Calculator Model:

ORCHID PARK SECONDARY SCHOOL
Preliminary Examination 2024

CANDIDATE NAME			
CLASS		INDEX NUMBER	

ADDITIONAL MATHEMATICS **4049/02**

Paper 2 **23 August 2024**

Secondary 4 Express / 5 Normal (Academic) **2 hours 15 minutes**

Setter: Mr Mohd Salim Bin Ramli **90 Marks**

Additional Materials: NIL

READ THESE INSTRUCTIONS FIRST
Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
Use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer. Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks of this paper is 90.

For Examiner's Use	
Total	

This document consists of **19** printed pages.

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\text{Area of } \triangle = \frac{1}{2}bc \sin A$$

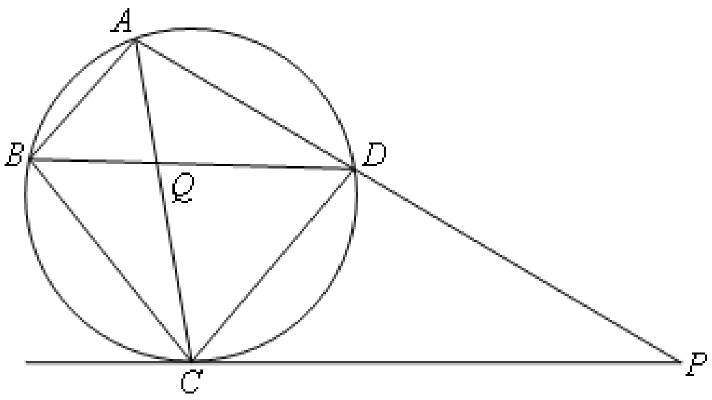
1 **(a)** Expand $\left(x+\frac{1}{x}\right)^4$ in descending powers of x. [2]

(b) Hence, given that $\left(\boldsymbol{x}+\frac{\mathbf{1}}{\boldsymbol{x}}\right)^{\mathbf{4}}-\left(\boldsymbol{x}-\frac{\mathbf{1}}{\boldsymbol{x}}\right)^{\mathbf{4}}=\boldsymbol{a}\boldsymbol{x}^{\mathbf{2}}+\frac{\boldsymbol{b}}{\boldsymbol{x}^{\mathbf{2}}}$, find the value of a and of b. [3]

(c) Given that there is no x term in the expansion of $\left(\frac{4}{3}x + \frac{k}{x} + \frac{x^3}{k}\right)\left(x + \frac{1}{x}\right)^4$, find the value of k.

[3]

2 In a diagram, *A*, *B*, *C* and *D* are points on the circle. The tangent at *C* meets *AD* produced at *P*. The chords *AC* and *BD* intersect at *Q*. The line *BQD* bisects angle *ABC*.



Prove that

(a) $\angle DCP = \angle ACD$, [3]

(b) ΔPCD is similar to ΔPAC, [2]

(c) $PC^2 = PA \times PD$. [1]

3 In recent years, the release of greenhouse gases has accelerated the melting of glaciers and thus, resulting in a rise of global temperatures. With minimal actions taken to prevent global warming, the average temperature, $T°C$, projected to rise after x years from 2024, is given by $T = 31(1.5)^{kx}$, where k is a constant.

(a) Given that the projected average temperature in Yishun in 2027 is $35°C$, find the value of k correct to 1 decimal place. [2]

(b) Find the average temperature in Yishun in 2024. [1]

(c) In which year will the average temperature in Yishun first be at least 15% higher than its temperature in 2024? [4]

(d) Sketch the graph of T against x. [2]

4 (a) The function f is given by $f(x) = asin3x + b$, where a and b are positive integers and $0 \leq x \leq \pi$. The maximum and minimum value of f are 6 and -2 respectively.

(i) State the period of f. [1]

(ii) Find the value of a and of b. [2]

(iii) Sketch the graph of $y = -f(x)$. [3]

4(b) Prove the identity $\dfrac{\sin\theta - \sin 2\theta + \sin 3\theta}{\cos\theta - \cos 2\theta + \cos 3\theta} = \tan 2\theta$. [4]

5 A particle moving in a straight line is such that its displacement, s metres, from a fixed point O, is given by $s = 4 - 2e^{-t} - t$ where t is the time in seconds after passing through a point B on the line. Find

(a) the distance OB, [1]

(b) the initial velocity of the particle, [2]

(c) the value of t when the particle is instantaneously at rest, [2]

(d) the total distance travelled by the particle in the first two seconds. [3]

6 The equation of a polynomial is given by $P(x) = 2x^3 + mx^2 + x + n$, where m and n are constants.

(a) Given that $2x^2 + x - 1$ is a factor of $P(x)$, find the value of m and of n. [4]

(b) Hence solve the equation $P(x) = 0$. [3]

(c) Using the results in parts **(a)** and **(b)**, solve the equation $\frac{1}{4}y^3 + \frac{m}{4}y^2 + \frac{1}{2}y + n = 0$. [3]

7 Two simultaneous equations are given:

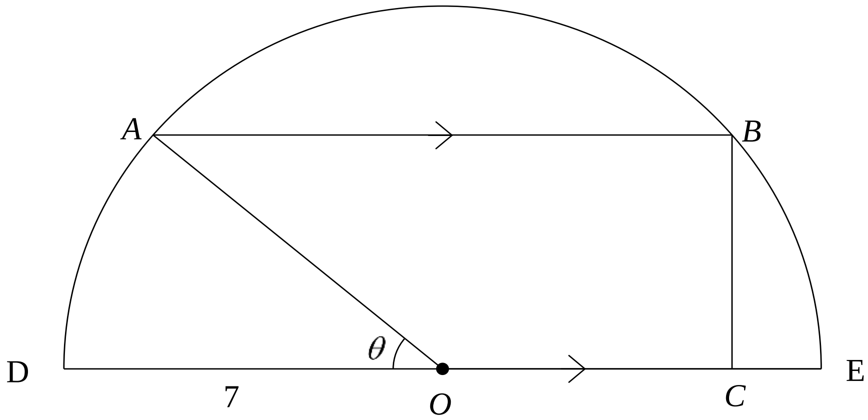
$$64^x \times 4^y = 1,$$
$$log_{\frac{1}{2}}(x - 3y - 6) + 2 = log_2 x.$$

(a) Show that $log_{\frac{1}{2}}(x - 3y - 6) + 2 = log_2 x$ can be written as

$$log_2(x - 3y - 6) = log_2 \frac{4}{x}$$ [3]

(b) Solve, for *x* and *y*. [7]

8



At the Singapore Book of Records event, the students taking Additional Maths need to paste their cranes in the shape of a trapezium *OABC* inscribed in a semicircle with centre *O*, and radius 7 cm. *OA* makes an angle θ with the diameter *DE*. *BC* is perpendicular to both *AB* and *CO*.

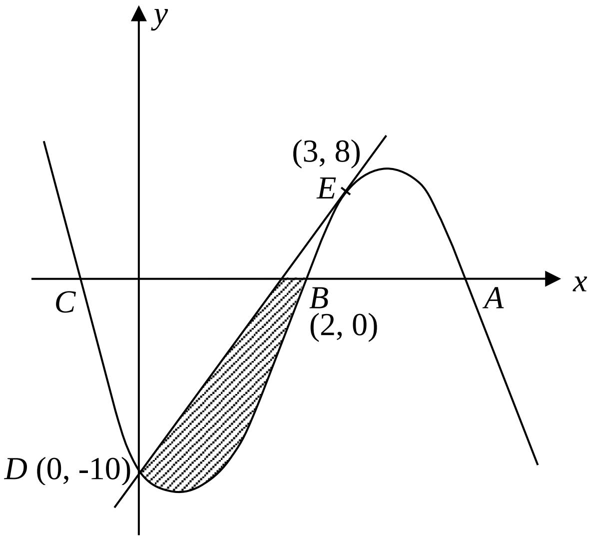
(a) State the property that shows that *AB* is twice the length of *OC*. [1]

(b) Show that *P* cm, the perimeter of the trapezium, can be expressed in the form $m + 21\ \cos\theta + 7\ \sin\theta$, where *m* is a constant to be found. [2]

(c) Express P in the form $m + R\ \cos(\theta - \alpha)$, where $R > 0$ and θ is an acute angle in degree mode. [3]

(d) Hence, find the maximum value of P and the corresponding value of θ at which this occurs. [3]

9



The diagram shows part of the curve $y = -x^3 + 6x^2 - 3x - 10$. The curve cuts the x-axis at A, B (2, 0) and C, and cuts the y-axis at D (0, -10).

(a) Show that the tangent to the curve at E (3, 8) passes through D. [5]

(b) Calculate the area of the shaded region. [7]

10 The table shows experimental values of two variables x and y when Arne Mo Salim was doing his O Level Physics Practical Exam.

x	0.2	0.6	1.0	1.4
y	0.5	1.4	2.5	4.0

It is known that x and y are related by the equation $y = ax\sqrt{x} + b\sqrt{x}$, where a and b are constants.

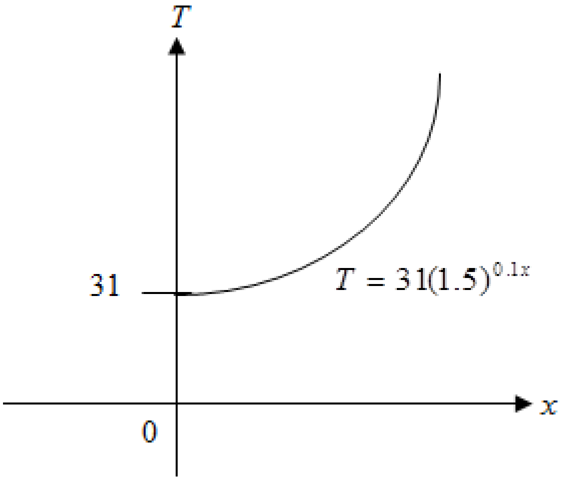
(a) Explain how a straight line graph can be obtained using the above data. [2]

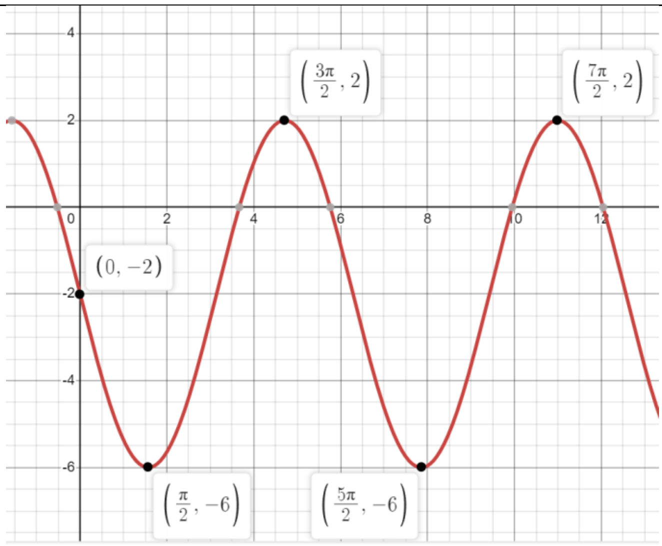
(b) Draw this graph on the next page for the given data and use it to estimate the value of a and of b. [6]

End of Paper

AMP2 Prelims 2024

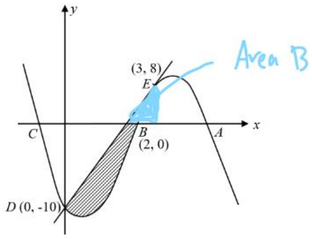
No	Answer	Marks
1(a)	$\left(x+\frac{1}{x}\right)^4 = x^4 + ({}^4_1C_1)(x^3)\left(\frac{1}{x}\right) + ({}^4_2C_2)(x^2)\left(\frac{1}{x}\right)^2 + ({}^4_3C_3)(x^1)\left(\frac{1}{x}\right)^3 + ({}^4_4C_4)(x^0)\left(\frac{1}{x}\right)^4$ $= x^4 + 4x^2 + 6 + 4x^{-2} + x^{-4}$	M1 A1
1(b)	$\left(x+\frac{1}{x}\right)^4 - \left(x-\frac{1}{x}\right)^4$ $= x^4 + 4x^2 + 6 + 4x^{-2} + x^{-4} - (x^4 - 4x^2 + 6 - 4x^{-2} + x^{-4})$ $= 8x^2 + 8x^{-2}$ $\therefore a = 8, b = 8$	M1 A2
1(c)	$\left(\frac{4}{3}x + \frac{k}{x} + \frac{x^3}{k}\right)\left(x+\frac{1}{x}\right)^4 = \left(\frac{4}{3}x + \frac{k}{x} + \frac{x^3}{k}\right)(x^4 + 4x^2 + 6 + 4x^{-2} + x^{-4})$ $= \ldots \frac{4}{3}x(6) + \frac{k}{x}(4x^2) + \frac{x^3}{k}(4x^{-2}) + \ldots = \ldots 8x + 4kx + \frac{4}{k}x + \ldots$ (find coefficient of x) Since there is no x term, $8x + 4kx + \frac{4}{k}x = 0$ $x\left(8 + 4k + \frac{4}{k}\right) = 0$ $8 + 4k + \frac{4}{k} = 0$ or $x = 0$ (NA) $4k^2 + 8k + 4 = 0$ $k^2 + 2k + 1 = 0$ $(k+1)^2 = 0$ $\therefore k = -1$	M1 M1 A1
2(a)	$\measuredangle DCP = \measuredangle CBD$ (tangent chord theorem) $\measuredangle ABD = \measuredangle CBD$ (bisected angles of triangle ABC) $\measuredangle ABD = \measuredangle ACD$ (angles in same segment) So $\measuredangle DCP = \measuredangle ACD$ (shown)	M1 M1 M1
2(b)	$\measuredangle CPD = \measuredangle APC$ (common angle) $\measuredangle PCD = \measuredangle PAC$ (tangent chord theorem) So by AA similarity test, $\triangle PCD$ similar to $\triangle PAC$	 M1 A1
2(c)	Since $\triangle PCD$ similar to $\triangle PAC$ from ii), $\frac{PC}{PA} = \frac{PD}{PC}$ $PC^2 = PA \times PD$ (shown)	B1
3(a)	When $x = 3$, $T = 35$, $35 = 31(1.5)^{k(3)}$ $k = 0.1(1dp)$	 M1 A1
3(b)	When $t = 0$, $T = 31(1.5)^0 = 31°C$	 B1

3(c)	115% of initial temp $= \frac{115}{100} \times 31$ $= 35.65°\text{C}$ When $T = 35.65°C$, $35.65 = 31(1.5)^{0.1x}$ $\frac{35.65}{31} = (1.5)^{0.1x}$ $\lg\left(\frac{35.65}{31}\right) = \lg(1.5)^{0.1x}$ $\lg(1.15) = 0.1x\lg(1.5)$ $0.1x = \frac{\lg 1.15}{\lg 1.5}$ $x = \frac{\frac{\lg 1.15}{\lg 1.5}}{0.1}$ $= 3.45(3.s.f)$ So the temperature will increase by at least 15% in 2028.	M1 M1 M1 A1
3(d)	 G1 Shape G1 y-intercept	
4(a)(i)	Period = $\frac{2\pi}{3}$	B1
4(a)(ii)	$a + b = 6$ $-a + b = -2$ Solve simultaneously, $a = 4$, $b = 2$	M1A1

4(a)(iii)	 1 mark for correct number of cycles 1 mark for correct endpoints, max points 1 mark for correct shape	
4(b)	$\dfrac{\sin\theta - \sin 2\theta + \sin 3\theta}{\cos\theta - \cos 2\theta + \cos 3\theta} = \tan 2\theta$ LHS $= \dfrac{\sin(2\theta-\theta) - \sin2\theta + \sin(2\theta+\theta)}{\cos(2\theta-\theta) - \cos2\theta + \cos(2\theta+\theta)}$ $= \dfrac{\sin2\theta\cos\theta - \cos2\theta\sin\theta - \sin2\theta + \sin2\theta\cos\theta + \cos2\theta\sin\theta}{\cos2\theta\cos\theta + \sin2\theta\sin\theta - \cos2\theta + \cos2\theta\cos\theta - \sin2\theta\sin\theta}$ $= \dfrac{2\sin2\theta\cos\theta - \sin2\theta}{2\cos2\theta\cos\theta - \cos2\theta}$ $= \dfrac{\sin2\theta(2\cos\theta - 1)}{\cos2\theta(2\cos\theta - 1)} = \tan 2\theta$ =RHS	M1 M1 M1 M1
5(a)	When $t = 0$, $s = 4 - 2e^0 = 2$ m	B1
5(b)	$s = 4 - 2e^{-t} - t$ $V = 2e^{-t} - 1$ When $t = 0, V = 2e^{-0} - 1 = 1$ m/s	M1 A1
5(c)	When $V = 0$, $0 = 2e^{-t} - 1$ $e^{-t} = 0.5$ $e^{t} = 2$ $t = \ln 2 = 0.693$ s (3 s.f.) (Accept $t = \ln 2$)	M1 A1
5(d)	When $t = 0$, $S = 2$ m When $t = 0.693$s, $S = 2.30685$ m When $t = 2$s, $S = 1.729329$ m M1 Total distance travelled = (2.30685 – 2) + (2.30685 – 1.729329) M1 = 0.884 m (3 s.f.) A1	Correct S values found Accept integration method
6(a)	$P(-1) = 0 \Rightarrow m + n = 3$ --------- (1) $P\left(\frac{1}{2}\right) = 0 \Rightarrow m + 4n = -3$ ----- (2) $m = 5$ and $n = -2$	M1 M1 A2
6(b)	$P(x) = 2x^3 + 5x^2 + x - 2 = (2x^2 + x - 1)(ax + b)$ By using the comparison method of the powers of x, $a = 1$ and $b = 2$ (or long division method)	M1 M1

	$P(x) = 0$ $\Rightarrow x = -2\ ,\ x = -1\ \ ,\ x = \frac{1}{2}$	A1 (all 3 values)
6(c)	$\frac{1}{4}y^3 + \frac{m}{4}y^2 + \frac{1}{2}y + n = 0$ $2\left(\frac{1}{2}y\right)^3 + m\left(\frac{1}{2}y\right)^2 + \frac{1}{2}y + n = 0$ Replace x by $\frac{1}{2}y$ $\therefore y = -4\ , y = -2\ , y = 1$	M1 A2 (all 3 values)
7(a)	$64^x \times 4^y = 1$ $log_{\frac{1}{2}}(\,x - 3y - 6) + 2 = log_2\, x$ $\dfrac{log_2(\,x - 3y - 6)}{log_2 \frac{1}{2}} = log_2\, x - log_2\, 4$ $-\,log_2(\,x - 3y - 6) = log_2\, x - log_2\, 4$ $log_2(\,x - 3y - 6) = log_2\, 4 - log_2\, x$ $log_2(\,x - 3y - 6) = log_2 \dfrac{4}{x}$	M1 M1 A1
7(b)	$64^x \times 4^y = 1$ $2^{6x} \times 2^{2y} = 2^0$ $6x + 2y = 0$ $3x + y = 0$ $y = -3x - - - (1)$ $x - 6 - 3y = \dfrac{4}{x} - - - (2)$ Sub (1) into (2) $x - 6 + 9x = \dfrac{4}{x}$ $10x - 6 = \dfrac{4}{x}$ $10x^2 - 6x = 4$ $5x^2 - 3x - 2 = 0$ $(5x + 2)(x - 1) = 0$ $x = -\dfrac{2}{5}$ or 1 $y = \dfrac{6}{5}$ or -3	M1 M1 M1 M1 M1 A1 A1
8(a)	$\perp$ from centre bisects chord	B1
8(b)	$\angle BAO = x$ (alt $\angle$) $\sin\theta = \dfrac{OX}{7} \Rightarrow OX = 7\sin\theta$	M1 M1

	$\cos\theta = \frac{AX}{7} \Rightarrow AX = 7\cos\theta$ $P = 7 + 14\cos\theta + 7\sin\theta + 7\cos\theta$ $P = 7 + 21\cos\theta + 7\sin\theta$	A1
8(c)	$R = \sqrt{21^2 + 7^2}$ $R = 7\sqrt{10}$ $\theta = \tan^{-1}\left(\frac{7}{21}\right) = 18.435^\circ$ $\therefore P = 7 + 7\sqrt{10}\cos\left(\theta - 18.4^\circ\right)cm$	M1 M1 A1
8(d)	Max $P = 7 + 7\sqrt{10}$ or 29.1 $\theta - 18.435^\circ = \cos^{-1}(1)$ $\theta = 18.4^\circ$	M1 M1 A1
9(a)	$\frac{dy}{dx} = -3x^2 + 12x - 3$ $x = 3, m_{\tan} = 6$ $y = 6x + c$ $c = -10$ $y = 6x - 10$ equation of tangent $sub\ x = 0$ in y = 6(0) $-10 = -10$ Since D(0, -10) satisfy the equation of tangent at E, therefore tangent passes through D.	M1 M1 M1 M1 A1
9(b)	Area $BCD = -\int_0^2 -x^3 + 6x^2 - 3x\ 10\ \mathrm{dx}$ $= \left[\frac{-1}{4}x^4 + 2x^3 - \frac{3}{2}x^2 - 10x\right]_0^2$ =14 units^2 Equation of line y = 6x-10 When y = 0, x = $\frac{5}{3}$ Area of triangle $BCD = \frac{1}{2} \times 10 \times \frac{5}{3} = \frac{25}{3}$ Shaded area = 14 $-\frac{25}{3} = 5\frac{2}{3}$ units^2	M1 A1 M1 A1 M1A1 A1
Or 9(b)	***area A =*** $\int_0^3 (6x - 10) - \left(-x^3 + 6x^2 - 3x - 10\right)dx$ $\int_0^3 \left(9x + x^3 - 6x^2\right)dx$ $= {}_0^3\left[\frac{9x^2}{2} + \frac{x^4}{4} - \frac{6x^3}{3}\right]$ $= 6.75$	Equation of line M1 M1 A1

	Area of B = $\frac{1}{2}\times 8\times\left(3-\frac{5}{3}\right)-\int_{2}^{3}\left(-x^{3}+6x^{2}-3x-10\right)dx$ $5\frac{1}{3}-\left[\frac{-x^{4}}{4}+2x^{3}-\frac{3}{2}x^{2}-10x\right]_{2}^{3}$ $=\mathbf{\frac{13}{12}}$  ***Answer:*** $\boldsymbol{6\frac{3}{4}-\left(\frac{13}{12}\right)=5\frac{2}{3}}$ ***units***^{2}	M1 M1A1 A1
10(a)	$y = ax\sqrt{x} + b\sqrt{x}$ $\frac{y}{\sqrt{x}} = ax + b$ $Plot\ \frac{y}{\sqrt{x}}\ against\ x$	M1 A1
10(b)	Graph $a = \text{gradient} \approx \frac{3.38-1.12}{1.4-0.2} \approx 1.57$ $b = \frac{y}{\sqrt{x}} - \text{intercept} \approx 0.75$	P1C1S1 M1A1 A1