ST ANDREW'S JUNIOR COLLEGE

PRELIMINARY EXAMINATION

MATHEMATICS
Higher 2 **9740/2**

Monday **15 SEP 2015** **3 hours**

READ THESE INSTRUCTIONS FIRST

Write your name, civics group and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.

Answer **all** the questions. Total marks is 100.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.
Unsupported answers from a graphic calculator are allowed unless a question specifically state otherwise.
Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematic steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

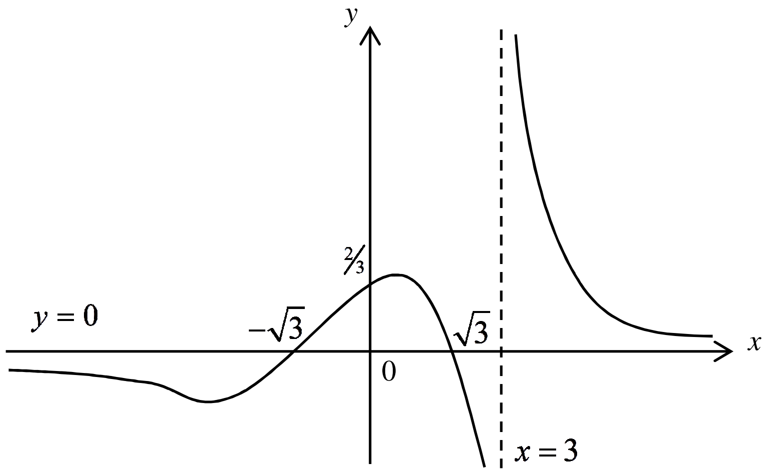
The number of marks is given in brackets [ ] at the end of each question or part question.
At the end of the examination, fasten all your work securely together.

This document consists of 6 printed pages including this page.

Pure Mathematics (40 marks)

1 It is given that $\mathrm{f}(x)=\dfrac{6(x^2-3)}{(x-3)(x^2+9)}$.

(i) Using partial fractions, find $\int \mathrm{f}(x)\ \mathrm{d}x$. **[5]**



The diagram above shows the curve with equation $y=\mathrm{f}(x)$.

(ii) Find the exact area bounded by the curve $y=\mathrm{f}(-x)$ and the x-axis, between the values of $x=0$ and $x=\sqrt{3}$. **[3]**

2 **(a)** Adrian has signed up at a driving centre to learn how to drive. His first lesson is 40 minutes long. Each subsequent lesson is 5 minutes longer than the previous lesson, so that the second lesson is 45 minutes long, the third lesson is 50 minutes long, and so on. The centre requires a student to have attended at least 60 hours of lessons before he is qualified to take the driving test. Find the minimum number of lessons that Adrian has to attend before he can take the test. **[4]**

(b) A sequence of real numbers $u_1, u_2, u_3, \ldots$, where $u_1 \neq 0$, is defined such that the $(n+1)$th term of the sequence is equal to the sum of the first n terms, where $n \in \mathbb{Z}^+$. Prove that the sequence $u_2, u_3, u_4, \ldots$ follows a geometric progression. **[3]**

Hence find $\sum_{r=1}^{N+1} u_r$ in terms of u_1 and N. **[2]**

3 The equations of lines l_1 and l_2 are given as follows:

$$l_1 : \mathbf{r} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix}, \ \lambda \in \mathbb{R}, \text{ and } l_2 : \ x = 3, \ \frac{3-y}{5} = \frac{z-6}{-1}.$$

(i) Find the coordinates of N, the foot of perpendicular of the point $A(4, 5, 3)$ onto l_1. **[3]**

(ii) The plane Π_1 contains the line l_1 and is parallel to l_2. Find the Cartesian equation of the plane Π_1. **[3]**

Another plane Π_2 is defined by $\mathbf{r} \bullet \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix} = 4$ and intersects Π_1 in a line l_3.

(iii) Find a vector equation of l_3. **[2]**

A plane Π_3 has an equation $tx - 2y + 6z = d$.

(iv) What can be said about t and d if the three planes Π_1, Π_2, Π_3 have exactly one point in common? **[2]**

4 **(a)** Solve the equation $w^4 + 1 - \sqrt{3}i = 0$, expressing the roots in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \le \pi$. Show the roots on an Argand diagram, showing the relationship between them clearly. **[5]**

(b) A complex number $z = x + \mathrm{i}y$ has modulus r and argument θ, where $0 < \theta < \dfrac{\pi}{2}$.

The complex numbers v and w are defined by $v = -y + x\mathrm{i}$ and $w = x^2 - y^2 + 2xy\mathrm{i}$.

(i) Express v and w in terms of z. **[2]**

(ii) Hence, or otherwise, express vw in exponential form in terms of r and θ. **[2]**

(iii) If $vw = -4 - 4\sqrt{3}\mathrm{i}$, solve for z in exponential form, giving your answer in exact form. **[4]**

Statistics (60 marks)

5 A school has a total of 1000 students. A company which sells stationery products in a bookstore inside the school, intends to select a sample of 50 students within the school to perform a survey with regards to its products, representative of the opinions of both male and female students.

(i) In the context of the question, describe how quota sampling could be carried out to select the 50 students, and explain a disadvantage of quota sampling. **[2]**

(ii) State the name of an appropriate sampling method that does not have this disadvantage and describe how it can be carried out. **[2]**

(iii) The stationery product also has retail outlets outside the school catering to the general public. Explain why it is not realistic for the company to carry out the sampling method in part (ii) to obtain a representative sample of 50 customers at their retail outlets at any given month. **[1]**

6 For events A and B, it is given that $P(B)=\frac{59}{100}, P(B'|A)=\frac{1}{3}, P\left(A|B'\right)=\frac{17}{41}$. Find

(i) $P(A)$, **[3]**

(ii) $P(A\cap B)$. **[2]**

7 Studies have shown that 74% of patients who suffer from an allergy are relieved of its symptoms after taking a new drug.

(i) A hospital tested the new drug on 15 randomly chosen patients with the allergy. Find the probability that at least half of the patients are relieved of the symptoms.

[2]

(ii) Another n patients were added to the group of 15 patients to form a new bigger group of patients. It is given that the probability of at least 2 patients from this new group not being relieved of the symptoms is at least 0.99. Express this information as an inequality in n, and hence find the least value of n. **[4]**

The drug was reformulated and the success rate of the improved drug was found to be 92%.

(iii) The hospital decides to test the reformulated drug on another group of patients. Using a suitable approximation, find the probability that, out of 36 patients, more than 30 patients are relieved of the symptoms. **[3]**

8 The table shows the number y (in millions) of cell-phone subscribers in a country from 2001 to 2010, where t represents number of years from 2000.

t	1	2	3	4	5	6	7	8	9	10
y	1.6	2.7	4.4	6.4	8.9	13.1	19.3	28.2	38.2	48.7

The relationship between y and t is given by the formula $y = ab^t$, where a and b are constants.

(i) Using the substitution $I = \ln y$, show that the relation between I and t is linear. **[1]**

(ii) Find the equation of the estimated regression line of I on t and hence give estimates for a and b. **[2]**

(iii) Find the (product moment) correlation coefficient between I and t. **[1]**

(iv) Predict the number of cell-phone subscribers in the year 2015. Comment on the reliability of your prediction. **[3]**

(v) It is required to estimate the value of t for which I = 1.5. Explain which of the regression lines I on t or t on I, should be used. Use the equation of your choice to find the value of t when I = 1.5. **[3]**

9 Farmer Chan found that the mean mass of his previous crop of tomatoes was μ_0 grams. He decides to try a new type of fertiliser for the present crop of tomatoes. The manufacturers of the new fertiliser claim that it will increase the mean mass of tomatoes. The farmer intends to test their claim by taking a random sample of size 50 from the present crop. A random sample of 50 tomatoes gives the following data

$$\sum x = 3500, \quad \sum x^2 = 245220.5$$

where X is the random variable representing the mass of tomatoes in grams, after application of the new fertiliser.

(i) Calculate the unbiased estimates of the population mean and variance. **[2]**

(ii) If $\mu_0 = 69.4$, test at 5% level of significance, whether the fertiliser manufacturer's claim is valid. Explain what you understand by the phrase "at 5% level of significance" in the context of this question. **[5]**

(iii) The null hypothesis H_0: $\mu = \mu_0$ is tested against H_1: $\mu > \mu_0$, where μ is the population mean of X. Given that the null hypothesis is not rejected at 3% level of significance, find the set of values that μ_0 can take, giving your answer correct to one decimal place. **[3]**

[Turn Over

10 A code consists of a letter followed by three numbers, and then, followed by another letter. The code is generated by a computer and thus, each of the two letters generated is equally likely to be any of the twenty-six letters of the alphabet A-Z. Each of the three digits generated is equally likely to be any of the ten digits 0-9.

(i) Find the probability that a randomly chosen code has three different digits and two different letters. **[2]**

A palindrome code is defined as a code that reads the same backward and forward. Examples of a palindrome code are A121A, B343B, G111G.

(ii) Find the probability that a randomly chosen code is a palindrome code. **[2]**

(iii) Find the probability that a palindrome code contains both the digits 2 and 3. **[3]**

(iv) Showing all necessary calculations, determine if the events of a code being palindrome and a code containing the digits 2 and 3 are independent. **[3]**

11 The number of people joining a queue at a checkout counter in a shop in a period of 10 minutes is a random variable that follows a Poisson distribution of mean 2.7.

(i) Find the probability that in a period of 15 minutes, there are more than 5 people joining the queue at the checkout counter. **[2]**

(ii) The manager tracked the number of people joining the queue at the checkout counter from 0900 to 0915 daily for 14 consecutive days. If the probability that there are at least k days with more than 5 people joining the queue at the checkout counter is less than 0.02, find the least value of k. **[3]**

(iii) The number of people leaving the same queue at the checkout counter in a period of 10 minutes is a random variable that follows a Poisson distribution of mean 3.1. **[5]**

At 0900 hours on the Monday morning, it was observed that there are 18 people in the queue at the checkout counter. Use suitable approximations to find the probability that by 0940 hours, there are at most 10 people in the queue at the checkout counter.

(You may assume that the queue does not become empty during this period.)

(iv) Explain why Poisson model may not be appropriate if applied to a time period of several hours. **[1]**

End of Paper