Tutorial 2B: Sequences and Series (Part II) – AP & GP, Σ Notation, MOD

Basic Mastery Questions

1. Find the sum of all the even numbers from 20 to 100 inclusive.

$$20 + (n-1)(2) = 100$$

$$n = 41$$

$$\therefore S_{41} = \frac{41}{2}[20 + 100]$$

$$= 2460$$

2. Find the number of terms that must be taken for the sum of the arithmetic series $12 + 16 + 20 + \ldots$ to be equal to 672.

$$S_n = 672$$

$$\frac{n}{2}[2(12) + (n-1)(4)] = 672$$

$$n(24 + 4n - 4) = 1344$$

$$n^2 + 5n - 336 = 0$$

$$n = \frac{-5 \pm \sqrt{25 - 4(1)(-336)}}{2}$$

$$= \frac{-5 \pm 37}{2} = 16 \text{ or } -21 \text{ (N.A)}$$

$$\therefore n = 16$$

3. The rth term of an arithmetic progression is $(1+4r)$. Find, in terms of n, the sum of the first n terms of the progression.

$$T_r = 1 + 4r$$

$$\Rightarrow T_1 = 5, \quad T_2 = 9 \Rightarrow d = 4$$

$$S_n = \frac{n}{2}[5 + 1 + 4n]$$

$$= n(3 + 2n)$$

4. Find the number of terms that must be taken for the sum of the geometric series $48+24+12+\ldots$ to be equal to $95\frac{1}{4}$.

$r = \frac{24}{48} = \frac{1}{2}$

$S_n = 95\frac{1}{4}$

$48\left[1-\left(\frac{1}{2}\right)^n\right] = \left(95\frac{1}{4}\right)\left(\frac{1}{2}\right)$

$\left(\frac{1}{2}\right)^n = \frac{1}{128}$

$= \left(\frac{1}{2}\right)^7$

$n = 7$

5. [**N1987/I/11**] Find $\sum_{r=0}^{n}(2n+1-2r)$ in terms of n.

$$\begin{aligned}\sum_{r=0}^{n}(2n+1-2r) &= \sum_{r=0}^{n}(2n+1)-2\sum_{r=0}^{n}r\\ &=(2n+1)(n+1)-2\frac{n(n+1)}{2}\\ &=(2n+1)(n+1)-n(n+1)\\ &=(n+1)(2n+1-n)\\ &=(n+1)^2\end{aligned}$$

6. The nth term of a series is $2^{n-2}+3n$. Find the sum of the first N terms.

$T_n = 2^{n-2} + 3n$

$\sum_{r=1}^{N}\left(2^{r-2} + 3r\right) = \frac{1}{4}(2)(2^N - 1) + 3\left(\frac{N}{2}\right)(N+1)$

$= \frac{1}{2}\left(2^N - 1 + 3N(N+1)\right)$

7. Express $\dfrac{1}{(2n-1)(2n+1)}$ in partial fractions and use the method of differences to find $\displaystyle\sum_{n=1}^{N}\frac{1}{(2n-1)(2n+1)}$, giving your answer in terms of N.

$$\frac{1}{(2n-1)(2n+1)} = \frac{A}{2n-1} + \frac{B}{2n+1}$$

using 'cover-up' rule:

$$A = \frac{1}{(2(\frac{1}{2})+1)} = \frac{1}{2}$$

$$B = \frac{1}{2(-\frac{1}{2})-1} = -\frac{1}{2}$$

$$\therefore \frac{1}{(2n-1)(2n+1)} = \frac{1}{2(2n-1)} - \frac{1}{2(2n+1)}$$

$$= \frac{1}{2}\left[\frac{1}{2n-1} - \frac{1}{2n+1}\right]$$

$$\therefore \sum_{n=1}^{N}\frac{1}{(2n-1)(2n+1)} = \frac{1}{2}\sum_{n=1}^{N}\left[\frac{1}{2n-1} - \frac{1}{2n+1}\right]$$

$$= \frac{1}{2}\Big[\ 1 - \frac{1}{3}$$
$$+ \frac{1}{3} - \frac{1}{5}$$
$$+ \ldots$$
$$+ \frac{1}{2N-3} - \frac{1}{2N-1}$$
$$+ \frac{1}{2N-1} - \frac{1}{2N+1}\ \Big]$$

$$= \frac{1}{2}\left[1 - \frac{1}{2N+1}\right]$$