Mock Preliminary Examination 2024

NAME: ____

CLASS: __________ REGISTER NUMBER: (   )

ADDITIONAL MATHEMATICS **4047/01**

Paper 1 **20 August 2024**

Secondary 4 Express **2 hours**

Additional Materials: Nil

READ THESE INSTRUCTIONS FIRST

Write your name, registration number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers in the space provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question. The total of the marks for this paper is 80.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n \ ,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \tfrac{1}{2} bc \sin A$$

1 Find the range of values of the constant p for which the line $y = px - 1$ intersects the curve $y = 2x^2 - 5x + 1$ at two points. [4]

2 A trapezoidal prism has a base area of $(5+\sqrt{5})$ cm^2. The volume of the prism is $(20+9\sqrt{5})$ cm^3. Find the height of the prism in the form $(a+b\sqrt{5})$ cm, where a and b are integers. [3]

3 Express $\frac{x^3+4x^2+7}{x^2(x-1)}$ in partial fractions. [5]

4 The polynomial $f(x) = 2x^3 + ax^2 + bx - 6$, where a and b are constants, has two factors $(x + 1)$ and $(x - 2)$.

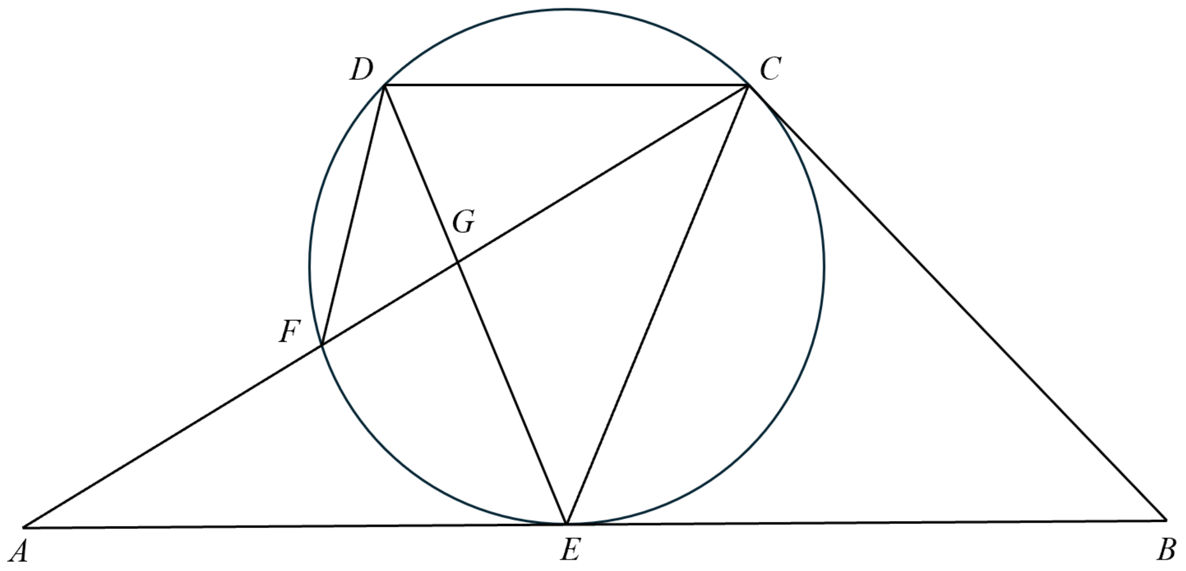
(i) Find the values of a and b. [4]

(ii) Using the values of a and b found in part **(i)**, explain why the equation f(x) has two distinct roots. Find the two roots. [4]

(iii) Hence, solve $2y^3 + ay^2 + by - 6 = 0$. [2]

5 Solve the equation $2\ln (x - 3) = \ln (x + 5) + \ln 4$. [5]

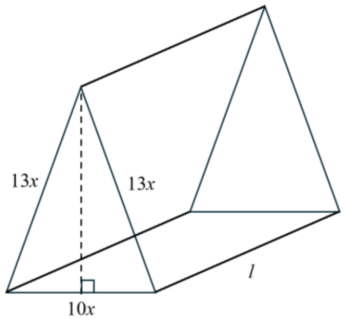
6 In the diagram, *AB* and *CB* are tangents to the circle at point *E* and *C* respectively. The two tangents both meet at point *B*. Points *C*, *D*, *F* and *E* lie on the circle. *FGC* and *DGE* are straight lines that intercept at point *G*. *ED* = *EC*.



(i) Prove that triangle *FDG* and triangle *CEG* are similar. [4]

(ii) What type of quadrilateral $DCBE$? Give reasons to support your answer. [5]

7 A triangular prism with a base area of an isosceles triangle whose sides are $13x$ cm, $13x$ cm and $10x$ cm. The height of the prism is l cm.



(i) Given that the volume of the 300 cm^3, show that $l = \frac{5}{x^2}$. [2]

(ii) Given that the total surface area is A cm^2, show that $A = \frac{310}{x} + 120x^2$. [2]

(iii) Find the value of x for which A has a stationary value, giving your answer to 2 decimal places. [4]

(iv) Find this stationary value of A, giving your answer to 3 significant figures. [1]

8 **(a)** Prove the identity $\frac{\cos\theta}{1-\sin\theta}+\frac{1-\sin\theta}{\cos\theta}=2\sec\theta$. [4]

(b) Hence, solve $\frac{\cos\theta}{1-\sin\theta}+\frac{1-\sin\theta}{\cos\theta}= -3$ for $0^{\circ} \leq \theta \leq 180^{\circ}$. [4]

9 It is given that $y = 2e^{2x+1} + 4x$.

Show that $y\left(\frac{d^2y}{dx^2} + \frac{dy}{dx}\right) = ae^{4x+2} + bxe^{2x+1}$, where a and b are integers.
Find the values of a and b. [7]

10 The equation of a circle is $(x-3)^2+(y+4)^2=25$.

(a) Find the centre and radius of the circle. [4]

(b) The circle intercepts the x-axis at point P and the y-axis at point Q.

Find the coordinates of P and of Q. [4]

(c) A line is tangent to the circle at point P.

Find the equation of the tangent. [4]

11 **(a)** In the binomial expansion of $\left(2x - \frac{3}{x}\right)^n$, where n is a positive integer, the coefficient of the third term is $\frac{270}{8}(2^n)$. Show that $n = 6$. [4]

(b) Hence, find the coefficient of x^4 in the expression of $(1 + x^2)\left(2x - \frac{3}{x}\right)^n$. [4]

Mock Preliminary Examination 2024

NAME: ____

CLASS: __________ REGISTER NUMBER: (   )

ADDITIONAL MATHEMATICS **4047/02**

Paper 2 **20 August 2024**

Secondary 4 Express **2 hours 30 minutes**

Additional Materials: Nil

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Mathematical Formulae

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where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)...(n-r+1)}{r!}$

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1 In the binomial expansion of $\left(x + \frac{k}{x}\right)^7$, where k is a positive constant, the coefficients of x and x^3 are the same. Find the value of k. [6]

2 Recorded values of the mass, m grams, of a radioactive substance, t hours after observations began, are shown in the table below.

t (hours)	2	4	6	8	10
m (mg)	48.2	41.5	35.7	30.8	26.5

It is known that m and t are related by the equation $m = m_0 e^{-kt}$, where m_0 and k are constants.

(i) Plot ln m against t and draw a straight line graph. [3]

Use your graph to estimate

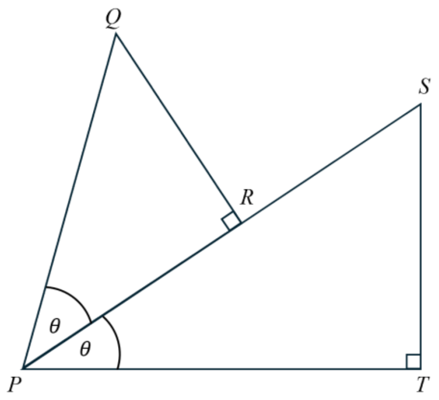
(ii) mass of the substance when the observation began, [2]

(iii) the value of k, [2]

(iv) the time taken for the substance to lose half of its original mass. [2]

Answer for **2 (i)**

3 The diagram shows a plot of land formed by two right-angle triangles *PQR* and *PST*. *PRS* is a straight line. Angle *PRQ* = angle *PTS* = 90°.
Angle *QPR* = angle *SPT* = θ, *PQ* = 10 m, *PS* = 15 m.

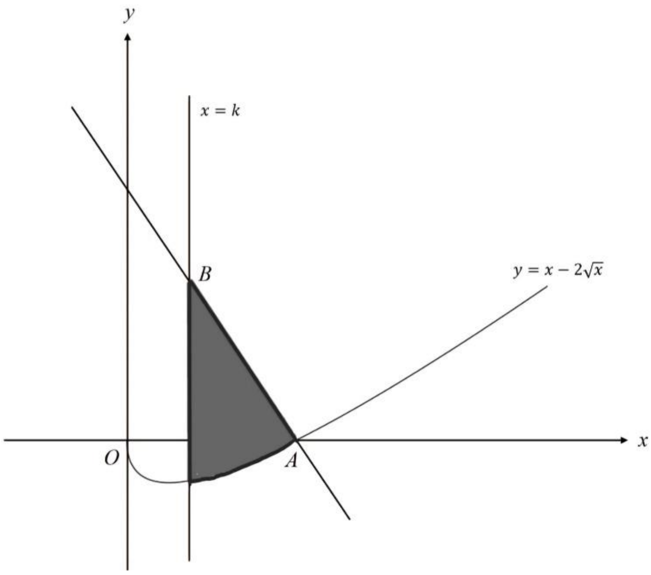


(i) Show that *P* m, the perimeter of the land, can be expressed in the form $25 + 25\sin\theta + 5\cos\theta$. [3]

(ii) Express *P* in the form $25 + R\sin(\theta + \alpha)$ where $R > 0$ and $0^\circ < \alpha < 90^\circ$. [3]

(iii) Find the value of θ when $P = 45$ m. [3]

4 The diagram below shows a curve $y = x - 2\sqrt{x}$. The curve passes through O and A. The line AB, the normal to the curve at point A, cuts the line $x = k$ at point B.



(i) Find the equation of the line AB. [4]

(ii) Given that $\frac{dy}{dx} = 0$ where $x > 0$, find the coordinates of B. [3]

(iii) Calculate the shaded area bounded by the line $x = k$, the normal line AB and the curve $y = x - 2\sqrt{x}$. [3]

5 Tea is poured into an empty cup. The temperature, T_c°C, of the tea in the cup, t minutes after it is poured, is modelled by the formula $T_c = 86e^{-0.06t}$.

(a) State the initial temperature of the tea. [1]

(b) Find the time taken for the temperature of the tea to drop to 37°C. [3]

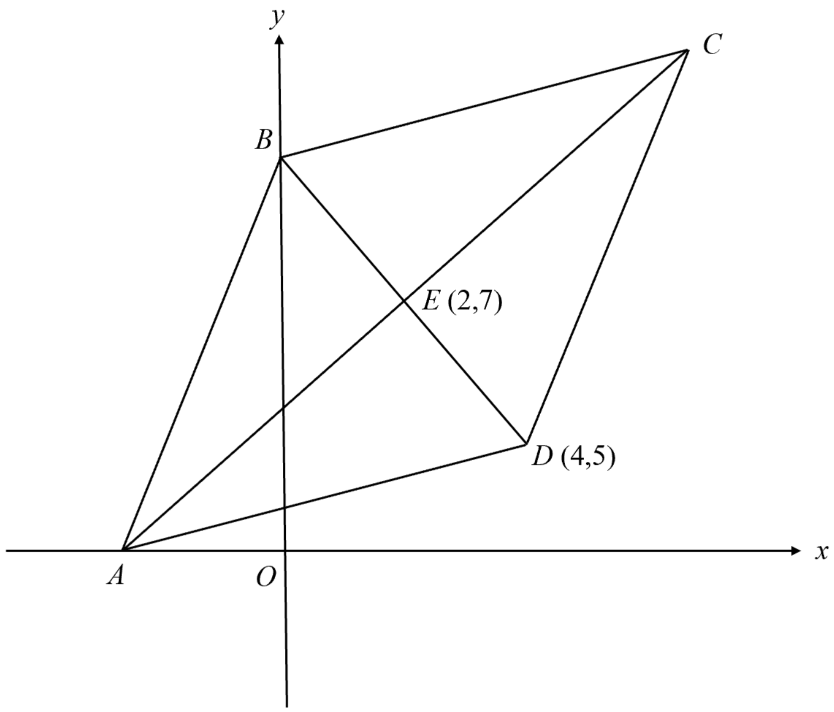
(c) Some tea is poured into an empty cup and at the same time the same volume of tea is poured into an empty flask. The temperature, T_f°C, of the tea in the flask at time t minutes after it is poured into the flask is modelled by $T_c = 86e^{-\lambda t}$ where λ is a constant. The formula for T_c still applies.

(i) After one hour the temperature of the tea in the flask is 82°C. Find λ. [2]

(ii) Using your answer from part **(c)(i)** find the time when the temperature of the tea in the cup is half the temperature of the tea in the flask. [3]

6 Solve the equation $6(4^y) + 2(2^{-y}) = 7(2^y) + 1$. [5]

7 The diagram shows a rhombus *ABCD* with points *A*, *B*, *C* and *D* (4,5). Point *A* lies on the *x*-axis and point *B* lies on the *y*-axis. Point *E* (2,7) lies on the interception of *AC* and *BD* such that *AC* is perpendicular to *BD*.



(a) Find the equation of *AC*. [4]

(b) Given that $BC = DC$ and the product of the x-coordinate and y-coordinate of point C is 84, find the coordinates of C. [5]

(c) Explain why the diagram is necessary. [1]

(d) Find the coordinates of A. [2]

(e) Find the area of rhombus $ABCD$. [2]

8 A ball is thrown upwards so that its height, y metres, above the ground after *t* seconds, is modelled by the equation $y = -2t^2 + 2t + 20$.

(a) Using **discriminant**, show that it is not possible for the ball to reach at a height of 39 metres. [2]

(b) Express $y = -2t^2 + 2t + 20$ in the form $y = -2(t - a)^2 + b$, where *a* and *b* are constants. Hence, sketch the graph $y = -2t^2 + 2t + 20$, indicating clearly the turning point and the *y*-intercept. [4]

(c) Find the value of the positive horizontal intercept and explain what the value represents. [2]

(d) State the maximum height the ball can reach. [1]

9 The function f is defined by $\mathrm{f}(x) = 4\cos 2x - 1$, for $0 \leq x \leq \pi$.

(a) State the amplitude and period of f(x). [2]

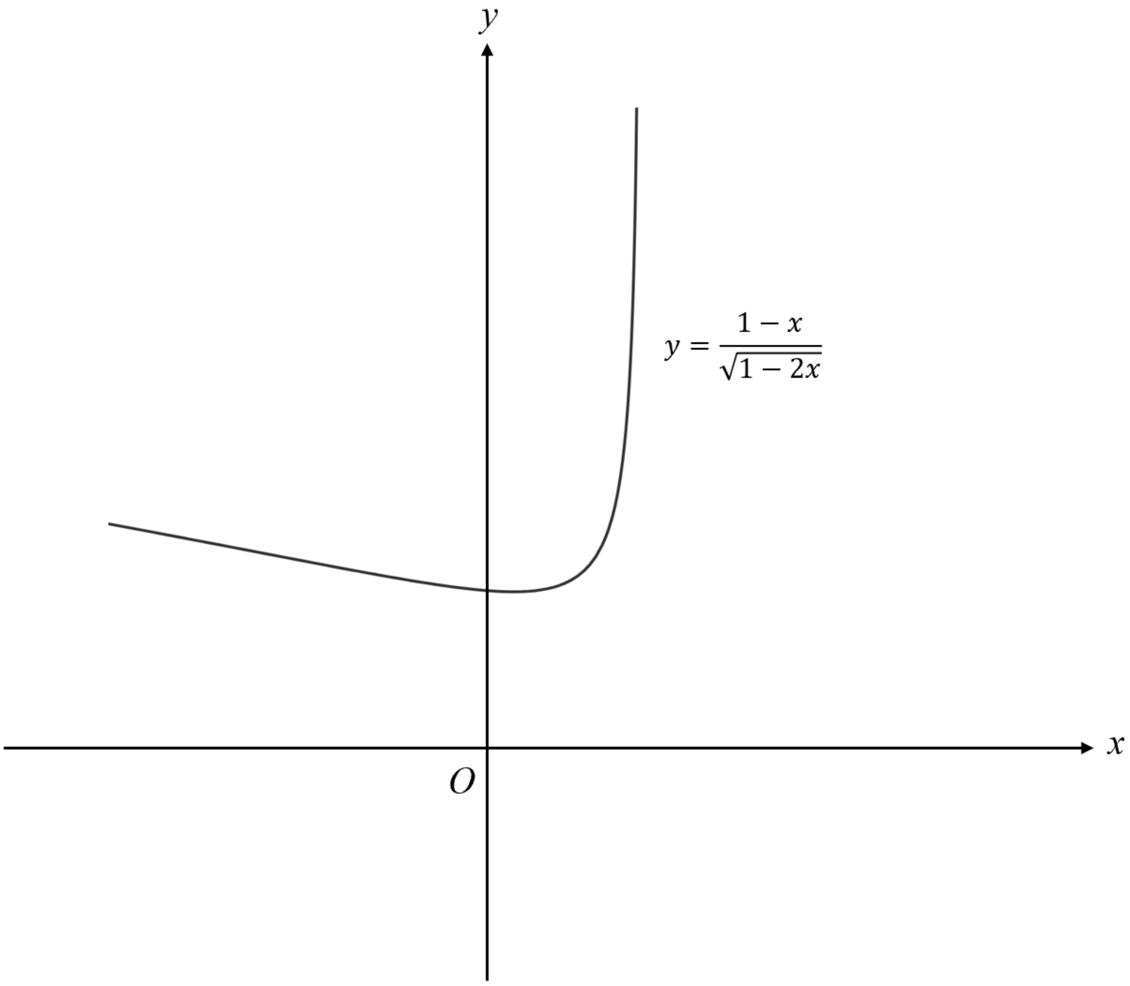
(b) Sketch the graph of y = f(x). [3]

(c) State the value(s) of k such that $4\cos 2x - 1 = k$ has exactly 1 solution. [1]

(d) On the same diagram, sketch the graph of $y = \frac{2x}{\pi} + 1$.

State the number of solutions which satisfy the equation $4\cos 2x = \frac{2x}{\pi} + 1$. [3]

10 The diagram shows a part of the graph $y = \frac{1-x}{\sqrt{1-2x}}$.



(i) Find $\frac{dy}{dx}$. [2]

(ii) Given that x is increasing at a constant rate of 0.08 units/s, find the rate of change of y when $x = -4$, stating if it is an increase or a decrease. [3]

11 Without using a calculator, prove that $\frac{\sin 75^{\circ}+\cos 15^{\circ}}{\cos 135^{\circ}-\sin 105^{\circ}}=a\sqrt{3}$, where a is a negative constant. [12]