Name:	Index No.:	Class:

PRESBYTERIAN HIGH SCHOOL

ADDITIONAL MATHEMATICS **4049/02**
Paper 2

21 August 2023 Monday 2 hours 15 mins

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2023 SECONDARY FOUR EXPRESS / FIVE NORMAL (ACADEMIC) PRELIMINARY EXAMINATIONS

DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

INSTRUCTIONS TO CANDIDATES
Write your name, index number and class in the spaces provided above.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided below the questions.
Give non-exact numerical answers correct to 3 significant figures or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

For Examiner's Use													
Qn	1	2	3	4	5	6	7	8	9	10				*Marks Deducted*
Marks														

Category	Accuracy	Units	Symbols	Others
Question No.				

TOTAL MARKS
90

Setter: Tan Chee Wee
Vetter: Tan Lip Sing

This question paper consists of **21** printed pages and **1** blank page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

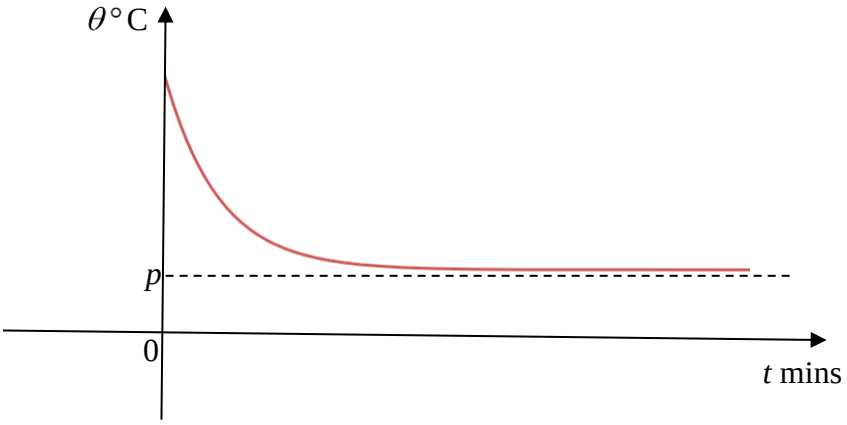
1 An object is heated in an oven until it reaches a temperature of X °C. It is then allowed to cool. Its temperature, θ °C, when it has cooled for time t minutes, is given by $\theta = 30 + 100(0.8)^{\frac{t}{6}}$.

(a) Find the value of X. [1]

(b) Find the value of θ when $t = 8$. [1]

(c) Find the value of t when $\theta = 95$. [3]

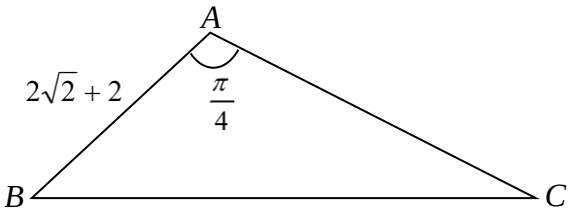
(d) A sketch of the graph of θ against t is given below.



State the value of p. [1]

A calculator must not be used in this question.

2 **(a)** In the diagram, triangle ABC has an area of $\left(8\sqrt{2}+4\right)$ cm^2, angle $BAC = \frac{\pi}{4}$ radian and $AB = \left(2\sqrt{2}+2\right)$ cm. Find the length of AC, leaving your answer in the form $\left(p\sqrt{2}+q\right)$ cm, where p and q are integers. [5]



(b) Find $\cos 75°$, giving your answer in the form $\frac{\sqrt{a}-\sqrt{b}}{4}$, where a and b are integers. [3]

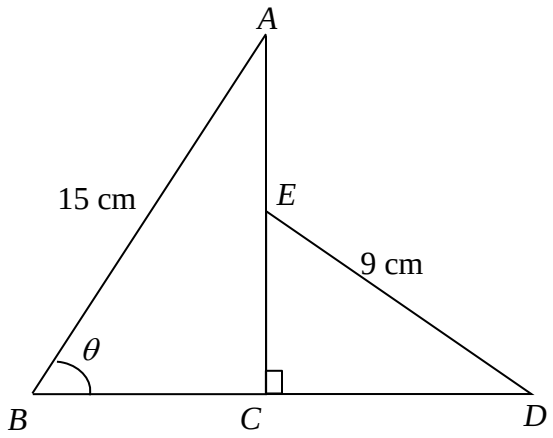
3 (**a**) Prove that $\text{cosec}2x - \cot 2x = \tan x$. [3]

(b) Hence solve $\text{cosec}2x - \cot 2x = 2\sec^2 x - 3$ for $0° \leq x \leq 360°$. [5]

4 **(a)** Solve $9^x + 5 = 2\left(3^{x+1}\right)$. [5]

(b) Solve $2\log_4\left[\log_{100}(x^2+9)-\log_{100} x\right]=-1.$ [5]

5 The diagram shows a quadrilateral *ABCDE* where triangle *ABC* is similar to triangle *DEC*. *AB* = 15 cm, *DE* = 9 cm, angle *ACD* = 90 ^{o} and angle *ABC* is a variable angle θ, where $0^{o} < \theta < 90^{o}$.

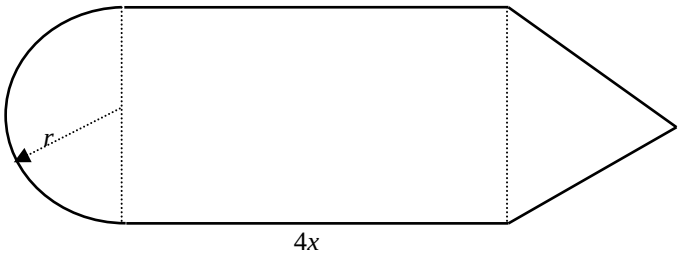


(a) Show that the perimeter, *P* cm, of the quadrilateral is given by $P = 24 + 24\sin\theta + 6\cos\theta$. [4]

(b) Express P in the form $R\sin(\theta+\alpha)+k$. [4]

(c) Find the value of θ when the perimeter is 38 cm. [2]

6 A piece of wire 60 cm long is bent to form the shape shown in the figure. This shape consists of a semi-circular arc, radius, r cm, and an equilateral triangle on the opposite ends of a rectangle of length $4x$ cm.



(a) Express x in term of r. [2]

(b) Hence show that the area enclosed, A cm^2, is given by

$$A = 60r + r^2(\sqrt{3} - 4 - \frac{\pi}{2}).$$ [3]

(c) Calculate the value of r for which A has a stationary value. Find this value of A and determine whether it is a maximum or a minimum. [5]

7 The equation of the curve is $y = (2x+1)\left(\sqrt{x-3}\right)$.

(a) Show that $\frac{dy}{dx}$ can be written in the form $\frac{6x-11}{2\sqrt{x-3}}$. [4]

(b) A particle moves along the curve in such a way that the x-coordinate is increasing at a constant rate of 3 units per second. Find the rate of change of y when $x = 7$. [2]

(c) Use the result from **(a)** to evaluate $\int_4^7 \frac{3(6x-11)}{\sqrt{x-3}}\,dx$. [4]

8 **(a)** Factorise $x^3 - 27k^3$ as a product of a linear and a quadratic factor. [2]

(b) Factorise $x^2 - (3k - 1)x - 3k$. [1]

(c) The equation $x^3 - 27k^3 = x^2 - (3k-1)x - 3k$ has only 1 real root. Find the set of values of the constant k. [6]

9 The equation of the circle, C, is $x^2 + y^2 - 6x + 10y - 66 = 0$.

(a) Find the coordinates of the centre of C and the radius of C. [4]

(b) Write down an equation of a vertical tangent to the circle. [1]

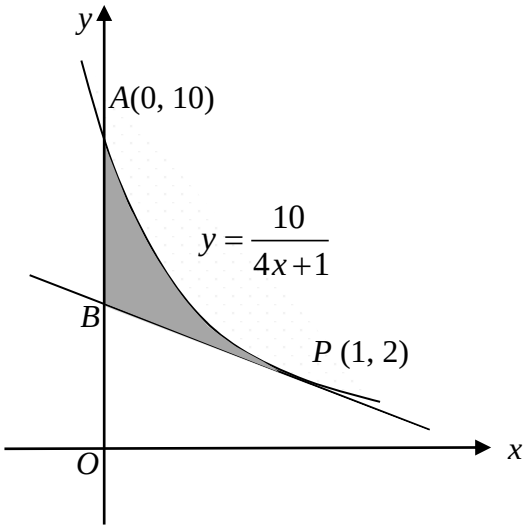
The point $A(-5, 1)$ lies on the circle.

(c) Find the equation of the tangent to the circle at point A. [3]

(d) AB is the diameter of the circle and P is the point $(0, 6)$. Explain why the angle APB is an acute angle. [2]

10 The diagram shows part of the curve $y = \frac{10}{4x+1}$ intersecting the y-axis at $A(0, 10)$.

The tangent to the curve at the point $P(1, 2)$ intersects the y-axis at B.



(a) Show that the coordinates of B is (0, 3.6). [4]

(b) Find the **exact** area of the shaded region. [5]

END OF PAPER

Answer Key

1a 130

b 104

c 11.6

d 30

2a $12\sqrt{2}-8$

b $\dfrac{\sqrt{6}-\sqrt{2}}{4}$

3b $45°, 153.4°, 225°, 333.4°$

4a $0, 1.46$

b $1, 9$

5b $\sqrt{612}\sin(\theta+14.0°)+24$

c $20.4°$

6a $x=\dfrac{60-4r-\pi r}{8}$

c $r=7.82$, $A=234$, A is a maximum value.

7b 23.25 unit/s

7c 126

8a $(x-3k)\left(x^2+3kx+9k^2\right)$

b $(x+1)(x-3k)$

c $k<-\dfrac{5}{9}$ *or* $k>\dfrac{1}{3}$

9a $(3,-5)$, 10 unit

b $x=-7$ or $x=13$

c $3y=4x+23$

10b $\dfrac{5}{2}\ln 5-2.8$ unit ^{2}