Chapter 1: Measurement

1. S.I. Units

- The seven **base units** are: metre (m), kilogram (kg), second (s), ampere (A), kelvin (K), mole (mol) and candela (cd).

- **Derived units** are defined in terms of base units. They are expressed as products or quotients of base units.

- An equation is said to be **homogeneous** or **dimensionally consistent** if every term on both sides of the equation has the same base units.

- A homogeneous equation may not be true or correct.

2. Errors and Uncertainties

Item	Accuracy	Precision
Instrument	Calibration of instrument	Smallest division of instrument
Measurements	Closeness of average to true value	Closeness of measurements to one another

- ***Systematic errors*** result in all readings or measurements being always smaller or always larger than the true value by a fixed amount.

- ***Random errors*** result in readings or measurements being scattered about the true value.

- ***Accuracy*** is the degree of closeness of the average value of the measurements to the true value. It is affected by systematic error.

- ***Precision*** is the degree of agreement between repeated measurements of the same quantity. It is affected by random error.

- Calculations:

  If $Q = aX \pm bY$, then $\Delta Q = |a|\,\Delta X + |b|\,\Delta Y$.

  If $Q = aX^m \times Y^n$ or $Q = a\dfrac{X^m}{Y^n}$, then $\dfrac{\Delta Q}{Q} = |m|\dfrac{\Delta X}{X} + |n|\dfrac{\Delta Y}{Y}$.

  Percentage uncertainty of Q = $\dfrac{\Delta Q}{Q} \times 100\%$.

  - Absolute uncertainty ΔQ should always be expressed to 1 s.f.
  - Quantity Q should be expressed up to the <u>same decimal place</u> as ΔQ.
    Eg: $g \pm \Delta g = (9.81 \pm 0.02)\ \text{m s}^{-1}$

  - Fractional and percentage uncertainty is expressed to 2 s.f.
    Eg: $\dfrac{\Delta g}{g} = \dfrac{0.02}{9.81} = 0.0020\ (2\text{ s.f.}) = 0.20\ \%\ (2\text{ s.f.})$

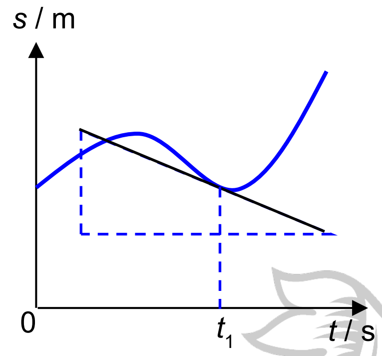
3. Scalars & Vectors

- A **scalar** quantity has a magnitude only.
- A **vector** quantity has both a magnitude and a direction.

Chapter 2: Kinematics

1. Interpretation of Graphs

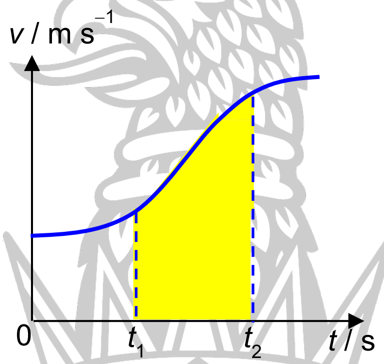
<u>$s - t$ graph</u>



Gradient $= \dfrac{ds}{dt} = v$

= instantaneous velocity

<u>$v - t$ graph</u>



Gradient $= \dfrac{dv}{dt} = a$

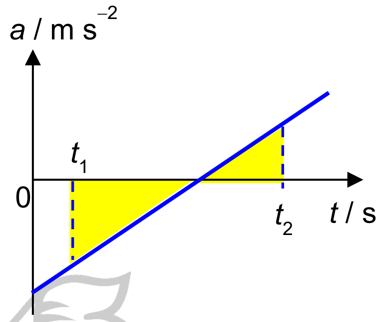
= instantaneous acceleration

Area under the graph

$= \int_{t_1}^{t_2} v\ dt = \Delta s$

= change in displacement

<u>$a - t$ graph</u>

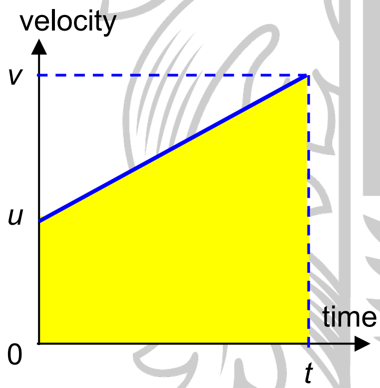


Area under the graph

$= \int_{t_1}^{t_2} a\ dt = \Delta v$

= change in velocity

2. Rectilinear Motion (1-D)



$$\text{gradient} = a = \frac{(v-u)}{t}$$

$$\therefore v = u + at \qquad \cdots(1)$$

$$\text{area} = s = \frac{1}{2}(u+v)t \qquad \cdots(2)$$

substituting (1) into (2),

$$s = ut + \frac{1}{2}at^2 \qquad \cdots(3)$$

$$\text{and } v^2 = u^2 + 2as \qquad \cdots(4)$$

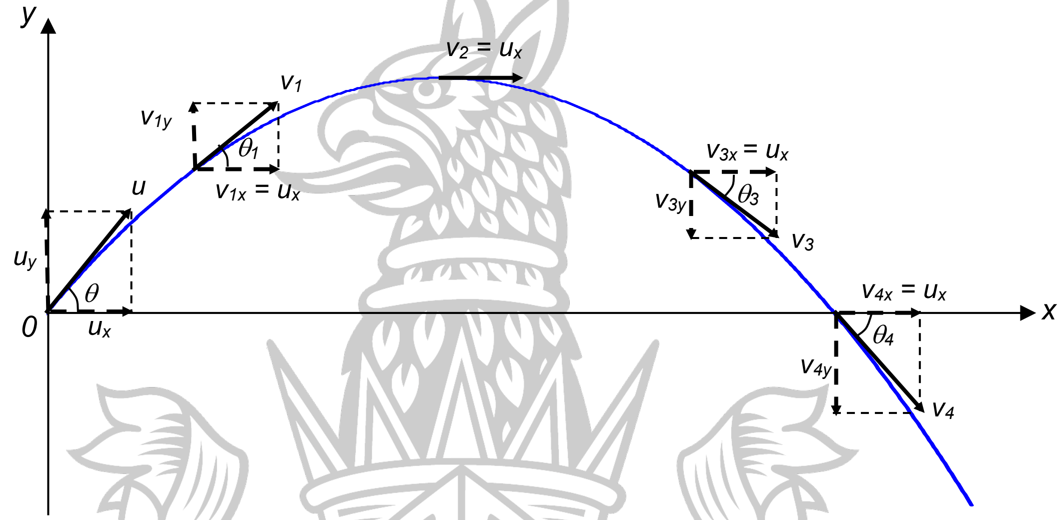
Equations (1) to (4) apply only to motion in a straight line at constant acceleration.

3. Solving kinematics problems

- Draw diagram(s) and transfer data from the question onto the diagram(s).
- Indicate positive displacements along the vertical and horizontal directions.
- Ensure that sign conventions are applied consistently to all quantities.
- Apply the above equations independently along the vertical and horizontal directions.

4. Projectile Motion (2-D)

In the absence of air resistance, the path/trajectory of a projectile is a parabola.
The magnitude of acceleration due to free fall (acting vertically downwards) is *g*.



		Vertically	Horizontal
Initial velocity	u	$u_y = u \sin \theta$	$u_x = u \cos \theta$
Acceleration	a	$a_y = g$ (downwards)	$a_x = 0$
Final velocity	v	$v_y = u_y + a_y t$ $v_y^2 = u_y^2 + 2a_y s_y$	$v_x = u_x + a_x t$ $\therefore v_x = u_x$
Displacement	s	$s_y = u_y t + \frac{1}{2} a_y t^2$ $s_y = \frac{1}{2}(u_y + v_y)t$ Maximum height (max s_y) is reached when $v_y = 0$	$s_x = u_x t$
Time	t	t	t

Chapter 3: Dynamics

1. Important Laws and Definitions

i. **Newton's First Law** states that every object continues in its state of rest or uniform motion in a straight line unless it is acted upon by a resultant external force.

ii. **Newton's Second Law** states that the rate of change of momentum of a body is proportional to the resultant force acting on it and the change occurs in the direction of the force. That is,

$$F_{net} = \frac{dp}{dt}$$

iii. **Newton's Third Law** states that if a body A exerts a force on body B, then body B exerts an equal but opposite force on body A.

iv. Since the internal forces of a system always add up to zero vectorally, $F_{net} = 0$ as long as there is no resultant external force acting on the system. When this happens, $\frac{dp_{system}}{dt} = F_{net} = 0$ which means that the total momentum of the system is constant.

The **Principle of Conservation of Momentum** states that when a system of bodies interact, the total momentum of the system remains constant, provided no net external force acts on it.

v. **Impulse** is defined as the product of a force F acting on an object and the time Δt for which the force acts.

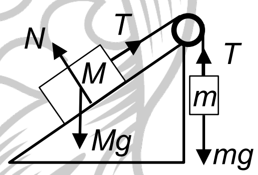
$$\text{Impulse} = F\,\Delta t$$

2. Applications of Newton's Second Law

i. A System of Objects

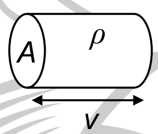
One example is shown in the figure on the right.

1. Identify forces acting on each object.
2. Write down the "$F = ma$" equation for each object.
3. Solve the simultaneous equations to get the answer.



ii. Force due to a Fluid Jet

Consider a continuous stream of fluid of density ρ moving with velocity v. After striking a surface, its velocity becomes v'.



Force experienced by the fluid

$= (\text{mass of fluid flow per unit time}) \times (\text{its change in velocity})$

$= (A\rho v)(v' - v)$

In most questions, v' is either 0 or $-v$. So $F = \rho A v^2$ or $F = 2\rho A v^2$

3. Momentum and its Conservation

i. Momentum is defined as the product of the mass of an object and its velocity.

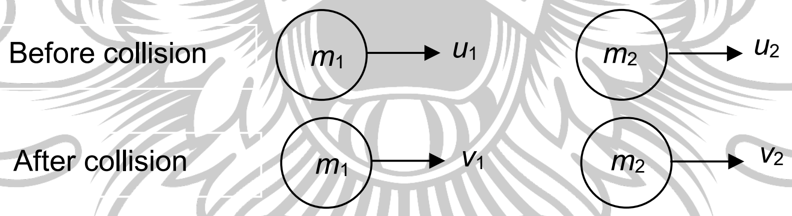
ii. Momentum is a vector quantity and its unit is kg m s^{-1} or N s.

iii. The impulse of a force = $\int F \, dt$ = area under the F−t graph. The impulse exerted on a body is equal to the change in momentum of the body.

iv. In collision problems in which external forces are absent or negligible, the total momentum is always conserved. On the other hand, the kinetic energy of the system is conserved only of the collision is perfectly elastic. We may classify the types of collisions as follows:

Type of collision	Momentum conserved?	K.E. conserved?
Elastic	✓	✓
Inelastic	✓	×
Completely inelastic (bodies coalesced)	✓	×

v. A head-on collision between two objects is shown below:



Since momentum is always conserved in such a collision, we can always write down:

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

It is important to remember that the equation is written down with the figure above in mind, i.e., all the velocities are rightward. If, for example, the question specifies that m_2 is moving at 5 m s^{-1} to the left, then you should substitute u_2 by –5.

If the collision is also perfectly elastic, then one can write down an additional equation

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

This new equation is slightly difficult to solve due to the squared terms. However, we can combine the two equations to obtain a third, linear equation:

$$u_1 - u_2 = v_2 - v_1$$

relative speed of approach = relative speed of separation

Chapter 4: Forces

1. Important Formulae and Definitions

i. Hooke's Law states that the extension x of a spring (or wire) is proportional to the applied force F, if the limit of proportionality is not exceeded.

$$F = kx$$

where k is the force constant.

ii. The weight of a body may be taken as acting at a single point known as its centre of gravity.

iii. Pressure is the normal force acting per unit area, where the force is acting at right angles to the area.

$$p = \frac{F}{A}$$

A scalar quantity.
S.I. units: N m^{-2} or Pascal (Pa)

At a given depth h, pressure *due to the fluid column above* is given by $p = \rho gh$, where ρ is the density of fluid. Note that the total pressure at depth h also includes the atmospheric pressure.

iv. Upthrust is the vertical upward force exerted on a body by a fluid when it is fully or partially submerged in the fluid due to the difference in fluid pressure.

Note: "Due to the difference in fluid pressure" part is important. A fluid exerts other kinds of forces on an object submerged, e.g. the drag force, which is not due to the fluid pressure but rather due to the motion of the object relative to the fluid.

v. Upthrust is equal to the weight of the fluid that is displaced by the body.

$$U = \rho gV$$

where V = volume submerged in the fluid,
ρ = density of fluid,
g = acceleration due to gravity.

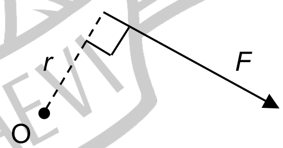
vi. Principle of floatation: For an object floating in equilibrium, the upthrust is equal in magnitude and opposite in direction to the weight of the object.

2. Moments and Torque

i. Turning effect of a force

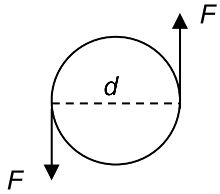
The **moment** (or **torque**) of a force about a point is defined as the product of the force and the perpendicular distance from the point to the line of action of the force.

Moment τ of force F about O, $\tau = F \times r$



A vector quantity (clockwise or anti-clockwise moment).
SI units: N m

ii. Torque of a couple



A couple consists of a pair of equal and opposite parallel forces whose lines of action do not coincide.

Torque of a couple
= magnitude of one force × the perpendicular distance between the two forces.
Torque of the couple shown, $\tau = d \times F$

iii. Principle of moments

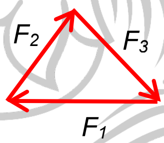
For a body in rotational equilibrium, the sum of all the clockwise moments about any point must equal the sum of all the anticlockwise moments about the same point.

3. Forces in Equilibrium

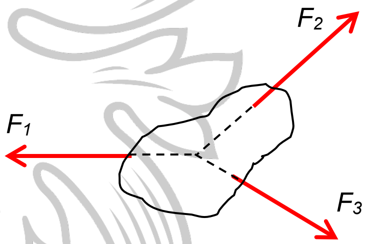
For a body in static or dynamic (constant velocity) equilibrium, the necessary conditions are:

i. The resultant force on the body is zero, $\Sigma F = 0$
ii. Resultant torque on the body about any point is zero, $\Sigma \tau = 0$

For 3 non-parallel forces acting on an object in equilibrium:



- The resultant force on the object is zero, $\Sigma F = 0$, implies that the forces, when taken in order, form a closed triangle.



- The resultant torque on the object about any point is zero, $\Sigma \tau = 0$, implies that their lines of action must all pass through a single common point (if they are not parallel).

4. Solving Problems Involving System in Equilibrium

i. Select a body or system of bodies for analysis.

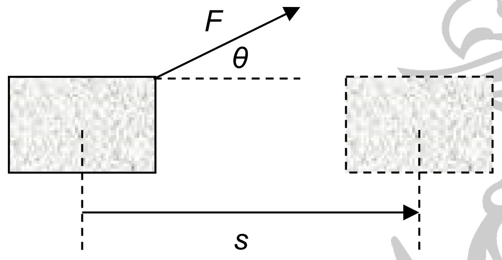
ii. Draw a free-body diagram showing all the forces acting on the body or system. Take care to draw the arrows for the forces starting from their correct points of action, as these will affect the moment calculation.

iii. Select a positive direction each for forces and moments. A wise choice of the point of reference (a.k.a. the pivot) simplifies the calculation for moments tremendously.

iv. Evaluate the type of problem to be solved and form the equations using:
$\Sigma F_x = 0$ and $\Sigma F_y = 0$ and $\Sigma \tau = 0$

Chapter 5: Work, Energy & Power

1. Work done (W) by a constant force is the product of the force and the displacement in the direction of the force.



$$W = Fs\cos\theta$$

Notes:
1. Work done becomes negative if $\theta > 90°$.
2. The above formula applies to a constant force. For a variable force, W.D. is equal to the area under the **force-displacement graph**.

2. The **external** force needed to stretch a spring by an extension x (from its unstretched length) is proportional to the extension x.

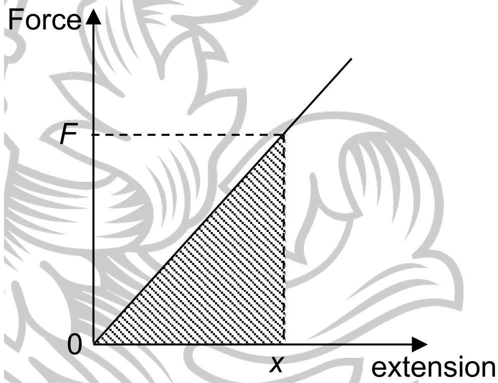
$$F = kx$$

where k is the force constant of the spring.

The work done by such an external force is stored in the spring as its elastic potential energy (U_E). It is given by the area under the **force-extension** graph (see right):

$$U_E = W = \frac{1}{2}kx^2$$

The same formula applies to compression.



3. Work done by a gas expanding against a **constant external** pressure p:

$$W = p\Delta V$$

where ΔV is the increase in volume of the gas. Note that W.D. **by** the gas (on the environment) is the negative of the W.D. **on** the gas by the environment. For example, if the gas contracts, ΔV is negative, W.D. by the gas is negative, and W.D. on the gas is positive.

4. Energy is the capacity to do work. It exists in various forms, including kinetic energy, (electric, gravitational, elastic) potential energy, nuclear energy, chemical potential energy and internal energy. Energy cannot be created or destroyed. It can only be converted from one form to another. This is known as the law of conservation of energy.

5. The sum of kinetic energy (E_k) and the (electric, gravitational, elastic) potential energies (E_p) together is called mechanical energy of a system. In a non-isolated system where there is work done by an external force, W_F, the energy equation is as follows:

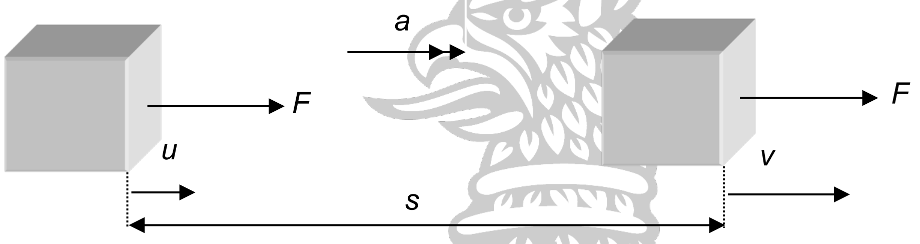
$$(E_p + E_k)_{\text{initial}} + W_F = (E_p + E_k)_{\text{final}}$$

One may understand this equation by considering W_F as the external energy input due to the external force, which becomes part of the final energy of the system. A system with no net external force acting on it is known as a closed system. For such a system, $W_F = 0$, the energy equation becomes

$$(E_p + E_k)_{\text{initial}} = (E_p + E_k)_{\text{final}}$$

6. Derivation of $E_k = \frac{1}{2}mv^2$

Consider a body of mass *m* that is moving with an initial velocity *u*. It is acted on by a **constant resultant force** *F* which is parallel to *u*. The body accelerates with a uniform acceleration *a* to a final velocity *v* over a displacement *s*.



Since the displacement of the body is in the same direction as the applied force,

$W = Fs = (ma)s$ ..... (1)

From $v^2 = u^2 + 2as$ (applicable since *a* is constant),

$s = \frac{v^2 - u^2}{2a}$ ..... (2)

Substituting equation (2) into (1),

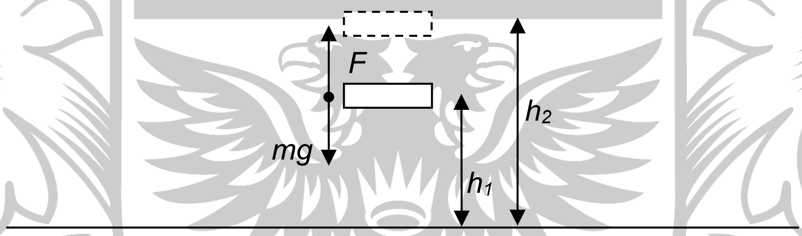
$W = (ma)\left(\frac{v^2 - u^2}{2a}\right) = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$ ..... (3)

Thus, the work done *W* is equal to the *change* in the quantity $\frac{1}{2}m \times (\text{velocity})^2$, which is termed the kinetic energy, E_k:

$$E_k = \frac{1}{2}mv^2$$

7. Derivation of gravitational $E_p = mgh$

Consider an object being raised upwards, at a **constant velocity**, from a height of h_1 to a height of h_2 near the Earth's surface. Since the velocity at which the object moves is constant, the **external** force, *F*, required to lift the object must be equal to its weight, *mg*.



Work done by *F* in displacing the center of mass of object vertically upward,

$$W = Fs = mg(h_2 - h_1) = mgh \quad \text{where } h = h_2 - h_1$$

In raising the object, the work done by the **external** force *F* (given by the expression above) is the gain in the gravitational potential energy of the object.

8. For a **field of force** (such as a gravitational field or an electric field), the relationship between the force *F* and the potential energy *U* for one-dimensional motion is given by

$$F = -\frac{\mathrm{d}U}{\mathrm{d}x}$$

From the relationship, we can infer:

1. The magnitude of the force at point *x* is equal to the gradient of the potential energy curve at *x*;
2. The direction of the force is the direction of decreasing potential energy.

9. Efficiency gives a measure of how much of the total energy may be considered useful and is not "lost". It is given by the expression:

$$\eta = \frac{\text{useful energy output}}{\text{total energy input}} \times 100\%$$

Which form of energy is "useful" depends on the scenario. The difference between total energy input and useful energy output is the energy loss.

10. Power is defined as the rate of work done or energy conversion with respect to time:

$$P = \frac{\mathrm{d}W}{\mathrm{d}t}$$

The above formula yields the instantaneous power. What is frequently useful is the average power, given by

$$P_{\text{avg}} = \frac{\Delta W}{\Delta t}$$

11. If a **constant force** *F* is applied and does work by moving its point of application a displacement *s* in time *t*, the power supplied is given by the following derivation:

$P = \dfrac{\mathrm{d}W}{\mathrm{d}t} = \dfrac{\mathrm{d}(Fs\cos\theta)}{\mathrm{d}t} = F\dfrac{\mathrm{d}s}{\mathrm{d}t}\cos\theta = Fv\cos\theta$, (Note: θ is the angle between F and v or s)

where $v = \dfrac{\mathrm{d}s}{\mathrm{d}t}$ is the velocity of the point of application.

$$P = Fv\cos\theta$$

12. Problems related to mass flow rate $\dfrac{dm}{dt}$ (such as energy conversion in a wind turbine):

1. Calculate the amount of mass passing through per unit time (E.g.,1 second).
2. Hence, calculate the amount of kinetic energy, passing through per unit time.
3. Write down the corresponding Conservation of Energy equations for the unit time.

Chapter 6: Circular Motion

1. Angular displacement: angle an object makes with respect to a reference line.

2. One radian : angle subtended by an arc length equal to the radius of the arc.

$\theta = \frac{s}{r}$ where s : arc length; r : the radius of the circle.

3. π radian $\equiv 180°$

4. Angular velocity ω, of an object in circular motion is defined as the rate of change of its angular displacement with respect to time.

$$\omega = \frac{d\theta}{dt} \qquad \text{(units: rad s}^{-1}\text{)}$$

5. Linear velocity $v = r\omega$ (units: m s^{-1})

6. Period $T = \frac{2\pi}{\omega} = 2\pi f$ (unit: s)

7. Frequency $f = \frac{1}{T}$ (unit: Hz)

8. Centripetal acceleration $a = v\omega = r\omega^2 = \frac{v^2}{r}$ (units: m s^{-2})

9. Centripetal force $F = ma = mv\omega = mr\omega^2 = \frac{mv^2}{r}$ (units: N)

10. An object in uniform circular motion is accelerating because the direction of its velocity is changing continuously although there is no change in its speed.

Problem-Solving Strategy for **Uniform** Circular Motions:

1. Draw a free-body diagram of the body under consideration. Label **all** forces acting on the body.
2. Identify the centre of the circular motion.
3. Resolve the forces along two perpendicular axes according to each scenario.
4. Find the expression for the resultant force towards the centre of the circular path.
5. Equate it to ma_c, where m is the mass of the body and $a_c = \frac{v^2}{r} = r\omega^2$.

Problem-Solving Strategy for **Non-Uniform** Circular Motions:

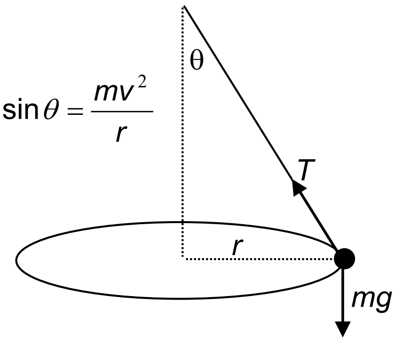
1. Draw a free-body diagram of the body under consideration. Label all forces acting on the body. Usually there are some special conditions such as "just in contact" which implies $N = 0$ or "string is just taut" which implies, $T = 0$.
2. Identify the centre of the circular motion.
3. Resolve the forces along two perpendicular axes according to each scenario.
4. Find the expression for the resultant force towards the centre of the circular path.
5. Equate it to ${mv^2}/{r}$ or $m\omega^2 r$.

   (Take note: for Non-Uniform circular motion, the speed v or angular velocity ω is NOT constant through the motion. Hence, when equating to ${mv^2}/{r}$ or $m\omega^2 r$, we are finding v or ω only at THAT instant).
6. Using Conservation of Energy, form another equation for the motion.
7. Solve them simultaneously to find the unknowns.

The centripetal force for the following **Uniform** Circular Motions are:

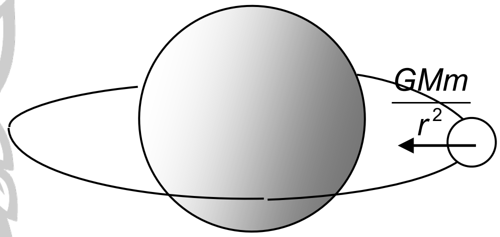
(i) Conical Pendulum:

$$T\sin\theta = \frac{mv^2}{r}$$



Centripetal force is the resultant force pointing towards the centre of rotation. Therefore, the centripetal force is provided by the horizontal component of tension $T\sin\theta$.

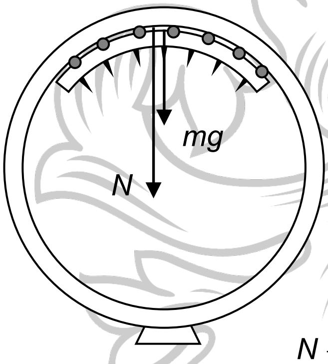
(ii) Celestial Objects:



$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

Centripetal force is the resultant force pointing towards the centre of rotation. Therefore, centripetal force is provided by gravitational force.

(iii) **Roller coaster (top):**



$$N + mg = \frac{mv^2}{r}$$

Centripetal force is the resultant force pointing towards the centre of rotation. Therefore, the centripetal force is the sum of the normal contact force and weight.

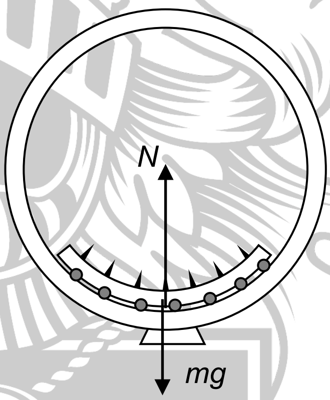
*** Minimum speed to stay in contact at the top**

For the roller coaster to just stay in contact, $N = 0$

$$\text{So } N + mg = \frac{mv^2}{r} \quad \Rightarrow mg = \frac{mv_{\min}^2}{r}$$

$$\therefore\ v_{\min} = \sqrt{rg}$$

(iv) **Roller coaster (bottom):**



$$N - mg = \frac{mv^2}{r}$$

Centripetal force is the resultant force pointing towards the centre of rotation. Therefore, the centripetal force is the normal contact force minus weight.

*** Minimum speed to enter to keep in contact at the top**

By Conservation of Energy,

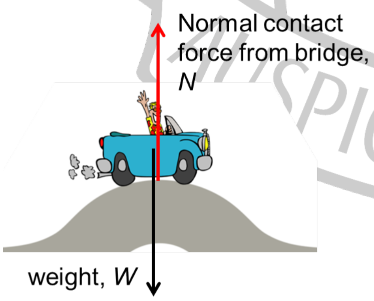
$$E_{k(bottom)} = E_{k(top)} + GPE_{(top)}$$

$$\frac{1}{2}mv_{bottom}^2 = \frac{1}{2}mv_{\min}^2 + mg(2r)$$

$$v_{bottom}^2 = v_{\min}^2 + 2g(2r)$$

$$v_{bottom} = \sqrt{rg + 4rg} = \sqrt{5rg}$$

(v) **A car going over an arch:**



$$mg - N = \frac{mv^2}{r}$$

Centripetal force is the resultant force pointing towards the centre of rotation. Therefore, the centripetal force is the weight minus normal contact force.