Chapter 16 ELECTROMAGNETISM

Content

- Concept of a magnetic field
- Magnetic fields due to currents
- Force on a current-carrying conductor
- Force between current-carrying conductors
- Force on a moving charge

Learning Outcomes

Candidates should be able to:

(a) show an understanding that a magnetic field is an example of a field of force produced either by current-carrying conductors or by permanent magnets

(b) sketch flux patterns due to currents in a long straight wire, a flat circular coil and a long solenoid

(c) use $B = \dfrac{\mu_0 I}{2\pi d}$, $B = \dfrac{\mu_0 NI}{2r}$ and $B = \mu_0 nI$ for the flux densities of the fields due to currents in a long straight wire, a flat circular coil and a long solenoid respectively **[Not in H1 syllabus]**

(d) show an understanding that the magnetic field due to a solenoid may be influenced by the presence of ferrous core

(e) show an understanding that a current-carrying conductor placed in a magnetic field might experience a force

(f) recall and solve problems using the equation $F = BIL \sin \theta$, with directions as interpreted by Fleming's left-hand rule

(g) define magnetic flux density

(h) show an understanding of how the force on a current-carrying conductor can be used to measure the flux density of a magnetic field using a current balance

(i) explain the forces between current-carrying conductors and predict the direction of the forces

(j) predict the direction of the force on a charge moving in a magnetic field

(k) recall and solve problems using $F = BQv \sin \theta$

(l) describe and analyse deflections of beams of charged particles by uniform electric and uniform magnetic fields **[Not in H1 syllabus]**

(m) explain how electric and magnetic fields can be used in velocity selection for charged particles **[Not in H1 syllabus]**

16.1 Concept of a magnetic field

Properties of magnets

- Natural magnets were discovered from ancient archaeological sites more than two thousand years old. The term 'magnet' comes from the name of one of the locations these stones were found.
- Experiments have shown that
  - (i) Magnetic poles are of 2 kinds: North (**N**) or South (**S**).
  - (ii) Like poles repel each other, unlike poles attract.
  - (iii) Poles occur in equal and opposite pairs (dipoles).
  - (iv) When no other magnet is near, a freely suspended magnet will align so that the line joining its poles is approximately parallel to the Earth's North–South axis.
  - (v) The pole of the magnet that points towards the Earth's Geomagnetic North is called the north pole of the magnet and the other the south pole.

Concept of a magnetic field

- Forces between magnets can be explained using the concept of a magnetic field.
  - o A magnet sets up a magnetic field in its vicinity.
  - o The force exerted by one magnet on another magnet is due to the interaction between one magnet and the magnetic field of the other.

Definition

> A **magnetic field** is a region of space in which a ***moving* charge** or a **current-carrying conductor** or any **ferromagnetic object** will **experience a magnetic force** when it is placed in it.
> Hence, it is known as a **field of force**.

- Quantitatively, the strength of a magnetic field is expressed by a quantity called the **magnetic flux density (B)**. Its unit is the **tesla (T)**. (The magnetic flux density and its unit will be defined later.) Magnetic flux density is a vector.

- It is important to note that a **moving charge** placed in a magnetic field will experience a magnetic force. (Static charge placed in a magnetic field will not experience a magnetic force.)

Note :
There is a subtle difference between the following two terms:

- Magnetic field strength (denoted by symbol H) (not in syllabus)
- **Magnetic flux density** (denoted by symbol $\boldsymbol{B}$) (in our syllabus)

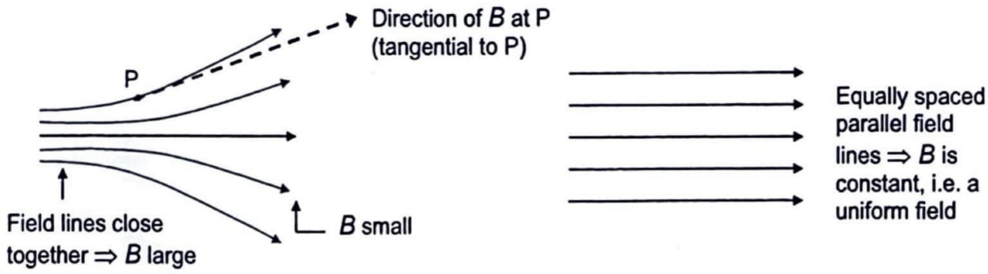
$B = \mu_0 H$ (equation not in syllabus)

In our syllabus, we focus on magnetic flux density (B).

Representation of a magnetic field

A magnetic field can be represented by field lines drawn such that:

- the tangent to a field line at a point gives the direction of B at that point
- the number of lines per unit cross–sectional area is proportional to the magnitude of B, i.e. if the lines are spaced closer, the magnitude of B is greater. If the field is uniform, the field lines are evenly spaced
- the arrows point away from the N–pole of a magnet to the S–pole. This is because the direction of the field is given by the direction of the force that acts on the north magnetic pole of a magnet



(a) non-uniform field (b) uniform field

Fig. 1

For a **two-dimensional view**, magnetic fields can be represented by dots and crosses, depending on whether it is perpendicularly pointing out or into the plane of the paper respectively.

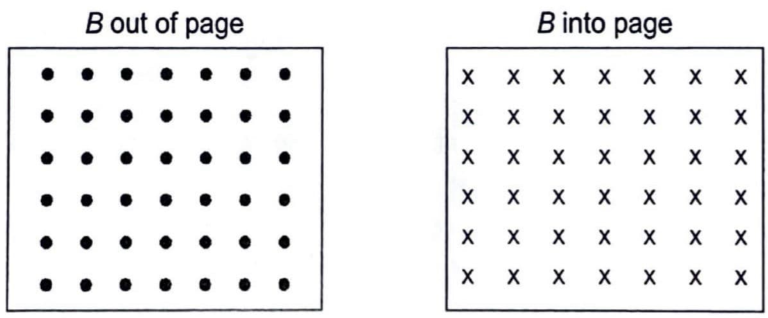
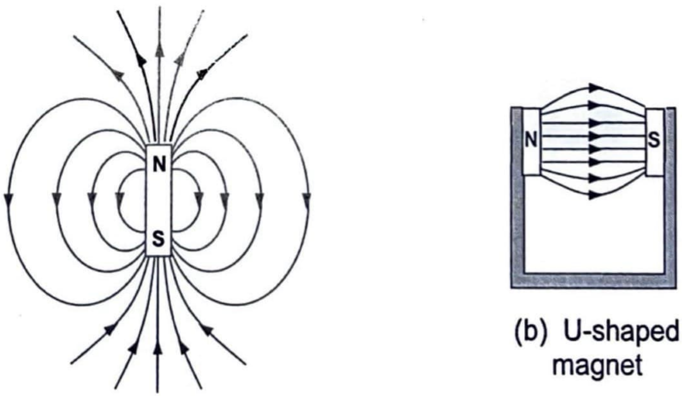


Fig. 2

Examples of field patterns



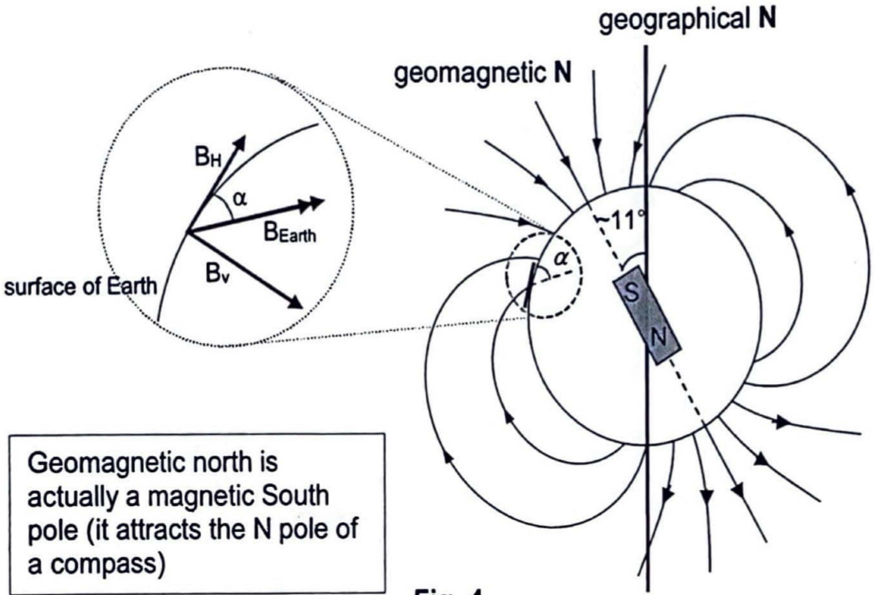
(a) Bar magnet

(b) U-shaped magnet

Fig. 3

Earth's magnetic field

- The Earth's magnetic field is a weak magnetic field believed to be caused by electric currents circulating within the core of the Earth.
- The magnitude and direction of this field varies with position over the Earth's surface and changes gradually with time.
- The axis of the Earth's magnetic field is approximately tilted 11 ° with respect to its rotational axis. Hence, the geographical North does not coincide exactly with the geomagnetic North.
- The field pattern is similar to that of a bar magnet embedded deep inside the centre of the Earth as shown in Fig. 4.



Geomagnetic north is actually a magnetic South pole (it attracts the N pole of a compass)

Fig. 4

- From the field pattern, it can be seen that, near the equator, the Earth's magnetic field, B_{Earth}, is almost horizontal.
- At all other positions, the B_{Earth} is inclined at various angles (called the *angle of inclination,* α) from the horizontal.

It is useful to resolve B_{Earth} into horizontal and vertical components.

$$B_H = B_{Earth} \cos \alpha$$
$$B_V = B_{Earth} \sin \alpha$$

Instruments such as compass needles whose motion is confined in a horizontal plane are affected only by B_H.

16.2 Magnetic fields due to currents

The birth of electromagnetism

In 1820, Hans Christian Oersted discovered the magnetic effect of an electric current. His findings saw the birth of electromagnetism.

He found that when compasses were placed on different sides of a current-carrying conductor, the needles of the compasses would be deflected in different ways as shown in Fig. 5.

Following Oersted's discovery, experiments deduced that there is a relationship between the magnetic field of a current carrying conductor and the current which flows through it.

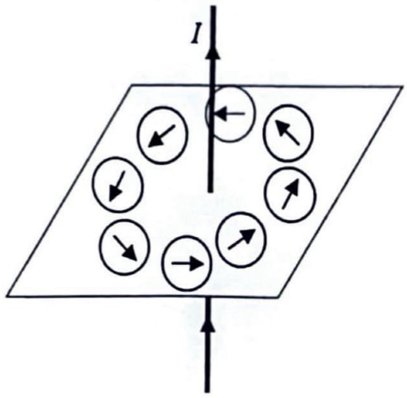


Fig. 5

The direction of magnetic field can be determined using Maxwell's *right-hand grip rule* as shown in Fig. 6.

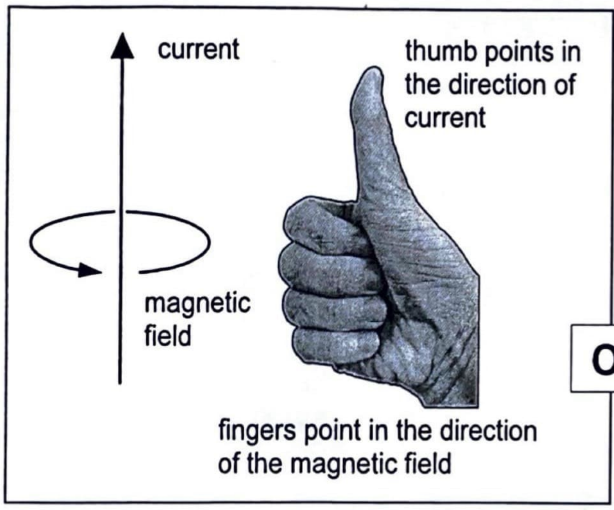


Fig. 6a
Right-hand grip for straight wire

OR

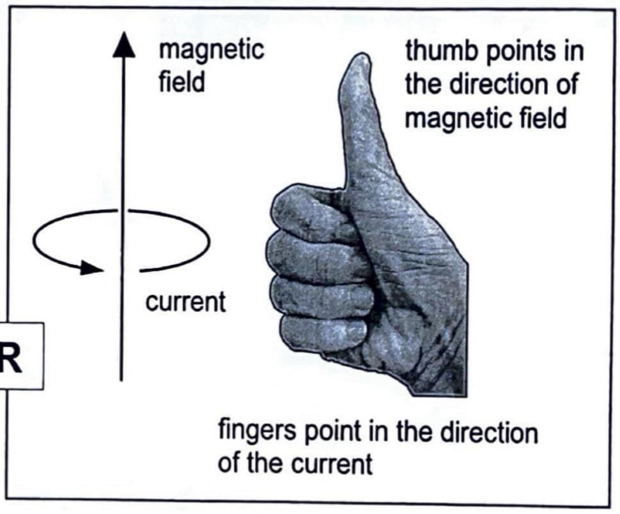
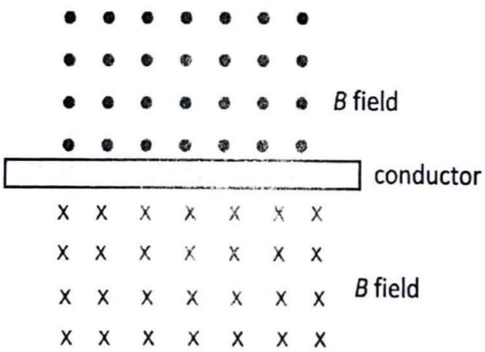


Fig. 6b
Right-hand grip for Coil and Solenoid

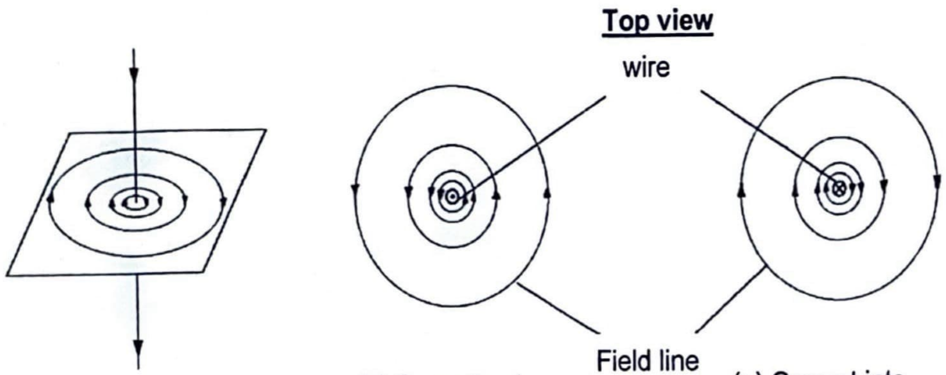
Quick Check
Determine the direction of the current in the straight conductor producing the magnetic field.



Magnetic field due to an infinitely long straight wire

Fig. 7 shows the field pattern around an infinitely long straight conducting wire.

- The magnetic field lines are concentric circles.
- The separation between the field lines increases with distance from the wire. This indicates that the magnetic flux density decreases in magnitude with distance.



(a) Straight wire　(b) Current out of paper　(c) Current into paper

Fig. 7

The direction of magnetic field in Fig. 7(b) can be determined using Maxwell's *right-hand grip rule* as shown in Fig. 8.

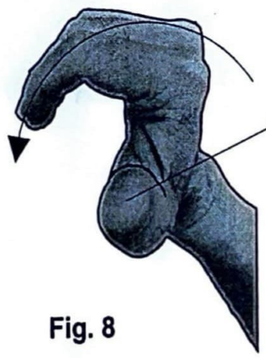


Fig. 8

Fig. 9 shows an infinitely long straight wire lying in the plane of the paper. At the point P, the magnetic flux density due to the current in the wire is directed into the paper and its magnitude is given by

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$ (provided in the formula list)

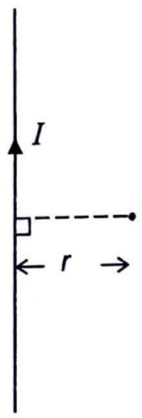
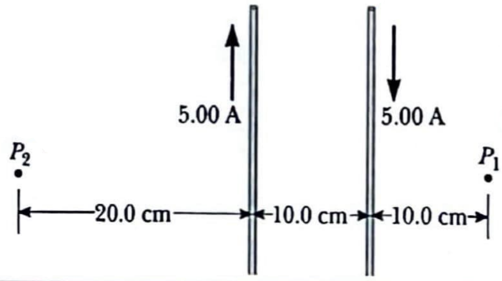


Fig. 9

where B = magnetic flux density
I = current in the wire
r = perpendicular distance of P from wire
μ_0 = permeability of free space (vacuum)
= $4\pi \times 10^{-7}$ H m^{-1}

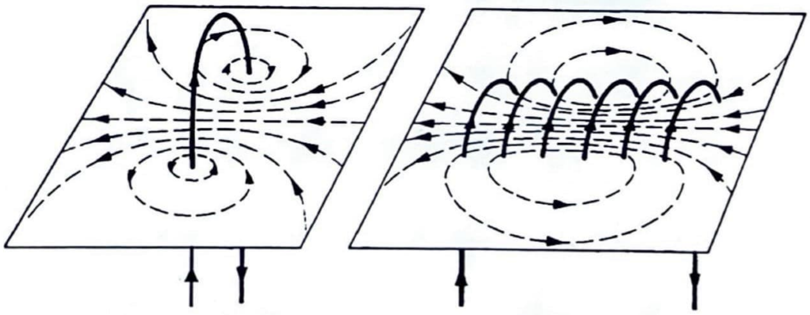
Quick Check
Determine the direction of the magnetic field at point P_1 and P_2.



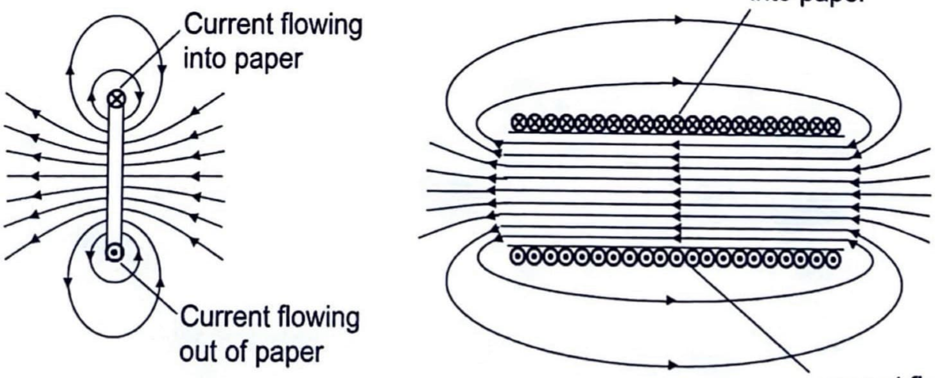
Magnetic fields due to a circular coil and solenoid

Fig. 10 shows the field patterns due to a flat circular coil and a long solenoid

- The direction of the field lines within the coil and the solenoid can be found using the *right-hand grip rule*: Grip the coil in your right hand with your fingers pointing in the direction of the current, then your thumb gives the direction of the magnetic field.
- When the turns of a solenoid are closely spaced, each can be regarded as a circular coil. Hence, the net field in a solenoid is due to the effect of many circular turns.
- The magnetic field inside a long solenoid is uniform.



(a) flat circular coil (b) solenoid



(a) flat circular coil Top view (b) solenoid

Fig. 10

Flat Circular Coil

Fig. 11 shows a flat circular coil lying in the plane of the paper. At the centre of the coil, the magnetic flux density B due to the current in the coil is directed into the paper (by the right-hand grip rule) and its magnitude is given by

$$B = \frac{\mu_0 N I}{2r}$$ (provided in formula list)

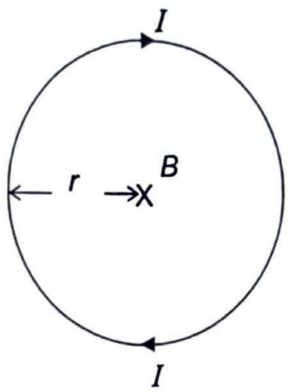


Fig. 11

where B = magnetic flux density
μ_0 = permeability of free space
N = Number of turns
I = current in the coil
r = radius of the coil

Long Solenoid

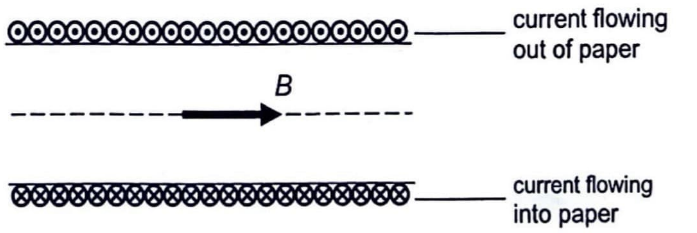


Fig. 12: Cross sectional view of a Solenoid

The magnetic flux density on the axis of an infinitely long solenoid is directed along the axis and its magnitude is given by

$$B = \mu_0 n I$$ (provided in formula list)

where B = magnetic flux density
μ_0 = permeability of free space
n = number of turns per unit length of solenoid
I = current in the solenoid

At either end of a solenoid of finite length, the magnetic flux density along the axis is $\frac{1}{2}\mu_0 n I$.

16.3 Force on a current-carrying conductor

Direction of the magnetic force

A current–carrying conductor in a magnetic field experiences a force.

This force is always perpendicular to the direction of the current and the direction of the magnetic field.

The relative directions of current, field and force are summarized by ***Fleming's Left–Hand Rule*** as shown in Fig. 13.

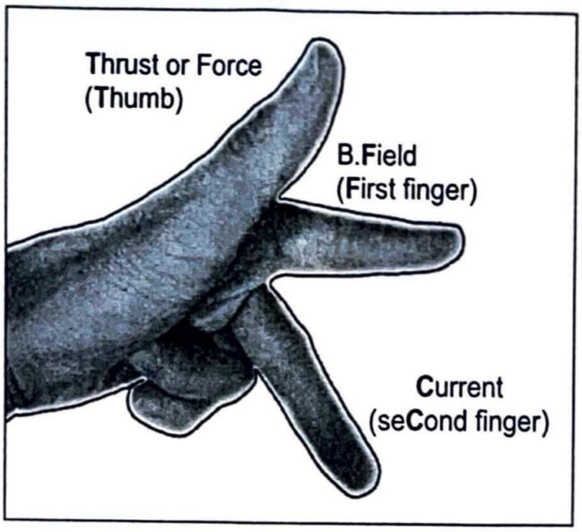
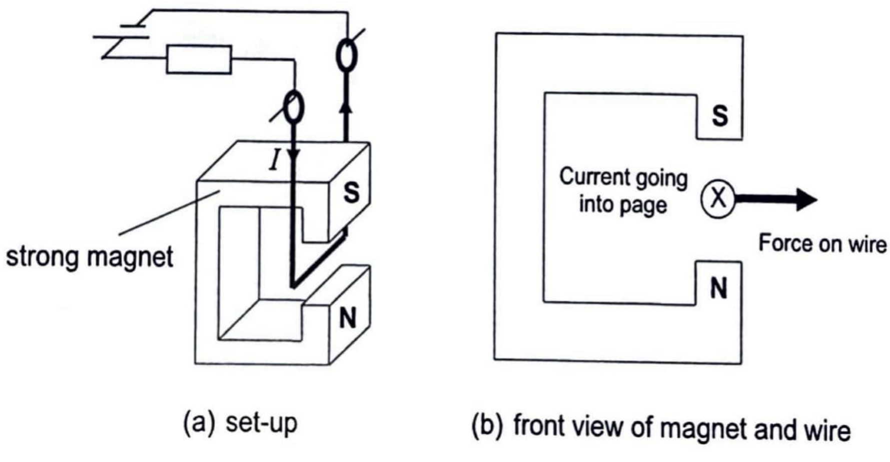


Fig. 13

The 'kicking wire' experiment



(a) set-up

(b) front view of magnet and wire

Fig. 14

The 'kicking wire' experiment in Fig. 14 shows that when a current passes through the wire, a magnetic force acts on the wire causing it to 'kick' in the direction shown.

Magnitude of the magnetic force and definition of magnetic flux density

Experiments with a wire placed at right angles to the magnetic field show that the magnitude of the magnetic force F is directly proportional to

- the **current** I in the wire
- the **length** L of the wire in the magnetic field
- the **magnetic flux density** B

This leads to a definition of magnetic flux density:

Definition

The **magnetic flux density** of a magnetic field is numerically equal to the force per unit length per unit current acting on a long straight conductor at right angles to the magnetic field.

$$B = \frac{F}{IL}$$

where B = magnetic flux density
F = magnetic force acting on the wire
I = current flowing through the wire
L = length of wire

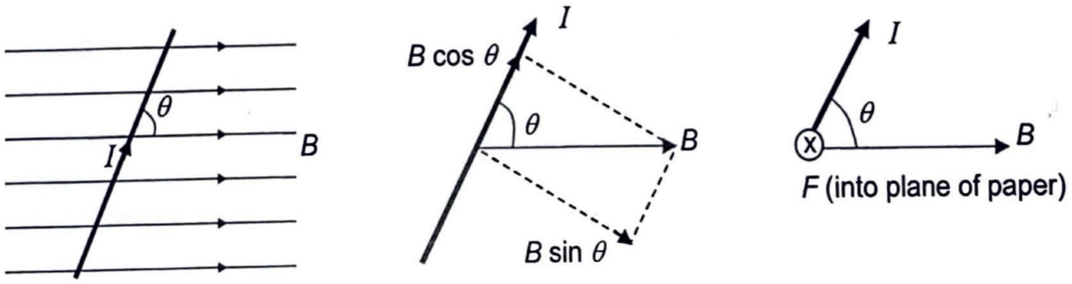
The S.I. unit for flux density is **tesla** (T).

$$1\ \text{T} \equiv 1\ \text{N A}^{-1}\ \text{m}^{-1} \equiv 1\ \text{kg s}^{-2}\ \text{A}^{-1}$$

Re–arranging the equation for flux density, the force F acting on a wire of length L carrying a current I when placed at right angles to the field is given by

$$F = BIL$$

In the general case, if the conductor makes an angle θ with the field as shown in **Fig. 15(a)**, the force can be regarded as due to the *component* of the field *perpendicular* to the current ($B \sin\theta$); the component parallel to the current produces no force.



(a) Top view of a conductor placed on a horizontal surface

(b) Components of magnetic field parallel and perpendicular to conductor

(c) Direction of force is perpendicular to plane containing B and I, found using Fleming's Left-hand Rule

Fig. 15

Formula

Thus, force F is given by $\boxed{F = BIL\sin\theta,}$ where angle θ is between B and I.

Example 1

A wire, 2.0 m in length, carrying a current of 10 A is placed in a field of flux density 0.15 T. Determine the magnitude of the force on the wire if it is placed

(a) at right angles to the field,

(b) at 30° to the field, and

(c) along the field.

Solution:

$F = BIL\sin\theta$ where θ is the angle between B and I

(a) $F = (0.15)(10)(2.0)\ \sin 90° = 3.0$ N
(b) $F = (0.15)(10)(2.0)\ \sin 30° = 1.5$ N
(c) $F = (0.15)(10)(2.0)\ \sin 0 = 0$ N

Measuring magnetic flux density using the current balance

A current balance is an arrangement that can be used to measure the magnetic flux density B of a magnetic field. It makes use of the principle of moments.

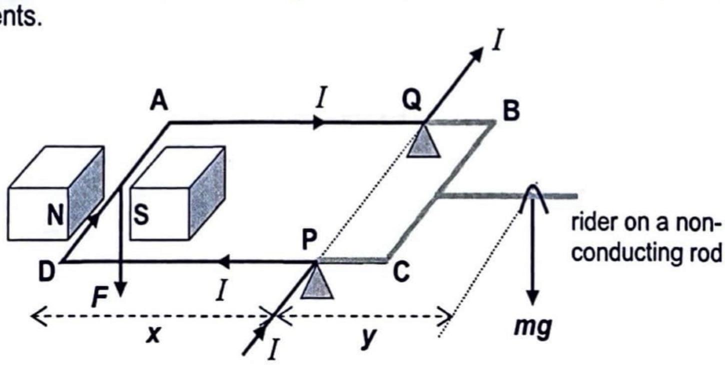


Fig. 16

- A wire frame ABCD is balanced on 2 pivots through which a current I from a d.c. source enters from P and leaves from Q.
- The frame is arranged such that the side AD of the frame (of length L) lies within a magnetic field whose flux density B is to be determined.
- When there is no current, the frame is horizontal.
- When current flows, a magnetic force acts on AD and pushes that side of the frame downwards (Fleming's left-hand rule).
- A mass m (known as a rider) is suspended on the right side to restore the frame to its horizontal position.

  By the principle of moments,

  sum of clockwise moments = sum of anticlockwise moments

$$mgy = Fx$$
$$= BILx$$
$$B = \frac{mgy}{ILx}$$

Hence, the magnetic flux density B of a magnetic field can be determined with the appropriate quantities known.

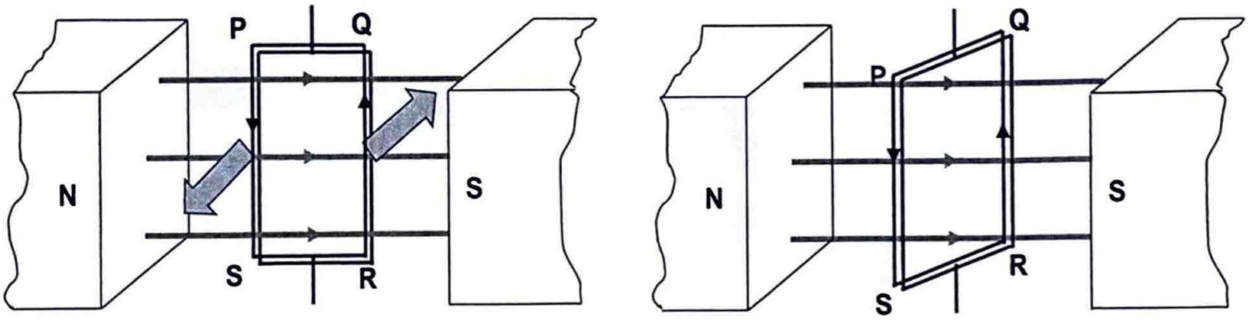
Hall Probe vs Curent Balance

Another device that is more commonly used to measure magnetic flux density is the Hall probe. The following table highlights some differences between the two devices.

Current Balance	Hall Probe
• Does not require calibration. Deduces B using force, current and length of test wire.	• Requires calibration by a magnetic field of known strength.
• Unable to measure weak fields.	• Can measure very weak fields.
• Apparatus is bulky. Not so practical and portable.	• Practical and convenient due to the small size of the probe.

Torque on a current-carrying coil in a magnetic field

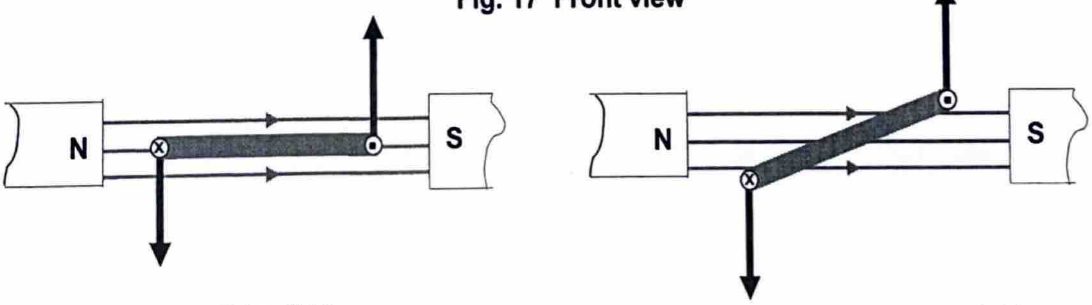
Consider a rectangular coil placed in the plane of a uniform magnetic field as shown in Fig. 17 (a). The plane of the coil is initially parallel to the field lines.



(a) Parallel to field

(b) at an angle to the field

Fig. 17 Front view



(a) Parallel to field

(b) at an angle to the field

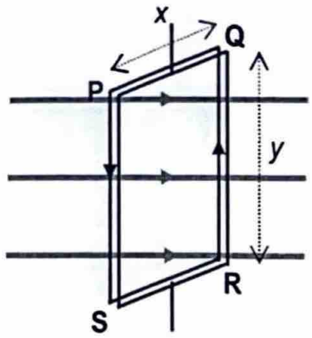
Fig. 18 Top view

When a steady current passes through the coil, a magnetic force acts on sides PS and QR, which are at right angles to the field lines.

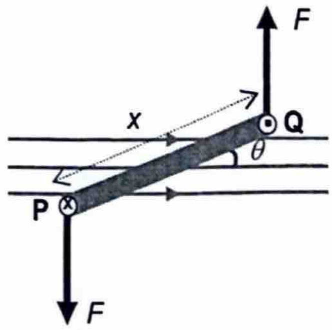
Using Fleming's Left Hand Rule, the force on PS is opposite to the force on QR. These two forces form a couple and provide a torque (turning effect) on the coil. Hence, the coil will rotate in the direction as shown in **Fig. 18**. This is the principle behind moving-coil meters and motors.

Let us take a closer look to determine an expression for the torque on a coil.

Fig. 19 shows the same coil PQRS placed in a uniform magnetic field *B* such that the vertical sides are 90° to the field while the horizontal sides make an angle θ to the field. Assume the coil has *N* turns and its horizontal and vertical sides are of length *x* and *y* respectively.



(a) side view



(b) top view

Fig. 19

- When current flows in the coil, each side of the coil experiences a force.
- The forces on the horizontal sides of the coil (PQ and SR) do not give rise to a turning effect but may distort the coil (if it is not rigid enough).
- The forces on the vertical sides (PS and QR), each of length *y*, are opposite in direction and equal in magnitude, given by $F = NBIy$.
- Whatever the position of the coil, its vertical sides are at right angles to the magnetic field and so the force *F* remains constant in magnitude.
- The forces constitute a couple whose torque τ is given by

$$\begin{aligned}\tau &= (\text{one force}) \times (\text{perpendicular distance between lines of action of the forces})\\ &= F(x\cos\theta)\\ &= (NBIL)(x\cos\theta)\\ &= NBIyx\cos\theta\\ &= NBIA\cos\theta\end{aligned}$$

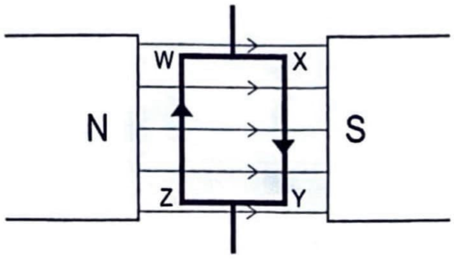
where A = area of the coil = xy

Note

- τ is maximum when $\theta = 0°$, when the plane of the coil is parallel to *B*.
- τ is zero when $\theta = 90°$, when the plane of the coil is perpendicular to *B*.

Example 2

In an electric motor, a rectangular coil WXYZ has 20 turns and is in a uniform magnetic field of flux density 0.83 T. The lengths of sides XY and ZW are 0.17 m and of sides WX and YZ are 0.11 m. The current in the coil is 4.5 A.



(a) Calculate maximum torque τ on the coil

(b) Calculate the torque on the coil when its plane makes an angle of 30° with the magnetic flux density.

(c) At what angle does the plane of the coil make with the magnetic field when the torque is zero?

[adapted from N2010/P1/31]

Solution:

(a) Maximum torque is when the plane of the coil is parallel to the magnetic field.

$$\tau = F \times L_{WX}$$
$$= NBIL_{WZ} \times L_{WX}$$
$$= NBIA$$
$$= (20)(0.83)(4.5)(0.17 \times 0.11)$$
$$= 1.4 \text{ Nm}$$

(b) When $\theta = 30°$

$$\tau' = NBIA\cos 30°$$
$$= 1.21 \text{ N m}$$

(c) $\tau = NBIA\cos\theta$

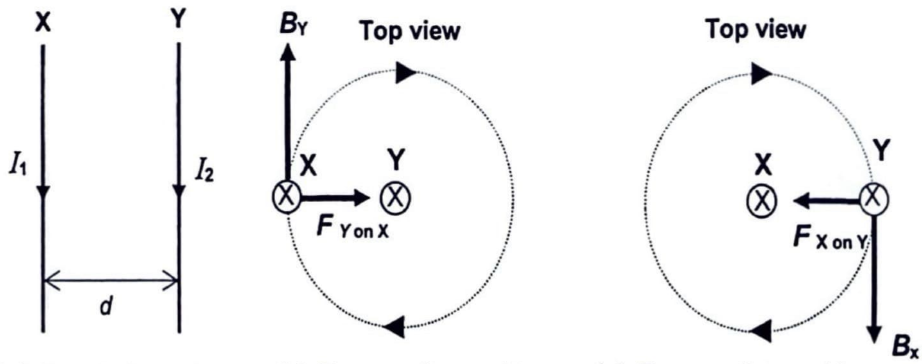
$= 0$ (when $\theta = 90°$)

Ans : 90°

16.4 Forces between current-carrying conductors

Currents flowing in the same direction

Consider two infinitely long parallel vertical wires X and Y carrying currents I_1 and I_2 flowing in the same direction, separated by a distance d as shown in Fig. 20(a).



(a) **Front view** of wire X and Y (b) Force acting on X (c) Force acting on Y

Fig. 20

Referring to Fig. 20(a), the current in X produces a magnetic field directed out of the page (using right-hand grip) at wire Y given by

$$B_X = \frac{\mu_o I_1}{2\pi d}$$

At Y, the direction of the field B_X due to X is perpendicular to Y as shown in Fig. 20(c). Using Fleming's Left-Hand Rule, the magnetic force F_{XonY} on a length L of Y would be **towards** X and has a magnitude of

$$F_{X\ on\ Y} = B_X I_2 L = \left(\frac{\mu_o I_1}{2\pi d}\right) I_2 L$$

Similarly, the magnetic force F_{YonX} on a length L of X is **towards** Y as shown in Fig. 20(b) and has a magnitude of

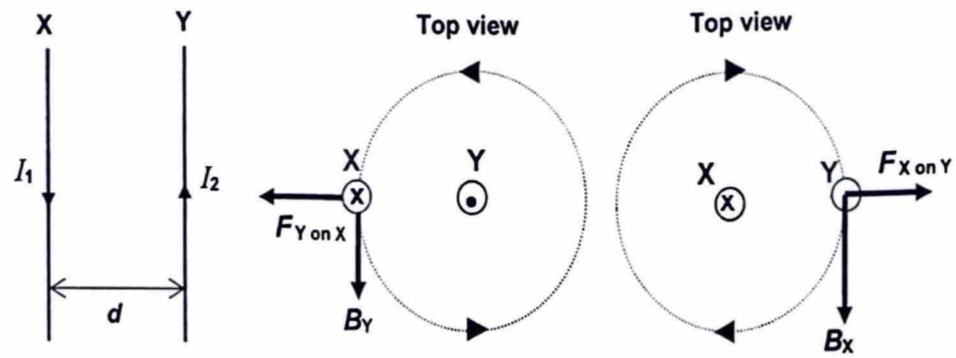
$$F_{Y\ on\ X} = B_Y I_1 L = \left(\frac{\mu_o I_2}{2\pi d}\right) I_1 L$$

By Newton's Third Law, $F_{X\ on\ Y}$ and $F_{Y\ on\ X}$ are equal and opposite forces. The two wires **attract** one another.

The force per unit length on each wire is given by $\frac{F}{L} = \frac{\mu_o I_1 I_2}{2\pi d}$.

Currents flowing in opposite directions

Consider 2 infinitely long parallel vertical wires X and Y carrying currents I_1 and I_2 flowing in opposite directions, separated by a distance d as shown in Fig. 21(a).



(a) Front view of wire X and Y (b) Force acting on X (c) Force acting on Y

Fig. 21

Referring to Fig. 21(a), the current in X produces a magnetic field directed out of the page (using right-hand grip) at wire Y given by

$$B_X = \frac{\mu_o I_1}{2\pi d}$$

At Y, the direction of the field B_X due to X is perpendicular to Y as shown in Fig. 21(c). Using Fleming's Left-Hand Rule, the magnetic force F_{XonY} on a length L of Y would be **away from** X and has a magnitude of

$$F_{X\ on\ Y} = B_X I_2 L = \left(\frac{\mu_o I_1}{2\pi d}\right) I_2 L$$

Similarly, the magnetic force on a length L of X is **away from** Y as shown in Fig. 21(b) and has a magnitude of

$$F_{Y\ on\ X} = B_Y I_1 L = \left(\frac{\mu_o I_2}{2\pi d}\right) I_1 L$$

By Newton's Third Law, $F_{X\ on\ Y}$ and $F_{Y\ on\ X}$ are equal and opposite forces. The two wires **repel** one another.

The force per unit length on each wire is given by $\frac{F}{L} = \frac{\mu_o I_1 I_2}{2\pi d}$

In summary,

for two parallel current-carrying conductors,

i) the **force per unit length** on each wire is given by the equation

$$\frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi d}$$

ii) **"Like currents attract, unlike currents repel"** as shown in Fig. 22.

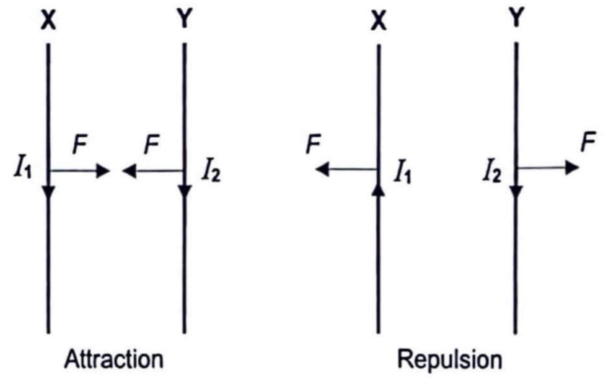
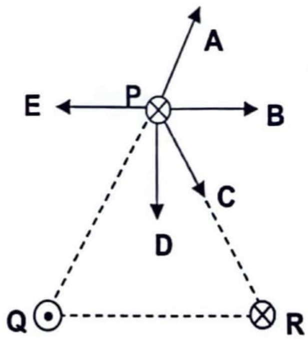


Fig. 22

Example 3

Three long vertical wires pass through the corners of an equilateral triangle PQR.



P and R carry currents directed into the plane of the paper and wire Q carries a current directed out of the paper.

All three currents have the same magnitude.

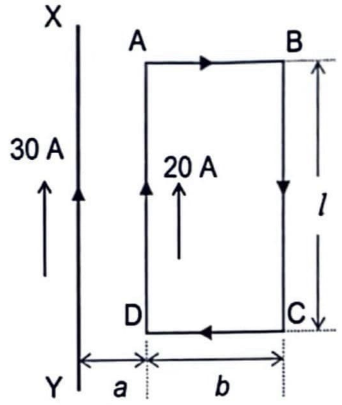
Which arrow shows the direction of the resultant force acting on P?

Solution:

Resultant force:

B.

Example 4 The figure shows a long wire XY carrying a current of 30 A. The rectangular loop ABCD carries a current of 20 A. The length of sides *a*, *b* and *l* are 1.0 cm, 8.0 cm and 30 cm respectively.



(a) Calculate the magnetic flux density due to the current in XY along
(i) AD
(ii) BC

(b) Calculate the resultant force acting on the loop.

Solution:

(a) (i) $B_{\text{on AD}} = \dfrac{\mu_o I_{XY}}{2\pi a}$

$= \dfrac{(4\pi \times 10^{-7})(30)}{2\pi(1.0\times10^{-2})}$

$= 6.0 \times 10^{-4}$ T

(ii) $B_{\text{on BC}} = \dfrac{\mu_o I_{XY}}{2\pi(a+b)}$

$= \dfrac{(4\pi \times 10^{-7})(30)}{2\pi(1.0\times10^{-2} + 8.0\times10^{-2})}$

$= 6.7 \times 10^{-5}$ T

(b) $F_{\text{resultant}} = F_{\text{on AD}} - F_{\text{on BC}}$

$= (B_{\text{on AD}} - B_{\text{on BC}}) I_{ABCD} l$

$= (6.0\times10^{-4} - 6.7\times10^{-5})(20)(0.30)$

$= 3.2\times10^{-3}$ N towards XY

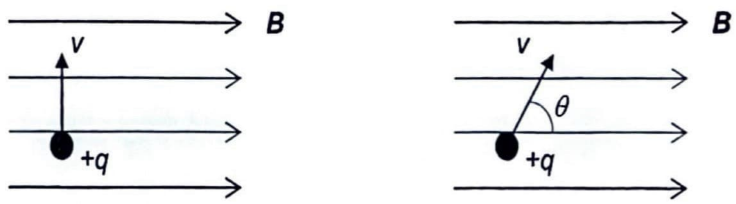
Think Oo∘ Is there any force acting on sides AB and CD?

16.5 Force on a moving charge

Force on a charged particle moving in a magnetic field

Since a current-carrying conductor can experience a force in a magnetic field, **a moving charge should also experience a force in a magnetic field.**

Consider a positive charge q moving at constant speed v at right-angles to a magnetic field of flux density B as shown in Fig. 23(a).



(a) Charge moving perpendicularly to the magnetic field (b) Charge moving at an angle to the magnetic field

Fig. 23

Assume the particle travels a distance L in time t, so its speed is $v = \frac{L}{t}$.

The moving charge constitutes a current of $I = \frac{q}{t}$.

Hence, the force on the charge is given by

$$F = BIL = B\left(\frac{q}{t}\right)L = Bq\left(\frac{L}{t}\right) = Bqv$$

If the velocity and field are inclined to each other by an angle θ as shown in Fig. 23(b), then

Formula

$$\boxed{F = Bqv\sin\theta}$$

where F = magnetic force acting on the charge
B = magnetic flux density
q = magnitude of charge
v = speed of charge
θ = angle the velocity makes with the field

Remember that Fleming's left-hand rule considers the direction of conventional current. Hence, for a

a) <u>**positive charge**</u>, the direction of the current is in the **same** direction of the motion of the charge

b) <u>**negative charge**</u>, the direction of the current is in the **opposite** direction as the direction of motion of the charge

and the direction of the magnetic force can thus be determined.

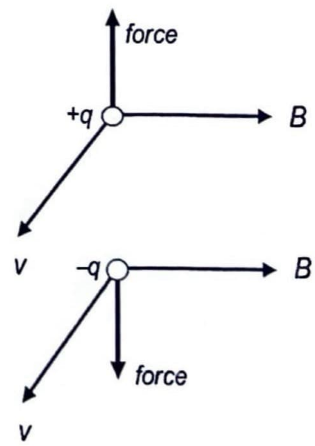


Fig. 24

Motion of a charged particle in a magnetic field

Uniform Circular Motion

When a charged particle is projected at a right angle into a magnetic field, the magnetic force *F* is always perpendicular to the direction of travel (or velocity) and the distance travelled in the direction of the force is zero, as shown in Fig. 25.

Therefore the work done on the charged particle is always zero.

This implies that no energy is gained or lost by the particle moving in the magnetic field and the particle's speed is always constant.

Since the force is of constant magnitude and is always at right angles to the velocity, the conditions are met for **circular motion**.

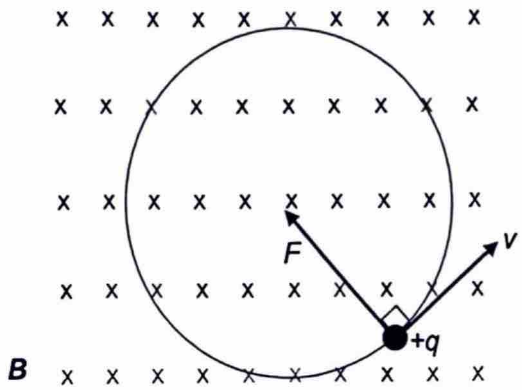


Fig. 25

The magnetic force on a moving charge provides for the *centripetal* force.

$$F_B = F_c$$

$$Bqv = \frac{mv^2}{r}$$

where m is the mass of the moving charge and r is the radius of circular path.

Helical path

For a charged particle entering a magnetic field at an angle θ such that $0° < \theta < 90°$, the particle would describe a **helix** (see Fig. 26).

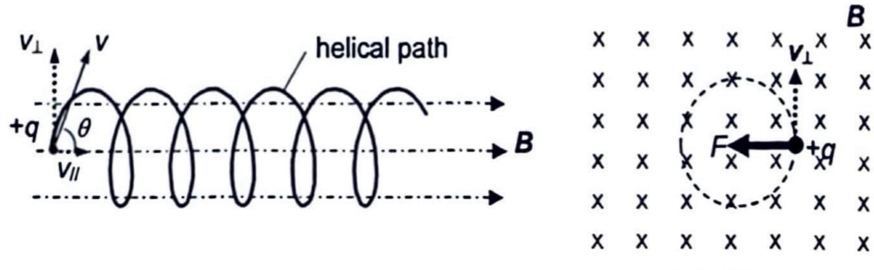


Fig. 26

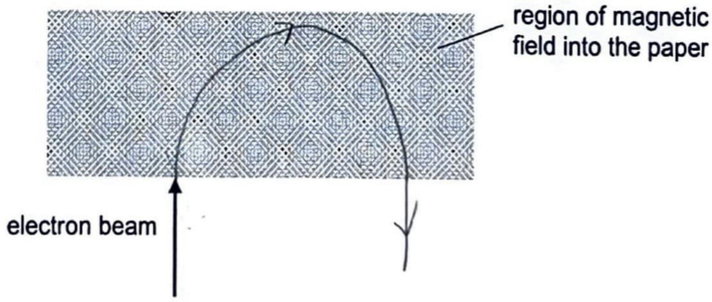
Explanation:
Resolve v into 2 components:

$$v_{//} = v \cos \theta$$
$$v_\perp = v \sin \theta$$

Motion of the charged particle is the result of superposing
(i) a uniform circular motion in which it has speed $v_\perp$ in a plane perpendicular to the direction of B.
(ii) a steady speed $v_{//}$ along the direction of B.

Example 5

A beam of electrons travelling with a velocity of 3.2×10^7 m s^{-1} enters a magnetic field of flux density 0.47 mT. The electrons are travelling at right angles to the field. ($e = 1.60 \times 10^{-19}$ C; $m_e = 9.11 \times 10^{-31}$ kg.)



(i) Calculate the force on each electron within the field.

(ii) Calculate the radius of curvature of each electron's path while in the field.

(iii) Sketch the path travelled by an electron within and beyond the field. Indicate clearly the direction of the electron's path, the field and the force.

Solution:

(i) $F = Bqv = (0.47 \times 10^{-3})(1.60 \times 10^{-19})(3.2 \times 10^7) = 2.4 \times 10^{-15}$ N

(ii) The magnetic force on the electrons provides for the centripetal force.

$$F_B = F_C$$

$$2.4 \times 10^{-15} = \frac{mv^2}{r}$$

$$r = \frac{(9.11 \times 10^{-31})(3.2 \times 10^7)^2}{2.4 \times 10^{-15}} = 0.39 \text{ m}$$

Example 6 An α–particle, of mass 6.6×10^{-27} kg and charge $+2e$, was injected at right angles into a uniform magnetic field of flux density 1.2 T. It travels in a circular path of radius 0.45 m. Calculate

(i) its speed,

(ii) its period of revolution,

(iii) its kinetic energy, and

(iv) the potential difference of an electric field through which it would have to be accelerated from rest to achieve this energy.

Solution:

(i) magnetic force provides for the centripetal force

$$Bqv = \frac{mv^2}{r}$$

$$v = \frac{Bqr}{m} = \frac{(1.2)(2 \times 1.60 \times 10^{-19})(0.45)}{(6.6 \times 10^{-27})} = 2.6 \times 10^{7} \text{ m s}^{-1}$$

(ii) $v = \dfrac{2\pi r}{T}$

From circular motion
$v = \omega r = (2\pi/T)\,r$

$$T = \frac{2\pi(0.45)}{2.6 \times 10^{7}} = 1.1 \times 10^{-7} \text{ s}$$

OR

$$Bqv = mr\omega^2 = mr\left(\frac{2\pi}{T}\right)^2$$

$$T = 1.1 \times 10^{-7} \text{ s}$$

(iii) $E_K = \frac{1}{2}mv^2 = \frac{1}{2}(6.6 \times 10^{-27})(2.6 \times 10^{7})^2 = 2.2 \times 10^{-12}$ J

(iv) Gain in KE = Loss in electric PE

$2.2 \times 10^{-12} = qV$ ← Topic of E-field

$$V = \frac{2.2 \times 10^{-12}}{2 \times 1.6 \times 10^{-19}}$$

$$= 6.9 \times 10^{6} \text{ V}$$

Applications involving motion of charged particles in uniform electric and magnetic fields.

Velocity selector

A velocity selector consists of a magnetic field and an electric field applied over the same region, allowing only charged particles of a particular velocity to pass through undeflected.

The two fields are applied perpendicular to each other in an orientation such that the electric force and the magnetic force on the particle act in opposite directions.

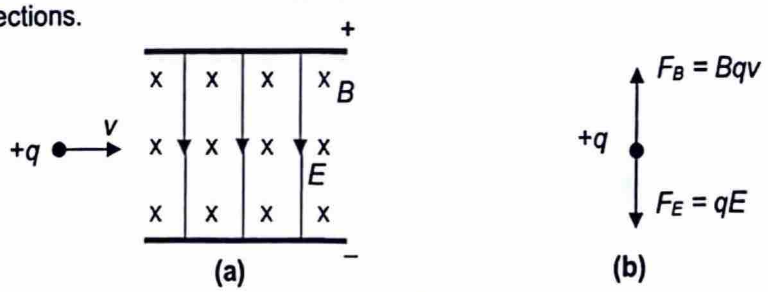


Fig. 27

Suppose a positive charge $+q$ enters the region where the two crossed fields are applied, with its velocity v perpendicular to both fields as shown in Fig. 27(a).

It experiences an upward magnetic force $F_B = Bqv$ and a downward electric force $F_E = qE$ as shown in Fig. 27(b).

NOTE!

[*The gravitational force acting on such charged particles is very small compared to F_B and F_E and hence can be ignored.*]

If $F_B > F_E$, the particle will deflect upward.
If $F_B < F_E$, the particle will deflect downward.

Particles whose velocities are such that $F_B = F_E$ will pass through undeflected.

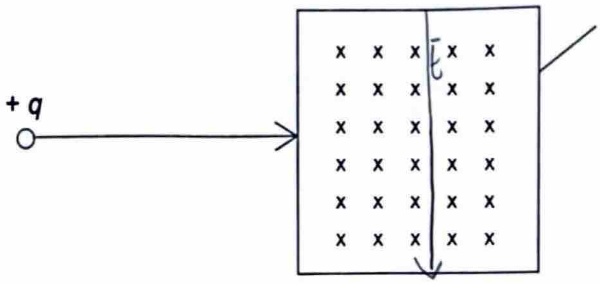
$$Bqv = qE$$

$$v = \frac{E}{B}$$

Hence only particles having this speed v will pass through undeflected.

The concept of charged particles moving through combined electric and magnetic fields has many real-life applications.

Example 7 A particle of charge $+q$ enters a uniform magnetic field of flux density B that is directed into the plane of the paper. The particle is travelling with velocity v at right angles to the magnetic field.



A uniform electric field is applied in the same region as the magnetic field so that the particle passes through undeflected.

(a) On the figure above, mark, with an arrow labelled E, the direction of the electric field.

(b) State and explain the effect, if any, on a particle entering the region of the fields if the particle has
(i) charge $-q$ and speed v,
(ii) charge $+q$ and speed $2v$.

Solution:

(b)(i) The particle remains undeflected. Magnetic force F_B is now downwards while electric force F_E is upwards.
Hence, there is no resultant force on the particle.

(ii) F_B increases with speed and hence $F_E < F_B$.
The particle **is deflected upwards.**

Comparison between deflections of beams of charged particles by uniform electric and uniform magnetic fields.

	Deflection in a magnetic field	Deflection in an electric field
1.	The magnetic field can exert a magnetic force only on a moving charged particle.	The electric field exerts an electric force on a stationary or moving charged particle.
2.	The magnetic force is perpendicular to the magnetic field and the direction of the motion of the charged particle.	The electric force acts in a direction parallel to the electric field.
3.	Magnetic force is dependent on the magnetic flux density and speed, charge, direction of motion of the particle.	Electric force is dependent on the charge of the particle and the electric field strength.
4.	When a charged particle enters a magnetic field perpendicularly, it moves in a circular path.	When a charged particle enters an electric field perpendicularly, it moves in a parabolic path.

Appendix

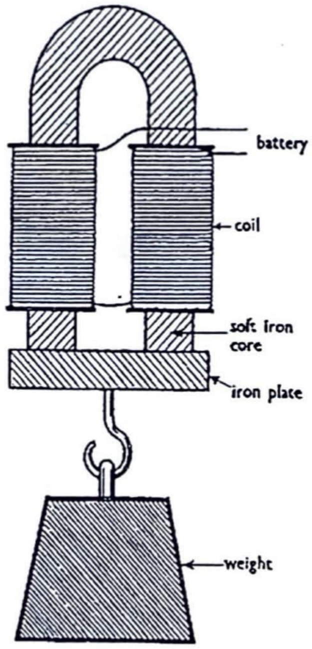
Solenoids and electromagnets

Effect of a ferrous core

A bar of iron can be magnetized by placing it inside a solenoid. When a current passes through the solenoid, it produces a magnetic field along its axis and the bar is magnetized accordingly. The resultant magnetic field is the sum of the field due to the current and that due to the iron core so that the magnitude of the resultant field can have a magnitude hundreds to thousands times that due to the current alone.

Uses of electromagnets

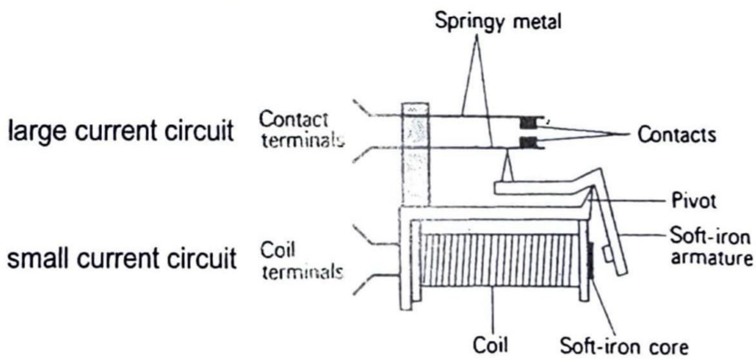
An iron core with a magnetizing coil wound around it is called an electromagnet. Its strength can be adjusted to exert large mechanical forces to lift heavy loads.



Relay

This is a switch worked by an electromagnet. It is useful if we want a small current in one circuit to control another circuit containing a device such as a lamp, electric bell or motor which requires a large current.

The structure of a relay is shown below. When the controlling current flows through the coil, the soft iron core is magnetized and attracts the L–shaped soft iron armature. This rocks on its pivot and closes the electrical contacts in the circuit being controlled.



Specific charge of electrons

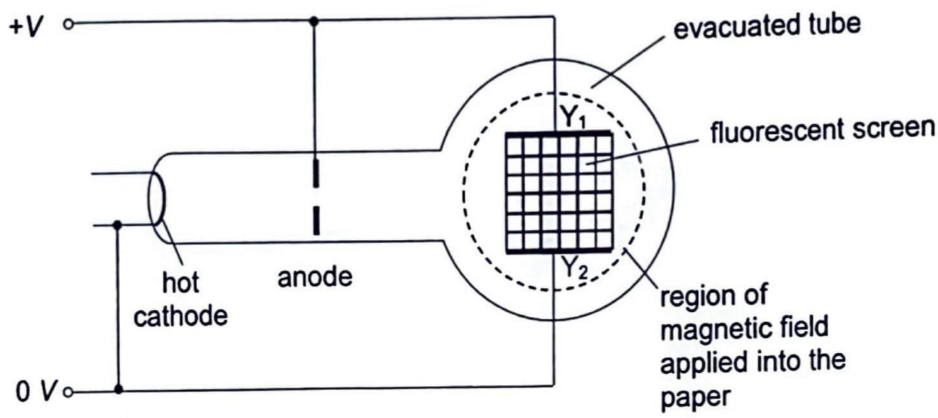
Determination of specific charge of electrons

The specific charge of a particle is the ratio of its charge to its mass:

$$\text{specific charge} = \frac{q}{m}$$

The charge-to-mass ratio provides a quick way to determine the mass of a particle by knowing its charge through theoretical derivations. In some experiments, it is easier to measure directly the charge-to-mass ratio of the particle.

An experiment significant to the discovery of the electron is the measurement of the specific charge of cathode rays by J.J. Thomson in 1897. The figure shows the set-up to find the specific charge of an electron.



Electrons produced by the hot cathode are accelerated in a vacuum towards an anode held at a potential $+V$ with respect to the cathode.

Assuming that the electrons are emitted with negligible speed from the cathode, they will leave the anode with a speed v which can be found:

Gain in K.E. of each electron = Loss in P.E. of the electron

$$\frac{1}{2}m_e v^2 = eV$$

$$v = \sqrt{\frac{2eV}{m_e}} \quad \text{.......... (1)}$$

where e and m_e are the charge and the mass of an electron respectively.

The beam of electrons emerging from the anode produces a narrow luminous trace when it hits a vertical fluorescent screen supported between two parallel plates Y_1 and Y_2. Y_1 is held at a potential $+V$ with respect to Y_2, thus creating an electric field E between the plates.

A uniform magnetic field B is applied perpendicular to the electric field and the magnitudes of the two fields are adjusted so that the beam passes through undeflected. Hence,

$$Bev = eE$$

$$v = \frac{E}{B} \quad \text{........ (2)}$$

Equating equations (1) and (2),

$$\frac{E}{B} = \sqrt{\frac{2eV}{m_e}}$$

$$\frac{e}{m_e} = \frac{E^2}{2B^2V}$$

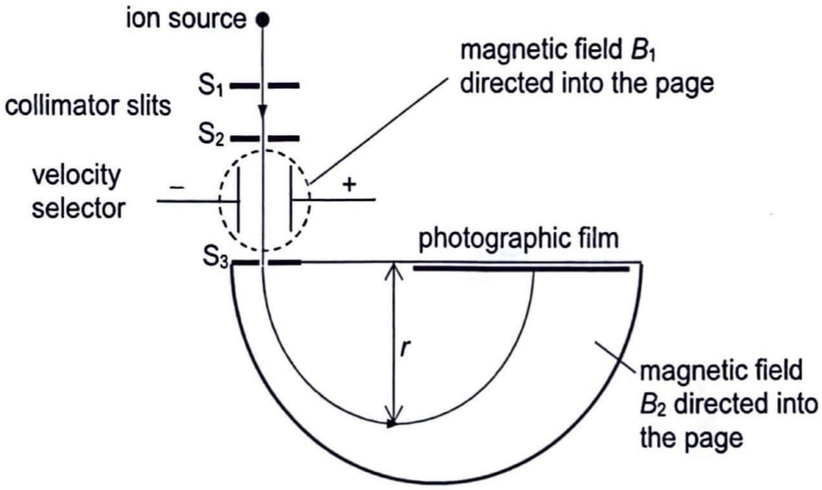
If the separation between plates Y_1 and Y_2 is d, then $E = \frac{V}{d}$ and the specific charge of an electron will be given by

$$\frac{e}{m_e} = \frac{V}{2B^2d^2}$$

Hence the specific charge of an electron can be found if V, B and d are known.

Mass spectrometer

A mass spectrometer separates ions according to their mass-to-charge ratio. The figure shows a Bainbridge Mass Spectrometer.



Ions of charge q and mass m with velocity $v = \frac{E}{B_1}$ pass undeflected through the velocity selector and are then deflected into a semi-circular path with radius r in the magnetic field B_2.

The magnetic force acting on the ions in B_2 provides for the centripetal force

$$F_B = F_C$$

$$B_2qv = \frac{mv^2}{r}$$

$$r = \frac{mv}{B_2q} = \frac{mE}{B_1B_2q}$$

Since B_1, B_2 and E are constant, $r \propto \frac{m}{q}$

Tutorial 16 ELECTROMAGNETISM

Data

- permeability of free space, $\mu_0 = 4\pi \times 10^{-7}$ H m^{-1}
- elementary charge $e = 1.60 \times 10^{-19}$ C
- mass of an electron $m_e = 9.11 \times 10^{-31}$ kg
- acceleration of free fall $g = 9.81$ m s^{-2}
- magnetic flux density due to a long straight wire, $B = \dfrac{\mu_0 I}{2\pi d}$
- magnetic flux density due to a flat circular coil, $B = \dfrac{\mu_0 N I}{2r}$
- magnetic flux density due to a long solenoid, $B = \mu_0 n I$

Self - Check Questions

S1. Draw diagrams to illustrate the magnetic field of **(a)** a short bar magnet, **(b)** a long straight wire, **(c)** a flat circular coil and **(d)** a long solenoid.

S2. Write down the equation for the force acting on a current-carrying conductor placed in a uniform magnetic field, explaining clearly the quantities involved. Draw a clear diagram showing the directions of the vector quantities involved.

S3. Define *magnetic flux density.*

S4. Two infinitely long parallel wires carrying currents I_1 and I_2 are placed a distance r apart. Describe what happens to the wires if the currents are

(a) in the same direction and **(b)** in the opposite direction.

S5. Write down an equation for the magnitude of the force acting on a charged particle moving in a uniform magnetic field, explaining clearly the quantities involved. Draw clear diagrams to show the direction of the force in relation to the direction of motion of the charge and the field direction in the case of

(a) positive charge, **(b)** negative charge.

S6. Describe and explain the path of a charged particle injected at right angles into

(a) a uniform electric field, **(b)** a uniform magnetic field.

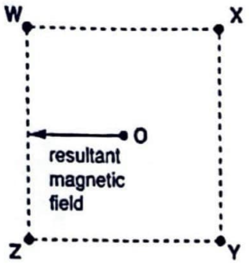
S7. Describe and explain the path of a charged particle when injected at an angle θ to a uniform magnetic field such that $0° < \theta < 90°$.

S8. Describe how a combination of an electric and magnetic field can be used in velocity selection of particles.

Self-Practice Questions

SP1

Four parallel conductors, carrying equal currents, pass vertically through the four corners of a square **WXYZ**. In two conductors, the current is directed into the page and, in the other two, it is directed out of the page.



It is required to produce a resultant magnetic field at O in the direction shown.

What must be the directions of the currents?

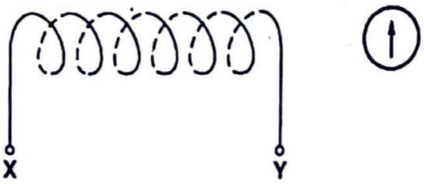
	into the page	*out of the page*
A	W and X	Y and Z
B	W and Z	X and Y
C	X and Z	W and Y
D	Y and Z	W and X

J91/I/17; J96/I/18

SP2

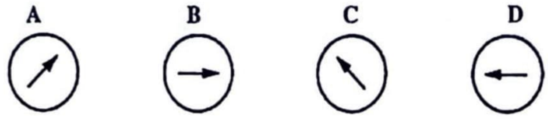
A plotting compass is placed near a solenoid.

When there is no current in the solenoid, the compass needle points due north as shown.



When there is a current from **X** to **Y**, the magnetic field of the solenoid at the compass is equal in magnitude to the Earth's magnetic field at that point.

In which direction does the plotting compass set?

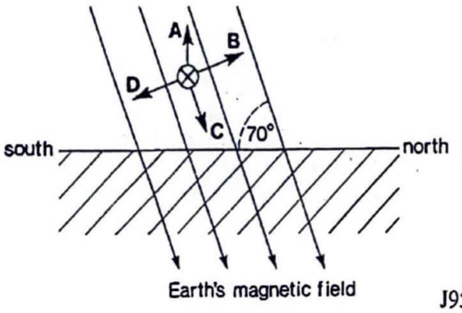


J94/I/17

SP3

A horizontal power cable carries a steady current in an east-to-west direction, i.e. into the plane of the diagram.

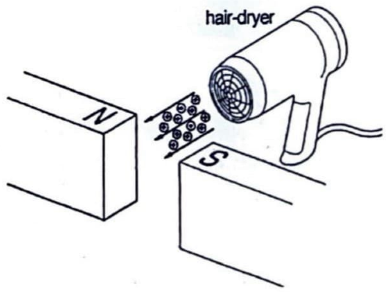
Which arrow shows the direction of the force on the cable caused by the Earth's magnetic field, in a region where this field is at 70° to the horizontal?



J95/I/18

SP4

Hot air from a hair-dryer contains many positively charged ions. The motion of these ions constitutes an electric current.



The hot air is directed between the poles of a strong magnet, as shown.

What happens to the ions?

They are deflected

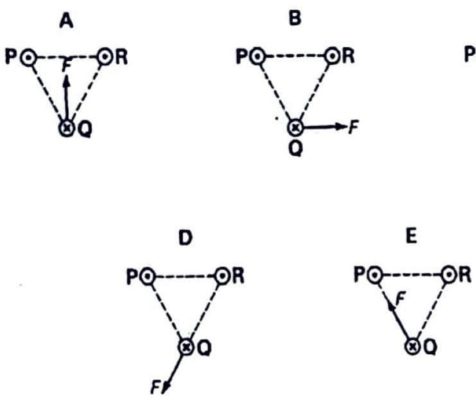
A towards the north pole N.
B towards the south pole S.
C downwards.
D upwards.

J97/I/19

SP5

Three long vertical wires pass through the corners of an equilateral triangle PQR. They carry equal currents into or out of the paper in the directions shown in the diagram.

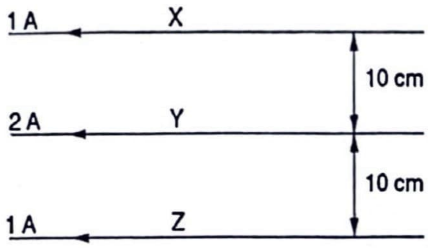
Which diagram shows the direction of the resultant force F on the wire at Q?



N93/I/16

SP6

Three long, parallel, straight wires X, Y and Z are placed in the same plane in a vacuum as shown. in the diagram below.



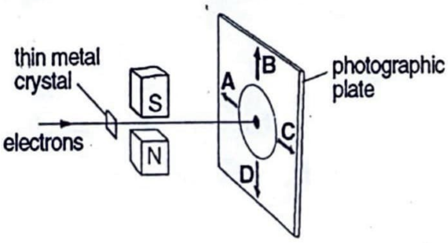
Given that the force per unit length between two long, parallel, straight wires placed 10 cm apart, each carrying a current of 1 A, is 2×10^{-6} N m^{-1}, what is the net force per unit length acting on Z?

A 3.0×10^{-6} N m^{-1}
B 3.5×10^{-6} N m^{-1}
C 4.0×10^{-6} N m^{-1}
D 4.5×10^{-6} N m^{-1}
E 5.0×10^{-6} N m^{-1}

J88/I/14

SP7

G P Thomson's early experiments on the diffraction of electrons by crystals were criticised on the grounds that the beams affecting the photographic plate might be X-rays. He proved that this was not so by placing bar magnets on each side of the beam as shown in the diagram.



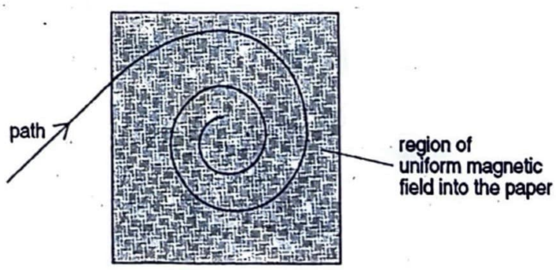
How would the magnetic field due to the magnets affect the diffraction ring?

A The ring would be deflected in the direction A.
B The ring would be deflected in the direction B.
C The ring would be deflected in the direction C.
D The ring would be deflected in the direction D.
E The diameter of the ring would decrease.

N90/I/19

SP8

A common way of investigating charged particles is to observe how they move in a plane at right angles to a uniform magnetic field. The diagram shows the path of a certain particle.



Which of the following gives a satisfactory explanation for the path?

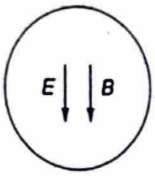
A The momentum of the particle is increasing steadily.
B The charge on the particle is decreasing steadily.
C The magnetic flux density is decreasing steadily.
D The mass of the particle is increasing steadily.
E The speed of the particle is decreasing steadily.

N93/I/26

SP9

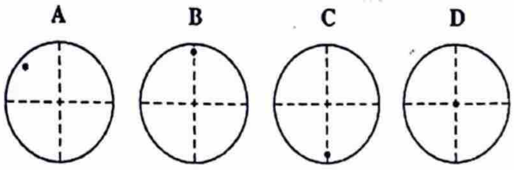
In a cathode-ray oscilloscope tube, the electron beam passes through a region where there are electric and magnetic fields directed vertically downwards as shown.

front view of screen



The deflections of the spot from the centre of the screen produced by the electric field E and the magnetic field B acting separately are equal in magnitude.

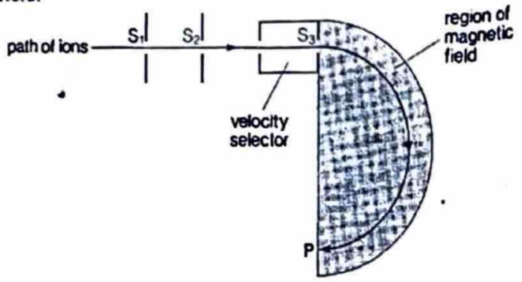
Which diagram shows a possible position of the spot on the screen when both fields are operating together?



J84/II/18; N91/I/27; J97/I/27

SP10

The diagram shows the principle of a simple form of mass spectrometer. Ions are passed through narrow slits S_1 and S_2 and into a velocity selector. The selected ions, after passage through the slit S_3, are deviated by the uniform magnetic field.



Which quantity must be the same for all ions arriving at point P?

A charge

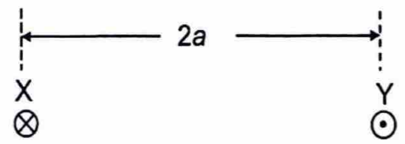
B $\frac{\text{charge}}{\text{mass}}$

C mass

D momentum

J91/I/29; N95/I/29

SP11 Two long straight parallel wires X and Y are separated by a distance 2*a*.



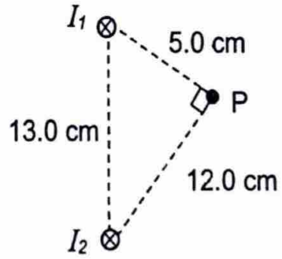
(a) If the wires carry equal currents I in opposite directions, determine in terms of *a*, μ_o and I, the magnitude of the magnetic flux density

(i) at a point midway between them, and

(ii) at a point a distance *a* from X and 3*a* from Y along the line joining the wires.

(b) For a case in which the wires carry equal currents I in the same directions, answer **(a)(i)** and **(ii)**.

SP12 Two long straight parallel conductors carry currents I_1 and I_2, both directed into the page as shown. $I_1 = I_2 = 3.00$ A.



Determine the magnitude and direction of the resultant magnetic field at P.

SP13 A circular coil X has 15 turns of radius 8.0 cm and carries a current of 1.2 A. Another coil Y with 10 turns and radius 4.0 cm is placed within coil X so that the two coils are coplanar and concentric. The current in Y is adjusted in magnitude and direction so that the resultant field at the common centre is zero.

Determine the current in coil Y.

SP14 A solenoid has 1500 turns m^{-1} and carries a current of 2.0 A.

(a) Calculate the magnetic flux density at the centre of the solenoid.

(b) Explain why no magnetic force acts on a straight current-carrying conductor placed along the axis of the solenoid.

SP15 **(a)** A proton (charge = 1.60×10^{-19} C, mass = 1.67×10^{-27} kg) is moving with velocity 7.00×10^{6} m s^{-1}. State the magnitude and direction of the force acting on it when the proton enters at right angles to each of the following fields:

(i) a gravitational field of field strength 9.81 N kg^{-1}

(ii) an electric field of field strength 1.50×10^{6} V m^{-1}

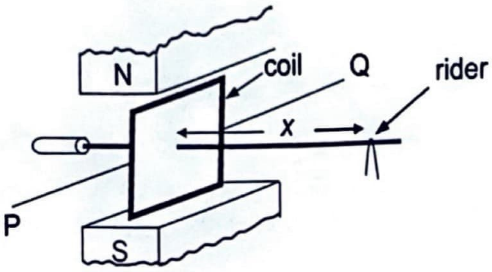
(iii) a magnetic field of magnetic flux density 0.125 T.

(b) State how your answers to the above would be affected if the proton were stationary instead of moving.

Discussion Questions

Force on a Current-Carrying Conductor

D1



A square coil of N turns has sides of length L and is mounted so that it can pivot freely about a horizontal axis PQ, parallel to one pair of sides of the coil, through its centre. The coil is situated between the poles of a magnet which produces a uniform magnetic field of flux density B. The coil is maintained in a vertical plane by moving a rider of mass M along a horizontal beam attached to the coil. When a current I flows through the coil, equilibrium is restored by placing the rider a distance x along the beam from the coil.

(a) Starting from the definition of magnetic flux density, show that B is given by the expression

$$B = \frac{Mgx}{IL^2N}.$$

[3]

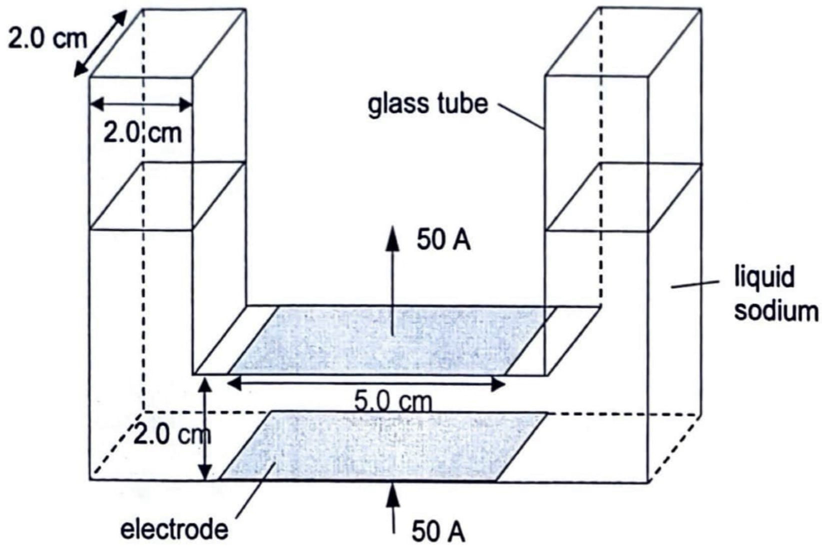
(b) If the current is supplied by a battery of constant e.m.f. and negligible internal resistance, discuss the effect on x if the coil is replaced by one wound with similar wire but having

(i) sides of length L with $2N$ turns, [3]

(ii) N turns with sides of length $\frac{L}{2}$. [2]

[J87/P3/Q12]

D2 **(a)** A glass U-tube is constructed from hollow tubing having a square cross-section of side 2.0 cm, as shown in the figure below.

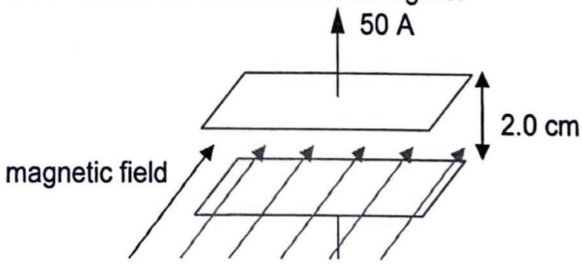


The U-tube has vertical arms and a horizontal section between the arms. Electrodes are set into the upper and lower faces of the horizontal section. Each electrode is of length 5.0 cm and width 2.0 cm. The U-tube contains liquid sodium of density 9.6×10^2 kg m^{-3} and of resistivity 4.8×10^{-8} Ω m.

(i) In these calculations, you may assume that the liquid sodium outside the electrodes has no effect on the resistance between the electrodes. Calculate

1. the resistance of the liquid sodium between the electrodes,
2. the potential difference between the electrodes required to maintain a current of 50 A in the liquid sodium. [4]

(ii) A uniform horizontal magnetic field of flux density 0.12 T is now applied at right angles to the axis of the horizontal section of the tube in the region between the electrodes as shown in the figure.



A force is exerted on the liquid due to the magnetic field. For this force,
1. state and explain its direction,
2. calculate its magnitude. [4]

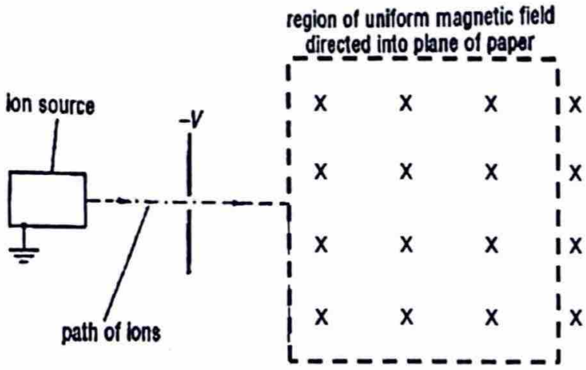
(iii) By considering the pressure difference which the force in **(ii)** causes, determine the difference in height of the surfaces of the liquid sodium in the vertical arms of the U-tube. [4]

(b) The technique outlined in **(a)** has been used as a means of pumping liquids. Suggest one advantage and one disadvantage of the technique when compared with conventional mechanical pumps. [2]

[N97/P3/4(part)]

Force on a Charge in a Magnetic Field

D3 Ions having charge +Q and mass M are accelerated from rest through a potential difference V. They then move into a region of space where there is a uniform magnetic field of flux density B, acting at right angles to the direction of travel of the ions, as shown in the figure.



(a) Show that v, the speed with which the ions enter the magnetic field, is given by

$$v = \sqrt{\frac{2QV}{M}}$$

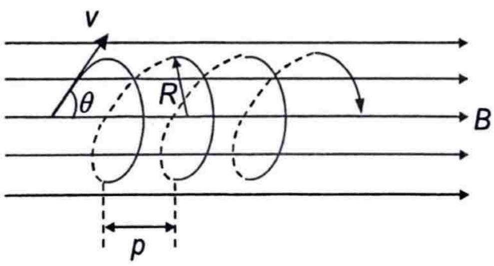
[3]

(b) Hence, derive an expression, in terms of M, Q, B and V, for the radius of the path of the ion in the magnetic field. [2]

(c) Briefly describe and explain any change in the path in the magnetic field of an ion of twice the specific charge (i.e. for which the ratio $\frac{Q}{M}$ is doubled). [2]

[J92/P3/5(part)]

D4 A uniform magnetic field B of flux density 3.0×10^{-5} T is directed along the positive x-axis. An electron is injected at a speed v of 6.7×10^{6} m s^{-1} and an angle θ of 40° to the x-axis. It describes a helical path as shown in the figure.



(a) Calculate

(i) the radius R of the helical path, [3]

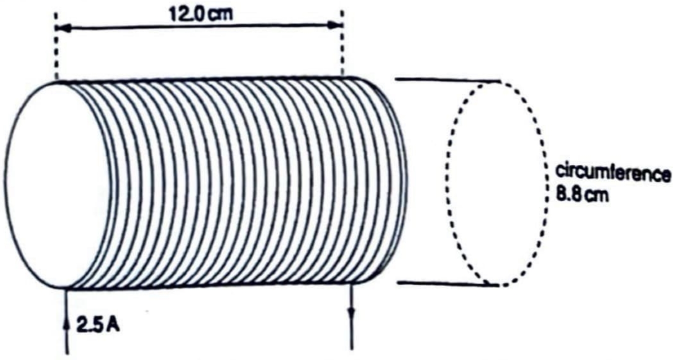
(ii) the time T for the electron to complete one revolution in the helix, [2]

(iii) the pitch p of the helix. [2]

(b) Deduce the pitch of the helix if the angle θ were very small. [2]

D5 A student makes a solenoid with insulated copper wire.

The solenoid has length 12.0 cm and the average length of one turn of wire on the solenoid is 8.8 cm, as shown in the figure below.



The copper wire has a circular cross-section of diameter 0.60 mm. The resistivity of copper is 1.6×10^{-8} Ω m.

It is found that the current in the solenoid is 2.5 A when the potential difference across its terminals is 4.5 V.

(a) **(i)** Calculate, for the solenoid, the resistance of the wire. [1]

(ii) Use your answer in **(i)** to calculate the total number of turns of wire on the solenoid. [3]

(iii) Use your answer in **(ii)** to show that the number of turns per metre length of the solenoid is 3000. [1]

(b) The magnetic flux density *B* (in tesla) inside the solenoid and parallel to its axis is given by the expression

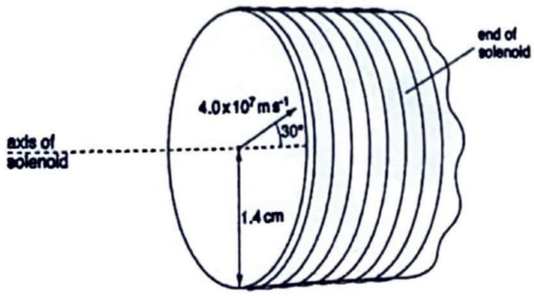
$$B = \mu_0 n I,$$

where *n* is the number of turns per metre length of the solenoid, I is the current in the solenoid expressed in amperes and μ_0 is the permeability of free space.

Calculate the magnetic flux density in the solenoid. [1]

(c) The solenoid in **(b)** is in a vacuum.

An electron is injected into the magnetic field of the solenoid with a speed of 4.0×10^7 m s^{-1} at an angle of 30° to its axis, as shown.



Calculate the magnitude of the component of the electron's velocity

(i) along the axis of the solenoid, [1]

(ii) normal to the axis of the solenoid. [1]

(d) A particle of mass m and charge q is moving with speed v normal to a magnetic field of flux density B.

Show that the particle will move in a circular path of radius r given by the expression

$$r = \frac{mv}{Bq}.$$

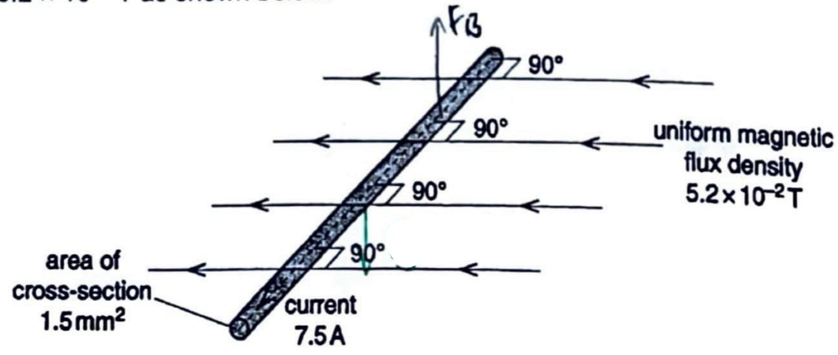
Explain your working. [3]

(e) The radius of the cross-section of the solenoid in **(c)** is 1.4 cm.

Use data from **(c)** and **(d)** to determine quantitatively whether the electron will travel down the length of the solenoid or will collide with its wall. [3]

[N2009/P3/2]

D6 A long straight copper wire is placed at an angle of 90 ° to a uniform magnetic field of flux density 5.2×10^{-2} T as shown below.



The current in the wire is 7.5 A.
There is a force on the current-carrying wire due to the magnetic field.

(a) For this force,

(i) on the figure above, mark its direction with an arrow, [1]

(ii) calculate the force per unit length on the wire. [2]

(b) The current in the wire is a movement of free electrons along the wire. The electrons may be assumed to be moving with speed *v* along the wire.
The number of free electrons per unit volume of the wire is 7.8×10^{28} m^{-3}.
The area of cross-section of the wire is 1.5 mm^2.

The force on the wire is equal to the total force on the free electrons as they move along the wire.

For a free electron that moves along the wire in the magnetic field,

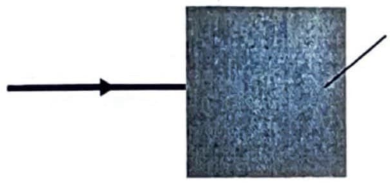
(i) use your answer in **(a)(ii)** to show that the magnitude of the force on each free electron is 3.3×10^{-24} N. [2]

(ii) determine the speed *v* of the free electron. [2]

[N2013/P3/3]

Applications involving motion of charged particles in uniform magnetic and electric fields

D7 **(a)** Neon ions (with charge +1.60 × 10^{-19} C and mass 3.32 × 10^{-26} kg) are accelerated from rest in a vacuum through a potential difference of 1400 V. They are then injected into a region of space where there are uniform electric and magnetic fields acting at right angles to the original direction of motion of the ions.



The electric field has field strength E and the flux density of the magnetic field is B.

(i) Copy the figure above and on your diagram indicate clearly the directions of the electric and magnetic fields so that the ions pass undeflected through the region. [2]

(ii) Calculate the speed of the accelerated ions on entry into the region of the electric and magnetic fields. [2]

(iii) The electric field strength E is 6.2 × 10^3 V m^{-1}. Calculate the magnitude of the magnetic flux density B so that the ions are not deflected in the region of the fields. [3]

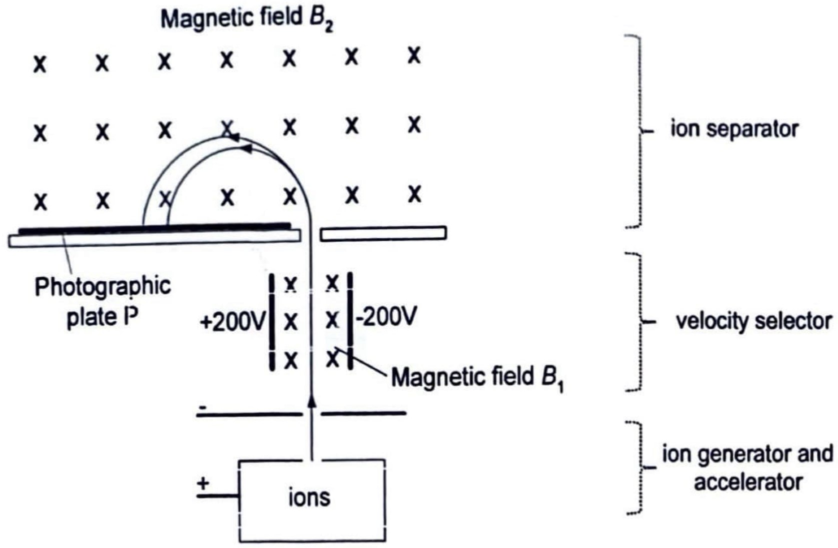
(b) The mechanism by which the neon atoms are ionized is changed so that each atom loses two electrons instead of one. State what change occurs in

(i) the speed of the ions entering the region of the electric and magnetic fields in **(a)**, [2]

(ii) the path of the ions in the two fields. [1]

[J99/P3/6(part)]

D8 The diagram shows a mass spectrometer used for separation of isotopes. It consists of an ion generator and accelerator, a velocity selector and an ion separator, all in a vacuum.



In one experiment, tin ions, each carrying a charge of +1.6 × 10^{-19} C, are produced in the ion generator and are then accelerated by a potential difference of 10000 V. Tin has a number of isotopes*, two of which are tin-118 (^{118}Sn) and tin-120 (^{120}Sn). The mass of the two isotopes are 2.01 × 10^{-25} kg and 2.04 × 10^{-25} kg respectively.

(a) Calculate the final speed after acceleration of an ^{120}Sn ion if it were at rest when generated. [2]

(b) In practice, all ions produced by the ion generator have a range of speeds. A velocity selector is used to isolate ions with a single speed.

(i) The plates producing the electric field have a separation of 2.0 cm. The potentials of the plates are marked in the figure above. Calculate the magnitude of the force on an ion due to this electric field in the velocity selector. [2]

(ii) Write down the equation which must be satisfied if the ions are to emerge from the exit hole of the velocity selector. Define the terms in the equation. [2]

(iii) Calculate the magnetic flux density B_1 required if ions traveling with a speed of 177 km s^{-1} are to be selected. [2]

(c) After selection of ions with the speed of 177 km s^{-1}, the ions are separated using only a magnetic field B_2, as shown in the diagram.

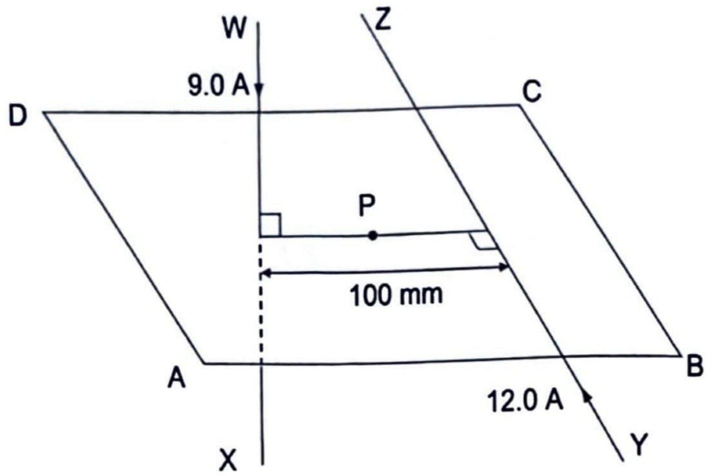
(i) Explain why the ions move in circular paths in this region. [2]

(ii) Show that the radius of the path is directly proportional to the mass of the ion. [2]

(iii) The ions are detected using the photographic plate P. Determine the distance between the points of impact on P of the two isotopes of tin when the magnetic flux density B_2 is 0.75 T. [2]

*Note: Isotopes are different forms of the same element which have the same number of protons but different number of neutrons in their nuclei.

Challenging Question



C1 The figure above shows a long straight, vertical wire WX, carrying a current of 9.0 A downwards. A second long, straight wire YZ is placed horizontally and carries a current of 12.0 A in the direction shown. ABCD is a horizontal, rectangular table-top: the wire YZ is parallel to the side BC of this table, and the wire WX passes through a small hole in the table. The perpendicular distance between the wires is 100 mm. P is the point 50 mm from YZ along the perpendicular between the wires.

(a) Calculate the magnitude and direction of the magnetic flux density at the point P. (Relate the direction of the flux density to the sides AB and BC of the table-top, as appropriate.)

(b) The wire YZ is to remain fixed in position, but the orientation and position of the wire WX can be changed. The currents in the two wires remain at the same values as before, and the point P remains 50 mm away from YZ in the plane ABCD. How should WX be arranged so as to produce zero magnetic flux density at P?

(Again relate the required position of WX to the table-top ABCD.)

[N93/P0/Q7]

Answers

D2 **(a)** **(i)** **1.** $9.6 \times 10^{-7}\,\Omega$ **2.** 4.8×10^{-5} V
(ii) **2.** 0.12 N **(iii)** 3.2 cm
D4 **(a)** **(i)** 0.817 m **(ii)** 1.19×10^{-6} s **(iii)** 6.12 m
(b) 7.99 m
D5 **(a)** **(i)** $1.8\,\Omega$ **(ii)** 360 turns **(b)** 9.4×10^{-3} T
(c) **(i)** 3.5×10^{7} m s^{-1} **(ii)** 2.0×10^{7} m s^{-1}
D6 **(a)** **(ii)** 0.39 N m^{-1} **(b)(ii)** 4.0×10^{-4} m s^{-1}
D7 **(a)** **(ii)** 1.16×10^{5} m s^{-1} **(iii)** 0.0534 T

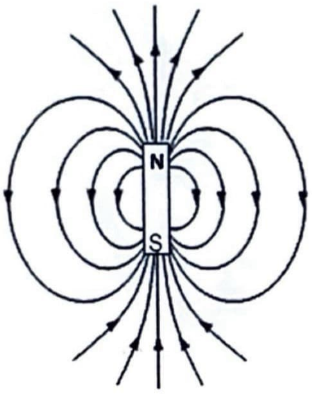
D8 **(a)** 1.25×10^{5} m s^{-1}
(b) **(i)** 3.2×10^{-15} N **(iii)** 0.113 T
(c) **(iii)** 8.85×10^{-3} m
C1 **(a)** 6.0×10^{-5} T, 53° upward from the horizontal plane
(b) WX should be placed 87.5 mm from YZ, parallel to YZ with current in the same direction as YZ

Suggested Solutions

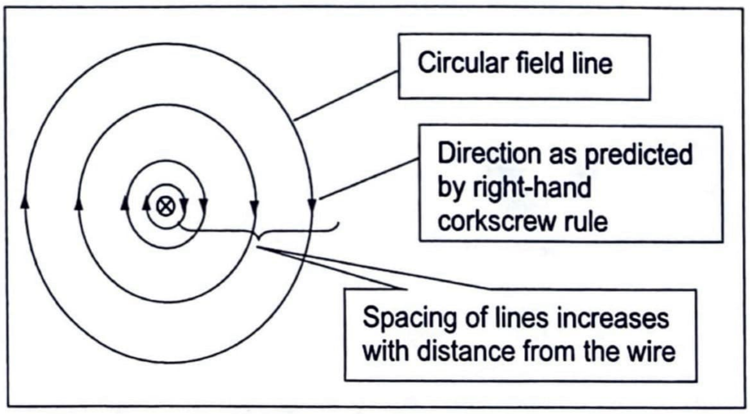
Self-Check Questions

S1 Flux pattern must show clearly and correctly
(1) Shape of field lines
(2) Direction of the field lines
(3) Spacing between field lines

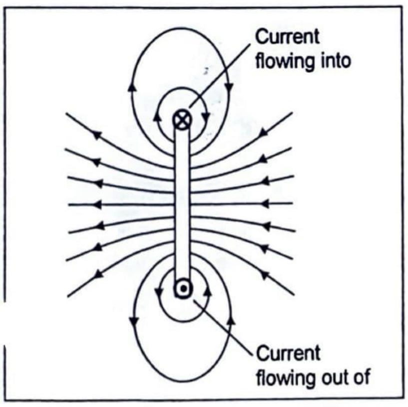
E.g. For flux pattern around a long straight wire: (1) circular field lines (2) direction as predicted by right-hand corkscrew rule and (3) spacing of lines increases with distance from the wire.



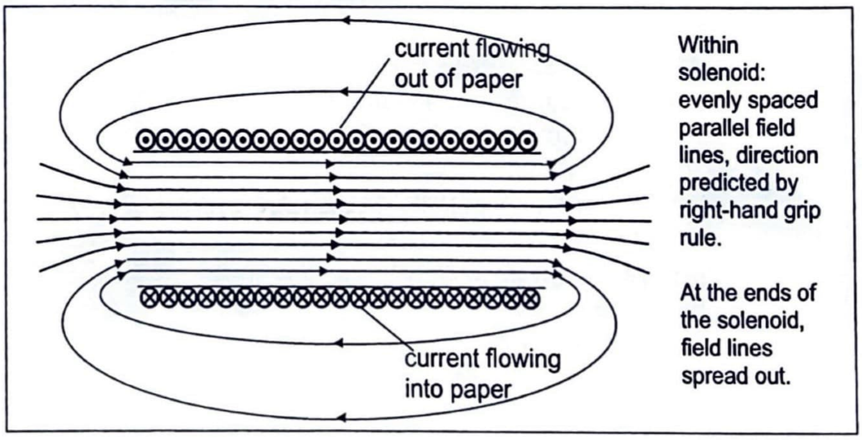
(a) Bar magnet



(b) Long straight wire



(c) flat circular coil



(d) long solenoid

S2 $F = BIL\sin\theta$

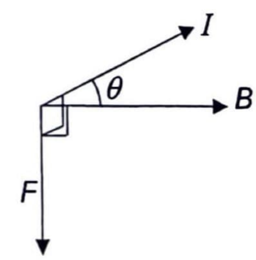
where F is the magnetic force on the conductor

B is the magnetic flux density of the magnetic field

I is the current in the conductor

L is the length of the conductor in the field

θ is the angle the conductor makes with the magnetic field.

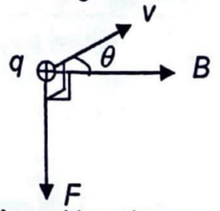


S3 The **magnetic flux density** of a magnetic field is numerically equal to the force per unit length per unit current acting on a long straight conductor at right angles to the magnetic field.

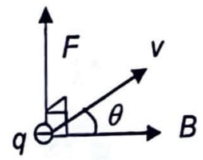
S4 **(a)** The wires will each experience a force of attraction towards each other.
(b) The wires will each experience a force of repulsion from each other.

In both cases, the force that each wire experiences is equal in magnitude but opposite in direction to that experienced by the other wire. The magnitude of the force per unit length is given by $\frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi r}$.

S5 $F = Bqv\sin\theta$
where F is the magnetic force on the charged particle
B is the magnetic flux density of the magnetic field
q is the charge of the particle
v is the velocity of the particle
θ is the angle the velocity of the particle makes with the magnetic field.



(a) positive charge



(b) negative charge

S6 **(a)** When a charged particle enters at a right angle to an electric field, it experiences a constant electric force that is perpendicular to the direction of its initial velocity. The particle will move in a parabolic path (similar to projectile motion in gravitational field).

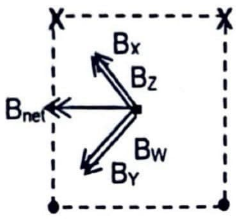
(b) When a charged particle enters at a right angle into a uniform magnetic field, it experiences a magnetic force that is at all times perpendicular to the velocity and constant in magnitude. The particle will move in a circular path with constant speed.

S7 When a charged particle enters at an angle θ to a uniform magnetic field such that $0° < \theta < 90°$, the component of its velocity perpendicular to the field, $v_\perp$, will cause it to move in a circular path with uniform speed $v_\perp$. The component of the velocity parallel to the magnetic field, $v_{//}$, remains unchanged and projects the charge forward (in a direction parallel to the field). The resultant path would be a helix.

S8 Refer to lecture notes page 23.

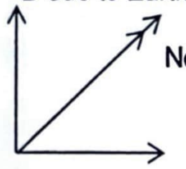
Self-Practice Questions

SP1



In order for the resultant magnetic field at O to be leftwards, the direction of the magnetic field at O due to each wire must be as shown (using the Right Hand Rule)
Answer: A

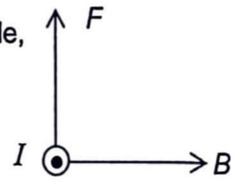
SP2



Answer: A

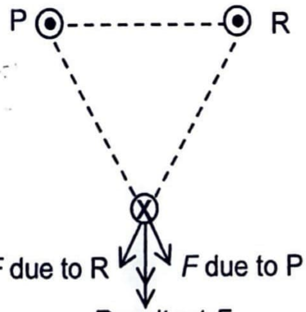
SP3 Using Fleming's Left Hand Rule.
Answer: D

SP4 Using Fleming's Left Hand Rule,



Answer: D

SP5



Answer: C

SP6

$$F_{X\text{ on }Z} = B_X I_Z L = \left(\frac{\mu_o I_X}{2\pi d}\right) I_Z L \qquad \therefore \frac{F}{L} = \left(\frac{\mu_o}{2\pi}\right)\left(\frac{I_X I_Z}{d}\right)$$

Since the currents are all in the same direction, the forces acting on Z due to X and Y are attractive.

Net force on Z = $F_{Y\text{ on }Z} + F_{X\text{ on }Z} = 2\,(2 \times 10^{-6}) + \left(\frac{2\times10^{-6}}{2}\right) = 5 \times 10^{-6}\ \text{N m}^{-1}$

Answer : E

SP7 Using Fleming's Left Hand rule, the electrons would experience a leftwards force due to the presence of the magnetic field. Hence they will be deflected in the direction A.
Answer : A

SP8

$$F_B = F_C$$

$$Bqv = \frac{mv^2}{r}$$

$$r = \frac{mv}{Bq}$$

The only possible explanation (based on the options given) for the decreasing radius is that the speed of the particle is decreasing.

Answer : E

SP9 The electric field deflects the electrons upwards while the magnetic field deflects the electrons to the left. Hence, the spot on the screen will be on the upper left quadrant (as viewed from the front of the CRO).

Answer : A

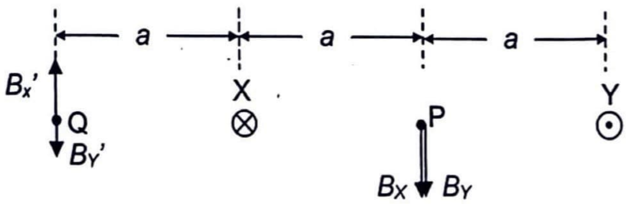
SP10

Inside the region of the magnetic field, $r = \frac{mv}{Bq}$

Since the ions that pass out of the velocity selector will have the same velocity, and the magnetic field is uniform, in order to be deflected to the same point P (ie, to have the same *r*), the particles must have the same $\frac{m}{q}$ or $\frac{q}{m}$ ratio.

Answer : B

SP11



(a) **(i)** At mid-point P between the wires, $B_X = B_Y = \frac{\mu_0 I}{2\pi a}$

$\because \overline{B_X}$ and $\overline{B_Y}$ are in the same direction,

resultant field $= B_X + B_Y = \frac{\mu_0 I}{\pi a}$

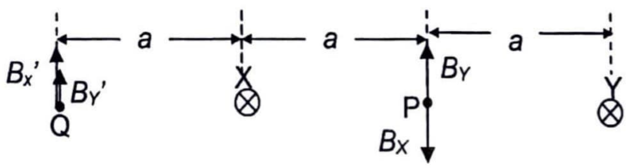
(ii) At a point *a* from one and 3*a* from the other, such as point Q,

$B_X' = \frac{\mu_0 I}{2\pi a}$; $B_Y' = \frac{\mu_0 I}{2\pi(3a)}$

$\because \overline{B_X'}$ and $\overline{B_Y'}$ are in opposite direction,

resultant field $= B_X - B_Y = \frac{\mu_0 I}{2\pi a}\left(1 - \frac{1}{3}\right) = \frac{\mu_0 I}{3\pi a}$

(b)



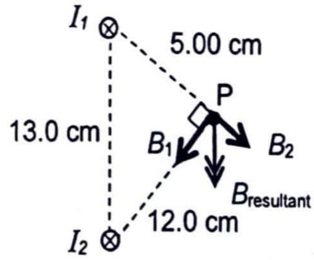
(i) At P, the two fields cancel out.

(ii) At Q, $B_X' = \dfrac{\mu_0 I}{2\pi a}$; $B_Y' = \dfrac{\mu_0 I}{2\pi(3a)}$

$\because \overrightarrow{B_X'}$ and $\overrightarrow{B_Y'}$ are in the same direction,

resultant field $= B_X + B_Y = \dfrac{\mu_0 I}{2\pi a}\left(1+\dfrac{1}{3}\right) = \dfrac{2\mu_0 I}{3\pi a}$

SP12



$$B_{resultant} = \sqrt{B_1^2 + B_2^2}$$
$$= \sqrt{\left(\frac{\mu_0 I_1}{2\pi x_1}\right)^2 + \left(\frac{\mu_0 I_2}{2\pi x_2}\right)^2}$$
$$= \sqrt{\left(\frac{\mu_o(3.00)}{2\pi(0.120)}\right)^2 + \left(\frac{\mu_o(3.00)}{2\pi(0.050)}\right)^2}$$
$$= \frac{\mu_o(3.00)}{2\pi}\sqrt{\left(\frac{1}{0.120}\right)^2 + \left(\frac{1}{0.050}\right)^2}$$

$= 1.3\times10^{-5}$ T, vertically down (use trigo to prove for yourself)

SP13 $\dfrac{\mu_o N_X I_X}{2 r_X} = \dfrac{\mu_o N_Y I_Y}{2 r_Y} \Rightarrow I_Y = \left(\dfrac{r_Y}{r_X}\right)\left(\dfrac{N_X}{N_Y}\right) I_X = \left(\dfrac{4.0}{8.0}\right)\left(\dfrac{15}{10}\right)1.2 = 0.90$ A

SP14 (a) $B = \mu_o n I = 4\pi \times 10^{-7} \times 1500 \times 2.0 = 3.8 \times 10^{-3}$ T

(b) Since the magnetic field and the current in the conductor are parallel to each other, there is no magnetic force acting on the conductor.

SP15 (a) (i) $F_g = m_p g = 1.67\times10^{-27}\times 9.81 = 1.64\times10^{-26}$ N, in the direction of the field

(ii) $F_e = qE = 1.60\times10^{-19}\times 1.50\times10^{6} = 2.40\times10^{-13}$ N, in the direction of the field

(iii) $F_B = Bev = 0.125\times1.60\times10^{-19}\times7.00\times10^{6} = 1.40\times10^{-13}$ N, at right angles to velocity and field

(b) F_g and F_e unaffected. $F_B = 0$.