Class	Index Number	Name

ANG MO KIO SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2019
SECONDARY FOUR EXPRESS / FIVE NORMAL ACADEMIC

ADDITIONAL MATHEMATICS **4047/02**
Paper 2

Wednesday **18 September 2019** **2 hours 30 minutes**

Answer on the question paper.
No additional materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class in the spaces at the top of this page.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is **100**.

For Examiner's Use
100

This document consists of **22** printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Delta = \frac{1}{2}ab\sin C$$

1 On 1st January 2010, a certain type of bacteria was found at the bottom of a seabed. It is known to grow with time, such that its population P, after t years is given by

$$P = 50\ 000\mathrm{e}^{kt}\text{, where } k \text{ is a constant.}$$

(i) Given that the population doubles in two years, show that $k = \frac{1}{2}\ln 2$. [2]

(ii) Find the size of the population of bacteria on 1st January 2015, giving your answer correct to the nearest 10 000. [2]

(iii) Find the year in which the population reaches 450 000. [3]

2 **(a)** Given that $y = e^x \sin x$, prove that $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + 2y = 0$. [4]

(b) A particle moves along the curve $y = \frac{2x+16}{x-1}$ in such a way that both its x-coordinate and y-coordinate are positive and vary with time. Find the x-coordinate of the particle at the instant that the rate of increase of its y-coordinate is twice the rate of decrease of its x-coordinate. [5]

3 **(a)** Solve $\lg x^2 - 2\log_x 10 = 3$. [5]

(b) Given that $\log_m\left(xy^3\right) = a$ and $\log_m\left(x^2\right) = b$, express $\log_m\left(\dfrac{x}{y}\right)$ in terms of a and b. [5]

(c) **Without using a calculator**, solve for *a* and *b*, the simultaneous equations

$$8 \times 4^a = 2^{2b-1},$$

$$3^a \sqrt{3^b} = 81.$$ [4]

4 **(a)** The equation of a curve is $y = \frac{5}{kx^2} + 10x^3$, where k is a constant. Given that its normal at $x = 2$ will never meet the y-axis, find the value of k. [3]

(b) The equation of a curve is $y = \ln\sqrt{3x+2}$.

(i) Determine the range of values of x for which the equation of the curve is defined. [1]

(ii) Find an expression for $\frac{dy}{dx}$, giving your answer in the simplest form. [2]

(iii) Find the equation of the normal to the curve at the point where the curve cuts the x-axis. [5]

5 A curve has the equation $y = 2x + \dfrac{4}{2x-1}$.

The point $A(p, q)$ is a stationary point on the curve where $p > 0$ and $q > 0$.

(i) Determine the value of p and of q. [5]

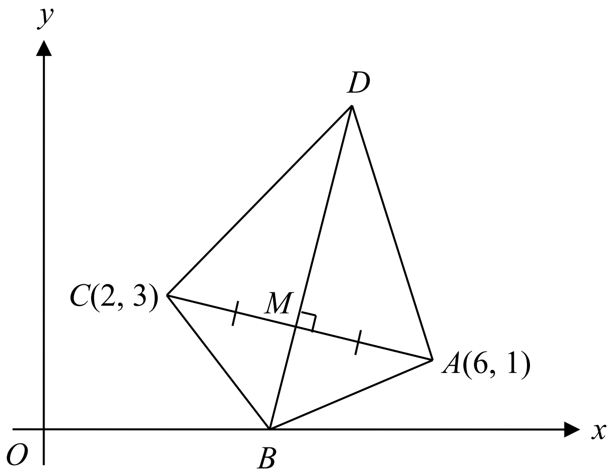
(ii) Find the coordinates of the other stationary point B on the curve and determine the nature of this stationary point B. [4]

6 **(i)** Write down the first three terms in the expansion, in ascending powers of x, of $(1-px)^6$, where p is a constant. [2]

(ii) Find, in terms of p, the coefficient of x^2 in the expansion of $(1-2x)^2(1-px)^6$. [2]

(iii) Given the coefficient of x^2 in the expansion of $(1-2x)^2(1-px)^6$ is twice the coefficient of x, find the values of p. [4]

7 The diagram below shows a quadrilateral $ABCD$ where A is (6, 1), B is on the x-axis. and C is (2, 3). The diagonal BD bisects AC at right angles at M such that $2BD = 5BM$.



(i) Find the equation of BD. [4]

(ii) Find the x-coordinate of B. [1]

(iii) Find the coordinates of D. [3]

(iv) Find the area of quadrilateral $ABCD$. [2]

8 The curve $y = a\cos bx + c$, where a is positive and $0 \le x \le 4\pi$ has an amplitude of 3 and period of 4π. The minimum value of y is -2.

(i) State the value of a, of b and of c. [3]

(ii) On the same axes, sketch the graphs $y = a\cos bx + c$ and $y = -|\sin x|$ for $0 \le x \le 4\pi$. [3]

(iii) Hence deduce the value of k, where k is an integer, for which the equation $a\cos bx + c = -|\sin x| + k$ has four solutions. [1]

9 **(i)** Differentiate $x\cos 2x$ with respect to x. [3]

(ii) Hence, use your result to evaluate $\int_0^{\frac{\pi}{2}}\left(3-2x\sin 2x\right)dx$. [5]

10 **(a)** **(i)** Determine the set of values of p for which the equation $p(x-2)=x^2-3$ has real and distinct roots. [4]

(ii) Hence state what can be deduced about the curve $y=x^2-3$ and the line $y=2x-4$. [2]

(b) Show that the roots of the equation $x^2+(k-2)x-2k=0$ are real for all values of k. [3]

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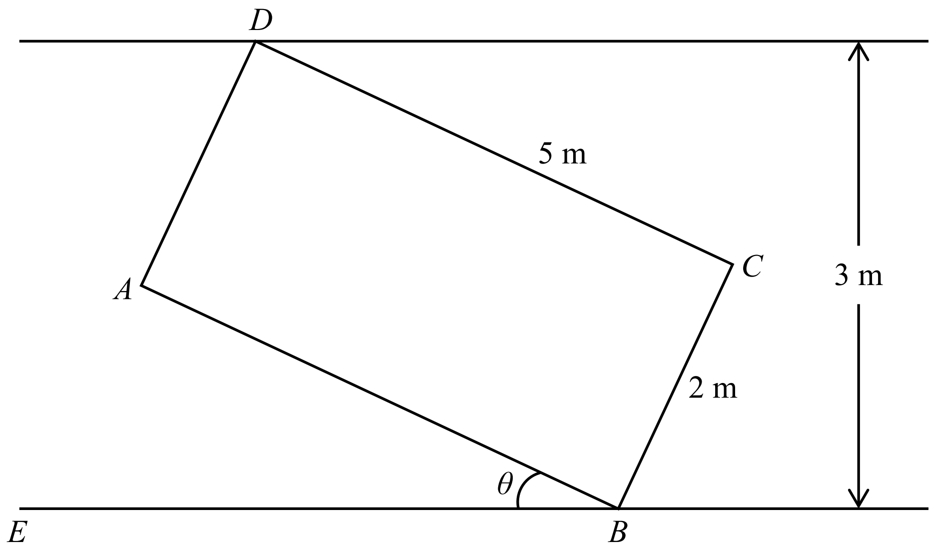

The diagram shows a rectangular table *ABCD* measuring 5 m by 2 m placed in a corridor 3 m wide such that *B* and *D* are in contact with the walls and $\angle ABE = \theta$, where $0° < \theta < 90°$.

(i) Show that $2\cos\theta + 5\sin\theta = 3$. [2]

(ii) Express $2\cos\theta + 5\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and α is acute. [4]

(iii) Hence find the value of θ. [2]

END OF PAPER