1i	$\begin{pmatrix}1\\1\\0\end{pmatrix}\times\begin{pmatrix}1\\-5\\-2\end{pmatrix}=\begin{pmatrix}-2\\2\\-6\end{pmatrix}=-2\begin{pmatrix}1\\-1\\3\end{pmatrix}$
1ii	Let θ be the required acute angle. $\cos\theta=\dfrac{\left\lvert\begin{pmatrix}1\\-1\\3\end{pmatrix}\cdot\begin{pmatrix}4\\5\\-6\end{pmatrix}\right\rvert}{\sqrt{1^2+(-1)^2+3^2}\sqrt{4^2+5^2+(-6)^2}}$ $\theta=\cos^{-1}\left(\dfrac{\lvert 4-5-18\rvert}{\sqrt{11}\sqrt{77}}\right)$ $\theta=\cos^{-1}\left(\dfrac{19}{11\sqrt{7}}\right)=49.2^\circ$ (1 d.p.)

2	$\dfrac{x^2}{1+x^2}+\dfrac{y^2}{1+y^2}=x^3y^5$ $1-\dfrac{1}{1+x^2}+1-\dfrac{1}{1+y^2}=x^3y^5$ Differentiate implicitly w.r.t. *x*. $\dfrac{2x}{\left(1+x^2\right)^2}+\dfrac{2y}{\left(1+y^2\right)^2}\dfrac{\mathrm{d}y}{\mathrm{d}x}=3x^2y^5+5x^3y^4\dfrac{\mathrm{d}y}{\mathrm{d}x}$. When $x=1, y=1$ and, $\dfrac{2}{(2)^2}+\dfrac{2}{(2)^2}\dfrac{\mathrm{d}y}{\mathrm{d}x}=3+5\dfrac{\mathrm{d}y}{\mathrm{d}x}$ $\dfrac{\mathrm{d}y}{\mathrm{d}x}=-\dfrac{5}{9}$ Hence the equation of the tangent is; $y-1=-\dfrac{5}{9}(x-1)$ $9y-9=-5x+5$ $5x+9y=14$.

3(i)	<u>Method 1</u> $\mathrm{f}'(x)=\dfrac{3\cos 3x}{1+\sin 3x}$ $\mathrm{f}''(x)=\dfrac{(1+\sin 3x)(-9\sin 3x)-(3\cos 3x)(3\cos 3x)}{(1+\sin 3x)^2}$ $\mathrm{f}''(x)=\dfrac{-9\sin 3x-9\sin^2 3x-9\cos^2 3x}{(1+\sin 3x)^2}$ $\mathrm{f}''(x)=\dfrac{-9\sin 3x-9}{(1+\sin 3x)^2}$ $\mathrm{f}''(x)=\dfrac{-9(1+\sin 3x)}{(1+\sin 3x)^2}$ $\mathrm{f}''(x)=\dfrac{-9}{1+\sin 3x}$ (Shown) Where $k=-9$.
3(ii)	$\mathrm{f}''(x)=\dfrac{-9}{1+\sin 3x}$ $\mathrm{f}'''(x)=\dfrac{9(3\cos 3x)}{(1+\sin 3x)^2}=\dfrac{27\cos 3x}{(1+\sin 3x)^2}$ When $x=0$, $\mathrm{f}(0)=\ln(1+0)=0,$ $\mathrm{f}'(0)=\dfrac{3}{1+0}=3,$ $\mathrm{f}''(0)=\dfrac{-9}{(1+0)^2}=-9,$ $\mathrm{f}'''(0)=\dfrac{27}{(1+0)^2}=27.$ $\mathrm{f}(x)=0+3x+\dfrac{-9}{2!}x^2+\dfrac{27}{3!}x^3+\ldots$ $=3x-\dfrac{9}{2}x^2+\dfrac{9}{2}x^3+\ldots$

4(i)	$z_1 = 1+\sqrt{3}\mathrm{i} = \sqrt{1^2+\left(\sqrt{3}\right)^2}\,\mathrm{e}^{\mathrm{i}\tan^{-1}\left(\frac{\sqrt{3}}{1}\right)} = 2\mathrm{e}^{\mathrm{i}\frac{\pi}{3}}$ $z_2 = 1-\mathrm{i} = \sqrt{1^2+(-1)^2}\,\mathrm{e}^{\mathrm{i}\tan^{-1}\left(\frac{-1}{1}\right)} = 2\mathrm{e}^{-\mathrm{i}\frac{\pi}{4}}$ $z_3 = 2\left(\cos\frac{1}{6}\pi+\mathrm{i}\sin\frac{1}{6}\pi\right) = 2\mathrm{e}^{\mathrm{i}\frac{\pi}{6}}$ $\left\|\frac{z_1}{z_2 z_3}\right\| = \frac{\|z_1\|}{\|z_2\|\|z_3\|} = \frac{2}{\left(\sqrt{2}\right)(2)} = \frac{1}{\sqrt{2}}$ $\arg\left(\frac{z_1}{z_2 z_3}\right) = \arg z_1 - \arg z_2 - \arg z_3$ $= \frac{\pi}{3}+\frac{\pi}{4}-\frac{\pi}{6}$ $= \frac{5\pi}{12}$ $\frac{z_1}{z_2 z_3} = \frac{1}{\sqrt{2}}\left(\cos\frac{5\pi}{12}+\mathrm{i}\sin\frac{5\pi}{12}\right).$
4(ii)	<u>Method 1</u>: As $\left\|\frac{z_1 z_4}{z_2 z_3}\right\| = 1$ and $\frac{z_1 z_4}{z_2 z_3}$ is purely imaginary $\Rightarrow \frac{z_1 z_4}{z_2 z_3} = \mathrm{i}$ or $-\mathrm{i}$. $\arg\left(\frac{z_1 z_4}{z_2 z_3}\right) = \frac{\pi}{2}$ or $-\frac{\pi}{2}$. $\arg z_4 + \arg\left(\frac{z_1}{z_2 z_3}\right) = \frac{\pi}{2}$ or $-\frac{\pi}{2}$ $\arg z_4 = \frac{\pi}{2}-\frac{5\pi}{12}$ or $-\frac{\pi}{2}-\frac{5\pi}{12} = \frac{\pi}{12}$ or $-\frac{11\pi}{12}$. As $\left\|\frac{z_1 z_4}{z_2 z_3}\right\| = 1 \Rightarrow \|z_4\| = \left\|\frac{z_2 z_3}{z_1}\right\| = \sqrt{2}$. Hence, $z_4 = \sqrt{2}\left(\cos\frac{\pi}{12}+\mathrm{i}\sin\frac{\pi}{12}\right)$ or $\sqrt{2}\left(\cos\left(-\frac{11\pi}{12}\right)+\mathrm{i}\sin\left(-\frac{11\pi}{12}\right)\right)$. <u>Method 2</u>: $\left\|\frac{z_1 z_4}{z_2 z_3}\right\| = 1 \Rightarrow \|z_4\| = \left\|\frac{z_2 z_3}{z_1}\right\| = \sqrt{2}$

	As $\frac{z_1 z_4}{z_2 z_3}$ is purely imaginary, $\arg\left(\frac{z_1 z_4}{z_2 z_3}\right)=\frac{(2k+1)\pi}{2}$, for integer *k*. $\arg z_4+\arg\left(\frac{z_1}{z_2 z_3}\right)=\frac{(2k+1)\pi}{2}$ $\arg z_4=\frac{(2k+1)\pi}{2}-\frac{5\pi}{12}=k\pi+\frac{\pi}{12}$ As *k* is an integer, we either have $\begin{cases}\arg z_4=\frac{\pi}{12}, & k \text{ is even}\\ \arg z_4=-\frac{11\pi}{12}, & k \text{ is odd}\end{cases}$ Hence, $z_4=\sqrt{2}\left(\cos\frac{\pi}{12}+\mathrm{i}\sin\frac{\pi}{12}\right)$ or $\sqrt{2}\left(\cos\left(-\frac{11\pi}{12}\right)+\mathrm{i}\sin\left(-\frac{11\pi}{12}\right)\right)$.

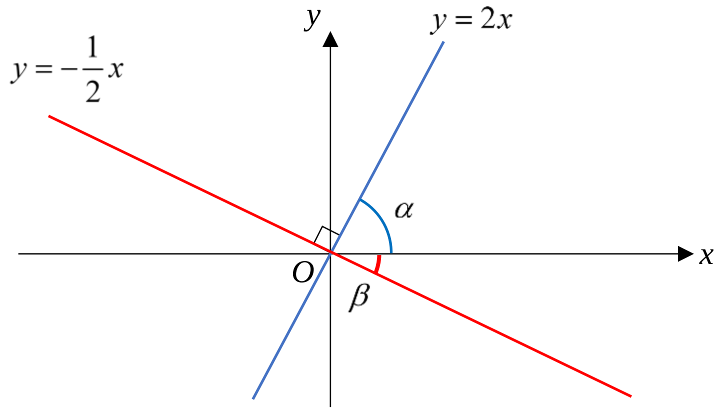
5(a)	$\mathbf{a}\times\mathbf{b}=\mathbf{b}\times\mathbf{a}$ $\mathbf{a}\times\mathbf{b}=-(\mathbf{a}\times\mathbf{b})$ $2(\mathbf{a}\times\mathbf{b})=\mathbf{0}$ $\mathbf{a}\times\mathbf{b}=0$ As both vectors are non-zero, this implies that **a** is parallel to **b**.
5(bi)	$(\mathbf{r}-\mathbf{p})\times\mathbf{q}=\mathbf{0}$ implies $\mathbf{r}-\mathbf{p}$ is parallel to **q** or that $\mathbf{r}-\mathbf{p}=\mathbf{0}\Rightarrow\mathbf{r}=\mathbf{p}$. $\mathbf{r}-\mathbf{p}=\lambda\mathbf{q},\lambda\in\mathbb{R}$ $\mathbf{r}=\mathbf{p}+\lambda\mathbf{q},\lambda\in\mathbb{R}$ The set of possible positions of *R* will be the line that is parallel to the vector **q** (or that it is parallel to *OQ*) that passes through point *P*.
5(bii)	$(\mathbf{r}-\mathbf{p})\cdot\mathbf{q}=0$ $\mathbf{r}\cdot\mathbf{q}-\mathbf{p}\cdot\mathbf{q}=0$ $\mathbf{r}\cdot\mathbf{q}=\mathbf{p}\cdot\mathbf{q}$ $\begin{pmatrix}x\\y\\z\end{pmatrix}\cdot\begin{pmatrix}3\\-5\\2\end{pmatrix}=\begin{pmatrix}-1\\2\\4\end{pmatrix}\cdot\begin{pmatrix}3\\-5\\2\end{pmatrix}=-3-10+8=-5$ $3x-5y+2z=-5$ The set of possible positions of *R* will be the plane that has a normal vector **q** (or that the normal vector is parallel to *OQ*) and it contains the point *P* $(-1,2,4)$.

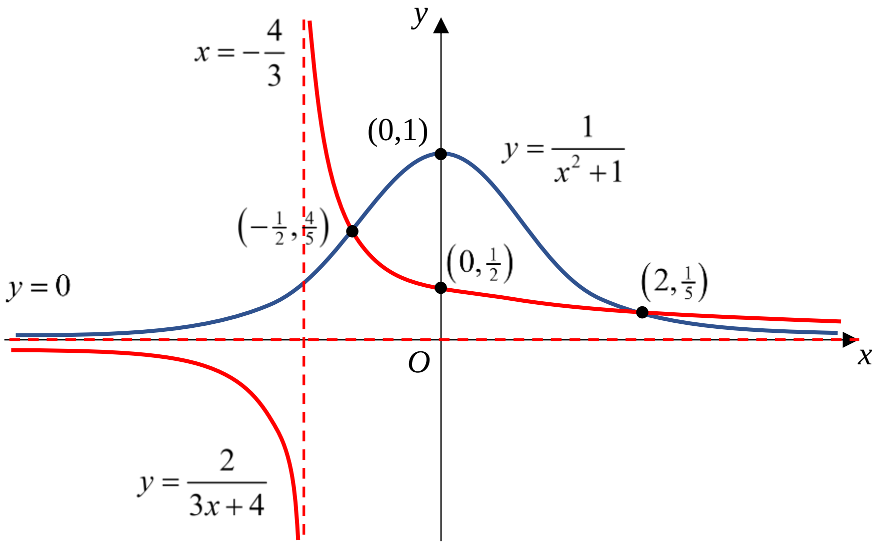
6	By factor theorem, if $z=k+k\mathrm{i}$, $(k+k\mathrm{i})^2(2+\mathrm{i})-8\mathrm{i}(k+k\mathrm{i})+t=0$ $k^2(1+2\mathrm{i}+\mathrm{i}^2)(2+\mathrm{i})-8k\mathrm{i}-8k\mathrm{i}^2+t=0$ $k^2(2\mathrm{i})(2+\mathrm{i})-8k\mathrm{i}+8k+t=0$ $4k^2\mathrm{i}+2k^2\mathrm{i}^2-8k\mathrm{i}+8k+t=0$ $(-2k^2+8k+t)+\mathrm{i}(4k^2-8k)=0$ We compare real and imaginary parts; Im: $4k^2-8k=0\Rightarrow k=2$. Re: $-2(2)^2+8(2)+t=0\Rightarrow t=-8$ <u>Method 1</u>: $z^2(2+\mathrm{i})-8\mathrm{i}z-8=0$ $z=\frac{8\mathrm{i}\pm\sqrt{(8\mathrm{i})^2-(4)(2+\mathrm{i})(-8)}}{2(2+\mathrm{i})}$ $z=\frac{8\mathrm{i}\pm(4+4\mathrm{i})}{2(2+\mathrm{i})}$ $z=\frac{12\mathrm{i}+4}{4+2\mathrm{i}}$ or $\frac{4\mathrm{i}-4}{4+2\mathrm{i}}$ $z=2+2\mathrm{i}$ or $-\frac{2}{5}+\frac{6}{5}\mathrm{i}$ Hence the other root is $z=-\frac{2}{5}+\frac{6}{5}\mathrm{i}$. <u>Method 2</u>: Let the other root be z_0. Sum of roots $=-\frac{-8\mathrm{i}}{2+\mathrm{i}}=\frac{8}{5}+\frac{16}{5}\mathrm{i}$. $2+2\mathrm{i}+z_0=\frac{8}{5}+\frac{16}{5}\mathrm{i}$ $z_0=-\frac{2}{5}+\frac{6}{5}\mathrm{i}$ (Note: One may also use product of roots.) Hence the other root is $z=-\frac{2}{5}+\frac{6}{5}\mathrm{i}$.

7(i)	$\int 2-\sin 4x \ \mathrm{d}x = 2x+\frac{1}{4}\cos 4x + c$.
7(ii)	$\int_0^{\frac{1}{2}\pi} 2x - x\sin 4x \ \mathrm{d}x = \left[x^2\right]_0^{\frac{1}{2}\pi} - \left(\left[-\frac{1}{4}x\cos 4x\right]_0^{\frac{1}{2}\pi} - \int_0^{\frac{1}{2}\pi} -\frac{1}{4}\cos 4x \ \mathrm{d}x\right)$ $= \frac{\pi^2}{4} - \left(\left(-\frac{1}{4}\left(\frac{\pi}{2}\right)(1)-0\right) + \left[\frac{\sin 4x}{16}\right]_0^{\frac{1}{2}\pi}\right)$ $= \frac{\pi^2}{4} - \left(-\frac{\pi}{8}+0-0\right)$ $= \frac{\pi^2}{4} + \frac{\pi}{8}$.
7(iii)	$\int_0^{\frac{1}{2}\pi} (2-\sin 4x)^2 \ \mathrm{d}x = \int_0^{\frac{1}{2}\pi} 4 - 4\sin 4x + \sin^2 4x \ \mathrm{d}x$ $= \left[4x+\cos 4x\right]_0^{\frac{1}{2}\pi} + \int_0^{\frac{1}{2}\pi} \frac{1-\cos 8x}{2} \ \mathrm{d}x$ $= \left(\frac{4\pi}{2} + \cos 2\pi - 0 - \cos 0\right) + \frac{1}{2}\left[x - \frac{\sin 8x}{8}\right]_0^{\frac{1}{2}\pi}$ $= 2\pi + 1 - 1 + \frac{1}{2}\left(\frac{\pi}{2} + \frac{\sin 4\pi}{8} - 0 - 0\right)$ $= 2\pi + \frac{\pi}{4}$ $= \frac{9\pi}{4}$.

8(ai)	Let a_n be the A.P. with first term 4 and common difference d . We have, $a_1 = 4, a_5 = 10$. $4+4d = 10$ $d = \frac{3}{2}$ $a_{30} = 4+29d = 4+\frac{29\times 3}{2} = 47.5$.
8(aii)	Method 1: Number of terms from 21 to $50 = 50-21+1 = 30$. And, $a_{21} = 4+20\left(\frac{3}{2}\right) = 34, a_{50} = 4+49\left(\frac{3}{2}\right) = 77.5$. Thus, $a_{21}+a_{22}+\ldots+a_{50} = \frac{30}{2}(34+77.5) = 1672.5$. Method 2: $a_{21}+a_{22}+\ldots+a_{50}$ $=(a_1+\ldots+a_{50})-(a_1+\ldots+a_{20})$ $=\frac{50}{2}\left(2(4)+49\left(\frac{3}{2}\right)\right)-\frac{20}{2}\left(2(4)+19\left(\frac{3}{2}\right)\right)$ $=1672.5$
8(bi)	Let b_n be the G.P. with first term 4 and common ratio r. $b_5 = 1.6384 \Rightarrow 4r^4 = 1.6384 \Rightarrow r = \frac{4}{5}$ (as $r > 0$) $S_\infty = \frac{4}{1-\frac{4}{5}} = 20.$
8(bii)	$b_1+b_2+\ldots+b_n > 19.6$ $\frac{4\left(1-\left(\frac{4}{5}\right)^n\right)}{1-\frac{4}{5}} > 19.6$ $20-20(0.8)^n > 19.6$ $-20(0.8)^n > -0.4$ $(0.8)^n < 0.02$ (Shown) Hence,

	$n\ln 0.8 < \ln 0.02$ $n > \dfrac{\ln 0.02}{\ln 0.8} = 17.531$ (5 s.f.) Least $n = 18$.

9(i)	Denote α, β be the angles inclined by the lines $y = 2x, y = -\frac{1}{2}x$ with respect to the positive x-axis.  As, $(2)\left(-\frac{1}{2}\right) = -1$, the lines are perpendicular and hence, $\alpha + \beta = \frac{\pi}{2}$. Moreover, by definition, $\tan^{-1}(2) = \alpha, \tan^{-1}\left(-\frac{1}{2}\right) = -\beta$. Therefore, $\tan^{-1} 2 - \tan^{-1}\left(-\frac{1}{2}\right) = \frac{\pi}{2}$. (Shown)
9(ii)	Note $k > 0$. For there to be intersection, the equation $\frac{1}{x^2+1} = \frac{k}{3x+4}$ has real solutions. $\frac{1}{x^2+1} = \frac{k}{3x+4}$ $kx^2 + k = 3x + 4$ $kx^2 - 3x + k - 4 = 0$ As $k > 0$ and that the equation has real solutions, it's discriminant ≥ 0. $(-3)^2 - 4(k)(k-4) \geq 0$ $9 - 4k^2 + 16k \geq 0$ $4k^2 - 16k - 9 \leq 0$ $\left(k - \frac{9}{2}\right)\left(k + \frac{1}{2}\right) \leq 0$ $-\frac{1}{2} \leq k \leq \frac{9}{2}$ Together with $k > 0$, we have $0 < k \leq \frac{9}{2}$. Hence, the required set is $\left\{k \in \mathbb{R} : 0 < k \leq \frac{9}{2}\right\}$.

9(iii)	
9(iv)	By the graph in **(9iv)**. Area required $=\int_{-\frac{1}{2}}^{2}\frac{1}{x^2+1}-\frac{2}{3x+4}\,dx$ $=\left[\tan^{-1}x-\frac{2}{3}\ln\lvert 3x+4\rvert\right]_{-\frac{1}{2}}^{2}$ $=\tan^{-1}2-\frac{2}{3}\ln 10-\tan^{-1}\left(-\frac{1}{2}\right)+\frac{2}{3}\ln\left(\frac{5}{2}\right)$ $=\tan^{-1}2-\tan^{-1}\left(-\frac{1}{2}\right)+\frac{2}{3}\ln\left(\frac{5/2}{10}\right)$ $=\frac{\pi}{2}+\frac{2}{3}\ln\left(\frac{1}{4}\right)$ (by (9i)) $=\frac{\pi}{2}-\frac{4}{3}\ln 2$ units^2.

10(i)	We note that Death rate > Birth rate, so Birth Rate – Death rate < 0. Therefore, $\frac{\mathrm{d}P}{\mathrm{d}t} = \text{Birth rate} - \text{Death rate}$ $\frac{\mathrm{d}P}{\mathrm{d}t} = -0.03P.$
10(ii)	$\frac{\mathrm{d}P}{\mathrm{d}t} = -0.03P$ $\frac{1}{P}\frac{\mathrm{d}P}{\mathrm{d}t} = -0.03$ $\int \frac{1}{P}\,\mathrm{d}P = \int -0.03\,\mathrm{d}t$ $\ln\lvert P\rvert = -0.03t + c$ $\ln P = -0.03t + c$ ($P \geq 0$ as it is the number of sheep.) $P = \mathrm{e}^{-0.03t+c} = A\mathrm{e}^{-0.03t}$, where $A = \mathrm{e}^{c}$. As $t \to \infty, \mathrm{e}^{-0.03t} \to 0 \Rightarrow P \to A\mathrm{e}^{-0.03t} = 0$. This means the population of sheep decreases to 0 after many years.
10(iii)	$\frac{\mathrm{d}P}{\mathrm{d}t} = -0.03P + \text{imported sheep per year}$ $\frac{\mathrm{d}P}{\mathrm{d}t} = -0.03P + n$ $\frac{\mathrm{d}P}{\mathrm{d}t} = n - 0.03P.$
10(iv)	$\frac{\mathrm{d}P}{\mathrm{d}t} = n - 0.03P$ $\frac{1}{n-0.03P}\frac{\mathrm{d}P}{\mathrm{d}t} = 1$ $\int \frac{1}{n-0.03P}\,\mathrm{d}P = \int 1\,\mathrm{d}t$ $-\frac{1}{0.03}\ln\lvert n-0.03P\rvert = t + c_0$ $\ln\lvert n-0.03P\rvert = -0.03t + c_1,$ where $c_1 = -0.03c_0$ $n - 0.03P = \pm\mathrm{e}^{-0.03t+c_1} = B\mathrm{e}^{-0.03t},$ where $B = \pm\mathrm{e}^{c_1}$ $P = \frac{1}{0.03}\left(n - B\mathrm{e}^{-0.03t}\right)$ $P = \frac{100}{3}\left(n - B\mathrm{e}^{-0.03t}\right)$ or $P = \frac{100}{3}\left(n + B_0\mathrm{e}^{-0.03t}\right)$, where $B_0 = -B$.

10(v)	Method 1: As $t \to \infty, \mathrm{e}^{-0.03t} \to 0 \Rightarrow P \to \frac{100}{3}\left(n - B\mathrm{e}^{-0.03t}\right) = \frac{100n}{3}$. This implies $\frac{100n}{3} = 500 \Rightarrow n = 15.$ Method 2: When it settles down, the rate of change is 0 as the value becomes constant. Hence, $\frac{\mathrm{d}P}{\mathrm{d}t} = 0$ when $P = 500$. $\frac{\mathrm{d}P}{\mathrm{d}t} = n - 0.03P$ $0 = n - 0.03(500)$ $n = 15.$

11(i)	We see that $\theta = \angle DKA - \angle CKA$. And, $\tan\angle DKA = \dfrac{a+4}{x}, \tan\angle CKA = \dfrac{a}{x}$. $\tan\theta = \tan\left(\angle DKA - \angle CKA\right)$ $= \dfrac{\tan\angle DKA - \tan\angle CKA}{1+\tan\angle DKA \tan\angle CKA}$ $= \dfrac{\frac{a+4}{x} - \frac{a}{x}}{1+\left(\frac{a+4}{x}\right)\left(\frac{a}{x}\right)}$ $= \dfrac{\frac{4}{x}}{\frac{x^2+a^2+4a}{x^2}}$ $= \dfrac{4x}{x^2+4a+a^2}$. (Shown)
11(aii)	Let $y = \tan\theta$. $y = \dfrac{4x}{x^2+4a+a^2}$ $\dfrac{dy}{dx} = \dfrac{\left(x^2+4a+a^2\right)(4)-(4x)(2x)}{\left(x^2+4a+a^2\right)^2}$ $\dfrac{dy}{dx} = \dfrac{4\left(4a+a^2-x^2\right)}{\left(x^2+4a+a^2\right)^2}$ As maximum $\tan\theta \Rightarrow \dfrac{dy}{dx} = 0$. $0 = \dfrac{4\left(4a+a^2-x^2\right)}{\left(x^2+4a+a^2\right)^2}$ $4\left(a+a^2-x^2\right)=0$ $x^2 = a^2+4a$ $x = \sqrt{a^2+4a}$ (reject $-$ve as $x \geq 0$) When $x = \sqrt{a^2+4a}$ (also, $x^2 = a^2+4a$)

	$\tan\theta = \frac{4x}{x^2+4a+a^2}$ $= \frac{4x}{x^2+x^2}$ $= \frac{2}{x}$ $= \frac{2}{\sqrt{a^2+4a}}$ Therefore, maximum $\tan\theta = \frac{2}{\sqrt{a^2+4a}}$ with $x=\sqrt{a^2+4a}$.
11(iii)	The distance from the optimal point to the interval between *CD* may be too far for the player to kick. Hence the player want want to move closer to *CD* on the line *AB*. This could happen if the player runs past the scoring line at a distance that is quite far from the interval *CD*.
11(iv)	$\tan\angle KDA = \frac{x}{a+4}$ At optimal point, $x=\sqrt{a^2+4a}$ $\tan\angle KDA = \frac{\sqrt{a^2+4a}}{a+4}$ $= \frac{\sqrt{a}\sqrt{a+4}}{a+4}$ $= \sqrt{\frac{a}{a+4}}$ (Shown) If a is much larger than 4, then $\frac{4}{a}\approx 0$, so $\tan\angle KDA = \sqrt{\frac{a}{a+4}} = \sqrt{\frac{1}{1+\frac{4}{a}}} \approx \sqrt{\frac{1}{1+0}} = 1$ $\tan\angle KDA \approx 1$ $\angle KCD \approx \tan^{-1}1$ $\angle KCD \approx \frac{\pi}{4}$.
11(v)	We note that $0 \le a \le \frac{50-4}{2} = 23$ We note that $\tan^{-1}\left(\frac{2}{\sqrt{a^2+4a}}\right)$ is a decreasing function. As $a\to 0$, θ is maximum and as $a=23$, θ is minimum.

	Moreover, $\tan^{-1}\left(\frac{2}{\sqrt{23^2+4(23)}}\right)=0.0800856\text{ (6 s.f.)}$. And $\tan^{-1}\left(\frac{2}{\sqrt{a^2+4a}}\right)\to\frac{\pi}{2}$ as $a\to 0$. Therefore we must have, $0.0801\text{ (3 s.f.)}\le\theta<\frac{\pi}{2}$.