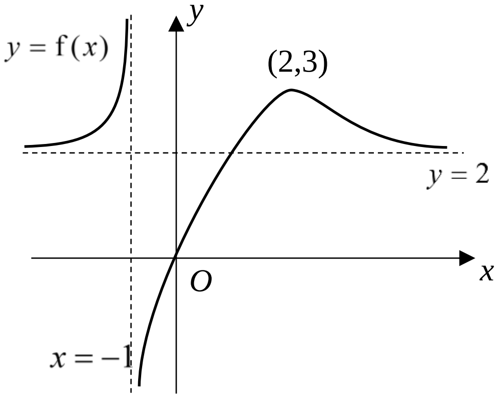
C: Reflection in the *x*-axis.

The equation of the resulting curve is $y=\dfrac{x^2}{4}+4$.

Determine the equation of the original curve *C* before the transformations were carried out.

[3]

3 The graph of $y=\mathrm{f}(x)$ has a maximum turning point at (2,3) and passes through the origin. The lines $x=-1$ and $y=2$ are asymptotes to the graph, as shown in the diagram below.



(a) State the range of values of *x* for which the graph of $y=\dfrac{1}{\mathrm{f}(x)}$ is increasing. [1]

(b) Sketch the graph of $y=\mathrm{f}'(x)$, showing clearly the axial intercepts and the asymptotes. [3]

4 The function f is defined by $\mathrm{f}: x \mapsto \dfrac{x^2+4}{x},\ x \in \mathbb{R}$.

(a) Give a reason why f does not have an inverse. [1]

(b) The domain is now restricted to $0 < x \leq 2$. Find $\mathrm{f}^{-1}(x)$ and state its domain, showing your working clearly. [4]

5 A curve C has equation $y = \dfrac{1}{\sqrt{bx^2 - 2bx}}$, where $b < 0$. Sketch C and give, in terms of b where appropriate, the equations of any asymptotes and the line of symmetry, and coordinates of turning point. [5]

6 The curve C has parametric equations

$$x = 5a\sec\theta, \qquad y = 3a\tan\theta, \qquad \text{where } -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

and a is a positive constant.

(i) Find the cartesian equation of C. [2]

(ii) Sketch C, giving the coordinates of any points of intersection with the axes and the equations of any asymptotes. [3]

7 **(a)** Given that both a and b are positive constants, on the same diagram, sketch the graphs of $y = \dfrac{2}{|x-a|}$ and $y = b|x-a|$. [3]

(b) Hence, or otherwise, solve the inequality $\dfrac{2}{|x-a|} > b|x-a|$. [3]

8 In this question you may use expansions from the List of Formulae (MF27).

(i) Find the Maclaurin expansion of $\ln(1+\cos 3x)$ in ascending powers of x, up to and including the term in x^4, for $0 \leq x < \dfrac{\pi}{3}$. [4]

(ii) Use your expansion from part **(i)** and to find an approximate value for $\int_0^{0.5} x\ln(1+\cos 3x)\,\mathrm{d}x$, giving your answer to 5 decimal places. [2]

(iii) Use your calculator to find the value of $\int_0^{0.5} x\ln(1+\cos 3x)\,\mathrm{d}x$ up to 5 decimal places. [1]

(iv) With the aid of a suitable diagram, comparing your answers in **(ii)** and **(iii)**, comment on the accuracy of your approximations. [2]

9 **(a)** **(i)** Find, in terms of n and x, an expression for $\sum_{r=0}^{n} \frac{(x+3)^r}{4^{r+1}}$. [2]

(ii) Give a reason why the infinite series $\sum_{r=0}^{\infty} \frac{(x+3)^r}{4^{r+1}}$ converges when $x = -5$ and determine its value. [2]

(b) **(i)** Given that $\sum_{r=1}^{n} r^2 = \frac{n}{6}(n+1)(2n+1)$, find in terms of k, the sum $\sum_{r=6}^{2k} r(3r-2)$. [3]

(ii) Using your result in part **(b)(i)**, evaluate $\sum_{r=10}^{66} (r-4)(3r-14)$. [2]

10 **(a)** Find $\int \frac{1-3x}{1+9x^2}\,\mathrm{d}x$. [3]

(b) Use the substitution $x = 2\sin\theta$, where θ is acute, to find $\int \frac{(x-1)^2}{\sqrt{4-x^2}}\,\mathrm{d}x$. [5]

(c) Find $\frac{\mathrm{d}}{\mathrm{d}x}\left(\sin^{-1}(2x^2)\right)$. Hence solve $\int \left(2x\sin^{-1}(2x^2)\right)\mathrm{d}x$. [5]

11 Betty received an SMS from a friend regarding a Bitcoin (BTC) investment that guarantees a fixed percentage of interest per month. Betty clicked on the link in the SMS and found out that the BTC investment has an account for each investor. An interest is added to the account at the end of each month at a fixed rate of 11% of the amount in the account at the beginning of the month.

(i) Betty decides to start an investment account with \$1500 on 1 Jan 2022. She invests with the intention of not withdrawing any money out of the account but just leave it for the interest to build up. Find the total profit at the end of one full year. [2]

Betty forwards the SMS to her friend, Carl who decides to invest \$200 on the first day of each month starting from 1 Jan 2022.

(ii) Show that the projected amount of money in Carl's investment account at the end of n months of investment is given by

$$\frac{22200}{11}\left[\left(\frac{111}{100}\right)^n - 1\right].$$ [3]

(iii) Find the projected amount of money in Carl's investment account at the end of one year. [1]

(iv) Determine with clear reasons, the date on which the projected amount of money in Carl's investment account first exceeds the projected amount of money in Betty's investment account. [4]

Unfortunately, the BTC investment is a scam. According to the Police, victims in Singapore lost \$633.3 million to scams in 2021. With public education campaigns, a researcher predicts that the amount lost to scams each year will drop by 5% of the amount lost in the preceding year.

(v) If the researcher is correct in his predictions, what would be the theoretical total amount of money lost to scams from 2021 (inclusive) onwards? [2]

12 A nature conservationist group studies the trend of population growth of an endangered animal species over the years. Denoting u_n as the population of the animals at the *n*th year, the initial population in the first year is observed to be $u_1 = a,$ where $a \in \mathbb{Z}^+$. The population at the (*n*+1)th year can be modelled by the relation

$$u_{n+1} = 2u_n - k, \text{ where } k \text{ is a constant.}$$

(a) For parts **(i)** and **(ii)** below, it is given that $a = 4$.

(i) Describe what happens to the population over time if

(A) $k = 1$, [1]

(B) $k = 5$. [2]

(ii) Find the value of *k* if there are 52 animals in the 3rd year. [2]

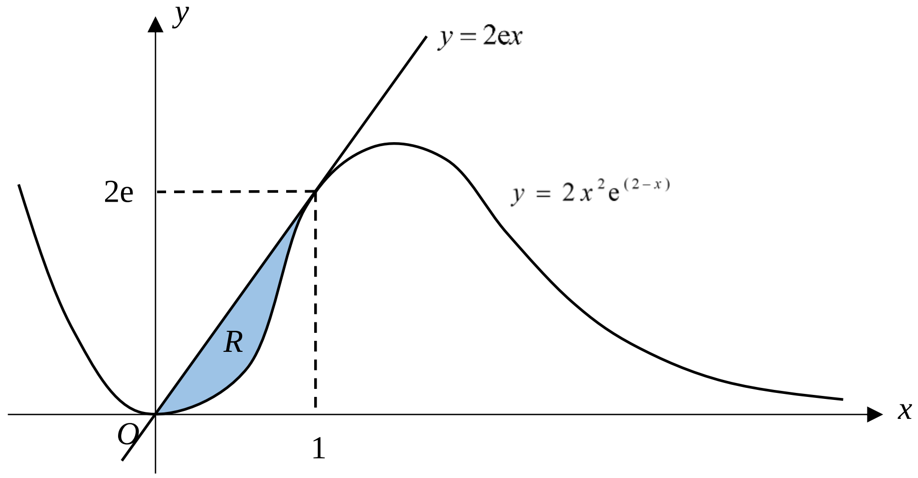
(b) Show that the sequence defined by $v_n = u_n - k$ is a geometric progression. By expressing v_n in terms of *n* or otherwise, show that u_{n+1} can be written in the form

$$u_{n+1} = (a-k)2^n + k.$$ [3]

(c) State the value of *a* such that the population will stay stabilized in the long term. [1]

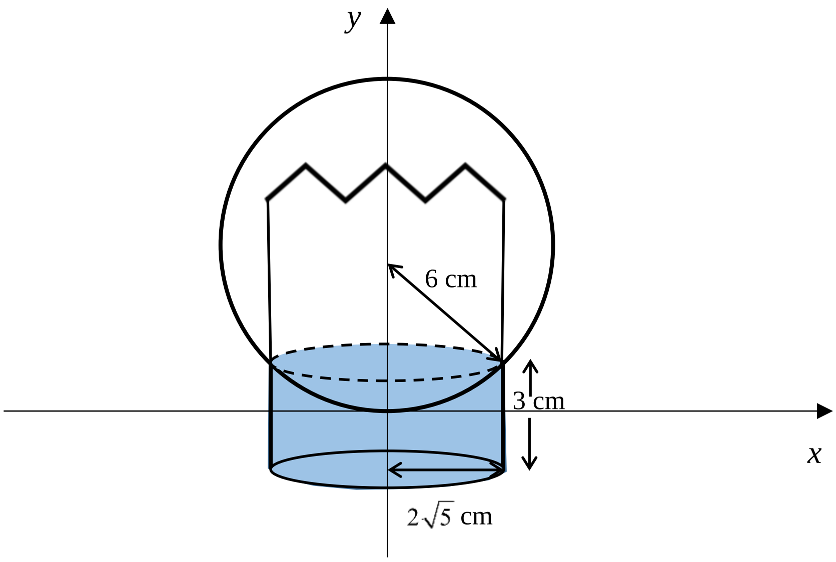
(d) The group intends to find out the sum of the population at every 5-year interval, starting from the 5th year and ending at the 80th year, i.e. $u_5 + u_{10} + \ldots + u_{80}$. Give your answer in the form $A(2^B - 1)\left(a-k\right) + Ck,$ where A, B and C are constants to be found. [3]

13 An art student wants to create an art installation "Glimpse of Hope" on the wall. The installation consists of a drawing, which is modelled by the curve $y = 2x^2 e^{(2-x)}$ and the line $y = 2ex$. The two graphs intersect at the origin and touch at the point $(1,\ 2e)$. The student intends to fit lightbulbs into the region R to signify that there is always hope in every "up-and-down" situation in life.



(a) Find the exact area of region R. [4]

(b)



The light bulbs that the student used are specially manufactured for the art installation. Each light bulb consists of part of an empty sphere with radius 6 cm and a solid cylinder of radius $2\sqrt{5}$ cm and height 3 cm as shown in the diagram. The bottom of the sphere is sliced off so that its edges touches the edges of the top of the cylinder. An inert gas is used to fill the spherical part of the light bulb to prevent the filament from decaying.

The sphere of the light bulb is modelled by rotating part of the curve $x^2 + (y-6)^2 = 36$ around the y-axis as shown in the diagram above.

(i) Assuming that the filament takes up negligible space and that the light bulb is of negligible thickness, calculate the exact volume of the spherical part of the light bulb that needs to be filled with the inert gas. [4]

(ii) To determine the amount of special coating to apply to the exterior of the spherical part of the light bulb, the manufacturer wishes to find the surface area of the spherical part of the light bulb.

It is given that the area of a surface of revolution formed by revolving the graph of $x = \mathrm{f}(y)$ around the y-axis over the interval from $y = a$ to $y = b$ is

$$\int_a^b 2\pi \mathrm{f}(y)\sqrt{1+\left(\mathrm{f}'(y)\right)^2}\,\mathrm{d}y.$$

Find numerical answer of the surface area of the spherical part of the light bulb. [4]

(iii) Each light bulb can effectively light up a circular area of 0.5 m^2. The art student wants to estimate the number of light bulbs needed for the art installation in part **(a)**. The art student uses the following formula to calculate the number of light bulbs to acquire.

$$\text{Number of light bulbs} = \frac{\text{Area of Region } R}{0.5}$$

Given that 1 unit on the art installation represents 100 m, determine the number of light bulbs that the art student would acquire. Suggest why the number of light bulbs using this formula may not be accurate. [2]