Answer **all** the questions.

1 **(a)** Find the range of values of p for which the equation $px^2 + x + p(x+1) = 0$ has real and distinct roots. [3]

(b) State the value(s) of p for which the curve $y = px^2 + x + p(x+1)$ is tangent to the x-axis. [1]

2 Solve the following equations

(i) $2\log_3 x + \log_x 3 = 3$, [4]

(ii) $3^{2y+1} - 3^{y+2} + 3 = 3^y$. [4]

3 In the expansion of $\left(x^2 - \frac{m}{2x}\right)^{12}$, where *m* is a positive constant, the independent term of *x* is 126720.

(a) Show that $m - 4$. [4]

(b) **Hence**, find the coefficient of x^9 in the expansion of $\left(x^2 - \frac{m}{2x}\right)^{12}\left(8x^9 + 5\right)$ [3]

4 **(a)** Express $\dfrac{-x-11}{(2x+1)(x-3)}$ in partial fraction. [3]

(b) **Hence**, evaluate $\displaystyle\int \frac{-2x-22}{(2x+1)(x-3)}\,\mathrm{d}x$. [3]

5 The curve $y = \dfrac{\ln x}{2x^2}$, for $x > 0$ has a stationary point at point *A*.

Find

(i) $\dfrac{\mathrm{d}y}{\mathrm{d}x}$, [2]

(ii) the *x*-coordinate of *A*, [2]

(iii) the nature of A. [3]

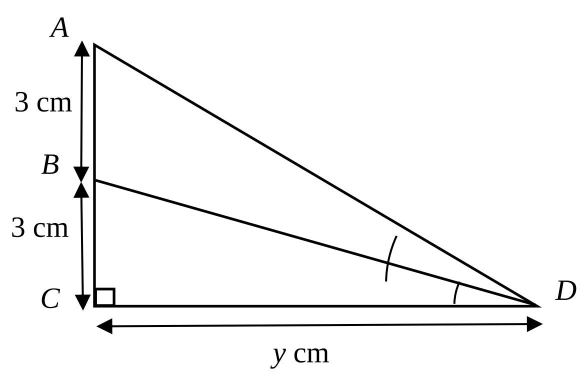
6 **(a)** Given that $5\sin x\cos x = 1$, find

(i) $\sin 2x$, [1]

(ii) $\cos 4x$. [2]

(b) The diagram below shows a right angled triangle *ACD*.

It is given that $\angle ADB = \beta$, $\angle BDC = \alpha$, $AB = BC = 3$ cm and $CD = y$ cm.



Express $\tan\beta$ in terms of *y*. [4]

7 A curve has the equation $y = \dfrac{x-2}{3x+2}$.

(a) Explain why the curve $y = \dfrac{x-2}{3x+2}$ has no turning point. [2]

(b) Given that y is increasing at a rate of 0.4 units per second at the instant when $y = \dfrac{1}{11}$, find the rate of change of x at that instant. [3]

(c) A curve is such that $\frac{d^2y}{dx^2} = 24x^2$ and has a turning point $(-1, -6)$.

Find the equation of the curve. [4]

8 The expression $f(x) = ax^3 + x^2 - bx + 3a$ has a factor of $(x+3)$ and leaves a reminder of -4 when divided by $(x-1)$.

(a) Find the value of *a* and of *b*. [4]

(b) **Using part (a),** solve $\mathrm{f}(x) = ax^3 + x^2 - bx + 3a$. [3]

(c) **Hence**, solve $\mathrm{f}(x) = ae^{3x} + e^{2x} - be^x + 3a$. [2]

9 **(i)** Differentiate $4xe^{3x}$ with respect to *x*. [2]

(ii) **Hence**, evaluate $\int_0^3 6xe^{3x}\,dx$, giving your answer in **exact form**. [4]

10 A circle passes through the points $P(-2, 8)$ and $Q(-4, 4)$.

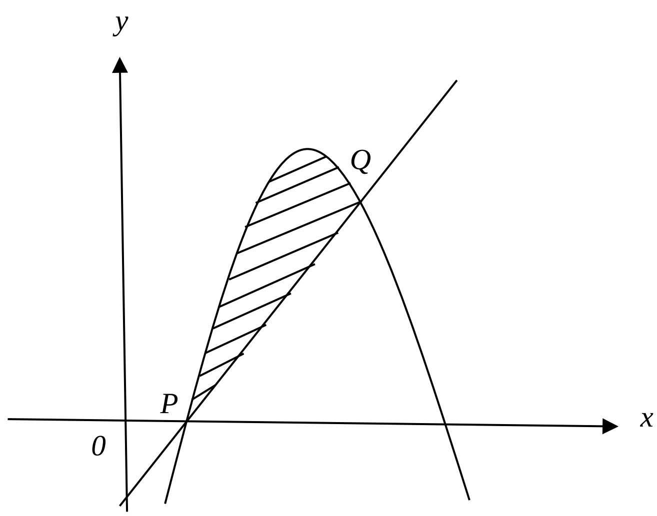
(i) Find the equation of the perpendicular bisector of PQ. [3]

The centre, *C*, of the circle lies on the line $2x + y = 6$.
Find the

(ii) coordinate of *C*, [3]

(iii) equation of the circle. [2]

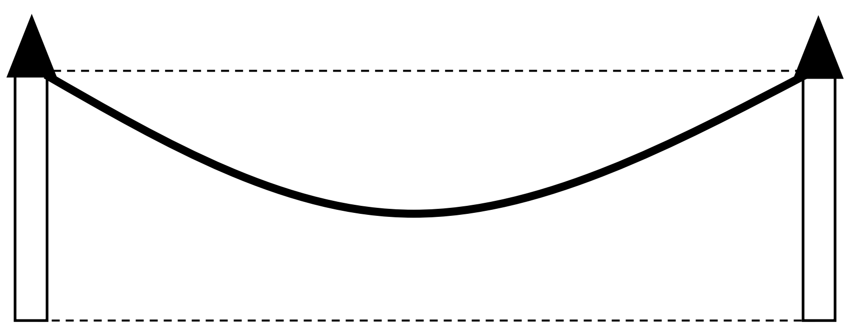
11 In the diagram, the curve $y=-x^2+5x-4$ cuts the line $y=x-1$ at two points P and Q.



Calculate the area of the shaded region bounded between the curve and line. [6]

[Continue your working here for Question 11]

12 The diagram below shows a zip line found in a particular Outward Bound School.
The height, h m, of a participant above the ground in a section of the zip line is given by $h=\frac{2}{3}x^2-4x+12$, where x m is the participant's horizontal distance from the start point.



(i) Express the function in the form $h=a\left(x-b\right)^2+c$. [2]

(ii) Find the participant's lowest height from the ground. [1]

(iii) If the participant is 9 m above the ground, find the 2 possible horizontal distance of the participant from the start point. [2]

(iv) One of the participant, Charlene claimed that at an angle of elevation of $55°$ from the lowest point along the zip line, she was able to see an owl standing at the top of the end point.

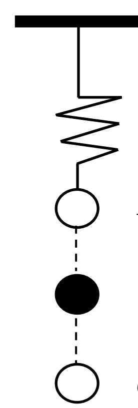
Verify, with reason, the validity of her claim. [2]

State one assumption made in your calculation.

13 A steel ball is attached to a spring is then pulled to position C from its equilibrium position B and released. The steel ball will oscillate up and down between A to C. It is assumed that there is no loss in energy or momentum of the steel ball.

Given that C is the initial position of the steel ball, the change in position, h cm, of the steel ball from B over time, t seconds, can be modelled as

$$h = -20\cos\left(\frac{\pi}{2}t\right).$$



(a) Find the vertical distance between A and B. [1]

(b) Find the time taken for the steel ball to move from *C* to *A*. [2]

(c) Find the vertical distance of the steel ball from *B* at $t = 5.5$. [1]

(d) Find the two earliest times when the mass is 2 cm above *B*. [2]