Solutions to 2023 TJC Prelim Paper 1

Solution to Q1

(a) $\int x\tan^{-1}x\,\mathrm{d}x$

$$= \tan^{-1}x\left(\frac{x^2}{2}\right) - \int \frac{1}{1+x^2}\left(\frac{x^2}{2}\right)\mathrm{d}x$$

$$= \tan^{-1}x\left(\frac{x^2}{2}\right) - \frac{1}{2}\int 1 - \frac{1}{1+x^2}\,\mathrm{d}x$$

$$= \tan^{-1}x\left(\frac{x^2}{2}\right) - \frac{1}{2}\left[x - \tan^{-1}x\right] + c$$

(b) $\int \sin 2x\cos 5x\,\mathrm{d}x$

$$= \frac{1}{2}\int \sin 7x - \sin 3x\,\mathrm{d}x$$

$$= \frac{1}{2}\left(-\frac{1}{7}\cos 7x + \frac{1}{3}\cos 3x\right) + C$$

$$= -\frac{1}{14}\cos 7x + \frac{1}{6}\cos 3x + C$$

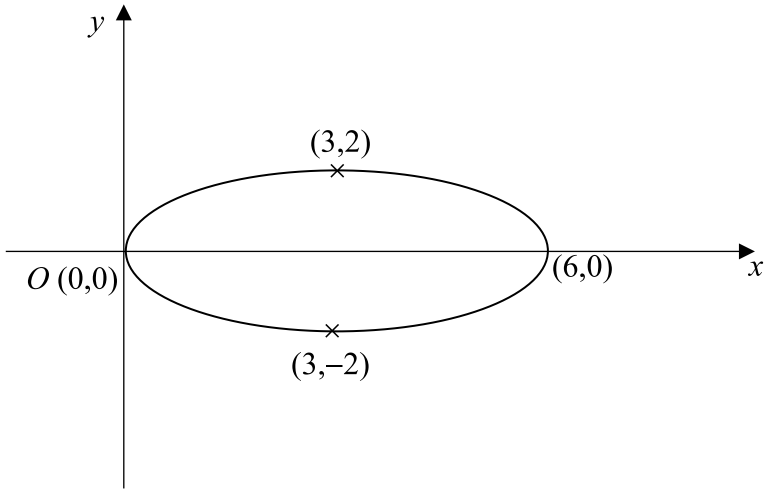
Solution to Q2

(a) $4x^2 - 24x + 9y^2 = 0$.

$$4\left(x^2 - 6x\right) + 9y^2 = 0$$

$$4\left(x-3\right)^2 + 9y^2 = 36$$

$$\frac{\left(x-3\right)^2}{3^2} + \frac{y^2}{2^2} = 1$$



(b)

The required sequence of transformations is

A: A translation of magnitude 3 unit in the negative direction of the x-axis.
B: A translation of magnitude 1 unit in the positive direction of the y-axis.
C: A scaling parallel to the y-axis by a scale factor of $\frac{1}{\alpha}$.

OR

A: A translation of magnitude 3 unit in the negative direction of the x-axis.
B: A **scaling parallel to the *y*-axis** by a scale factor of $\frac{1}{\alpha}$.
C: A **translation of magnitude** $\frac{1}{\alpha}$ unit in the positive direction of the y-axis.

(c) $$\frac{x^2}{9} + \frac{\left(\alpha y - 1\right)^2}{4} = 1$$

$$\frac{x^2}{3^2} + \frac{\left(y - 1/\alpha\right)^2}{\left(2/\alpha\right)^2} = 1$$

For shape of a circle, $\alpha = \frac{2}{3}$

Solution to Q3

(a) $$g(x)=\frac{x}{\sqrt{4-x}}$$

$$=x(4-x)^{-\frac{1}{2}}$$

$$=4^{-\frac{1}{2}}x\left(1-\frac{x}{4}\right)^{-\frac{1}{2}}$$

$$=\frac{1}{2}x\left(1-\frac{x}{4}\right)^{-\frac{1}{2}}$$

$$=\frac{1}{2}x\left(1+\left(-\frac{1}{2}\right)\left(-\frac{x}{4}\right)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2!}\left(-\frac{x}{4}\right)^{2}+\cdots\right)$$

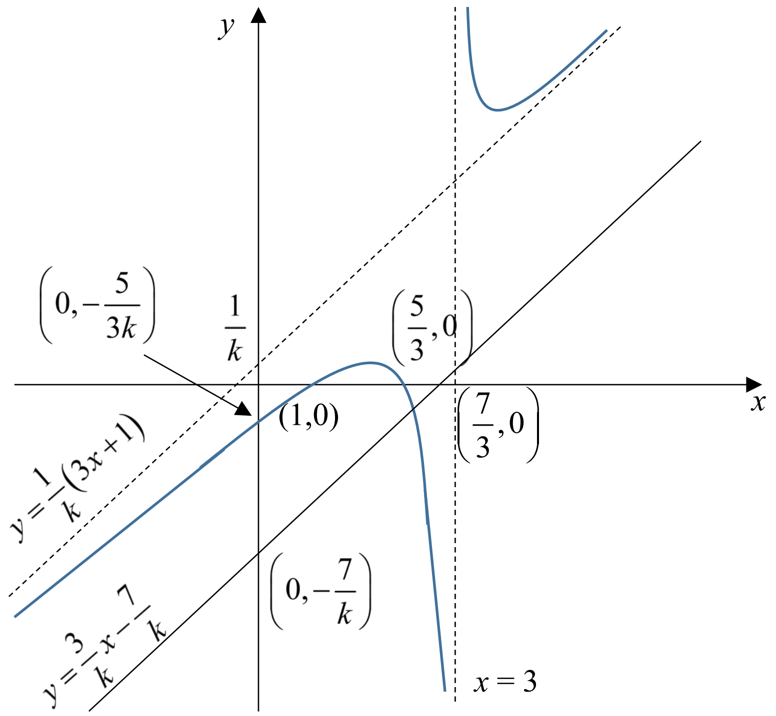
$$=\frac{1}{2}x+\frac{1}{16}x^{2}+\frac{3}{256}x^{3}+\cdots$$

Therefore $a=\frac{1}{2},\quad b=\frac{1}{16},\quad c=\frac{3}{256}$

(b) Percentage error $=\left|\frac{f(x)-g(x)}{g(x)}\right|\times 100\% < 4\%$

$$\Rightarrow \left|\frac{\left(\frac{1}{2}x+\frac{1}{16}x^{2}+\frac{3}{256}x^{3}\right)-\frac{x}{\sqrt{4-x}}}{\frac{x}{\sqrt{4-x}}}\right|<0.04$$

Using GC, $\Rightarrow\quad 0<x<1.87$ (corr. to 3 s.f.)

Solution to Q4
(a),(b) $y=\dfrac{-3x^2+8x-5}{k(3-x)}=\dfrac{1}{k}\left(3x+1-\dfrac{8}{3-x}\right)$ When $x=0$, $y=-\dfrac{5}{3k}$ When $y=0$, $-3x^2+8x-5=(-3x+5)(x-1)=0 \Rightarrow x=1$ or $\dfrac{5}{3}$ Equation of asymptotes: $y=\dfrac{1}{k}(3x+1)$ and $x=3$ 
(c) $\dfrac{1}{k}\left(3x+1-\dfrac{8}{3-x}\right)=\dfrac{3}{k}x-\dfrac{7}{k}$ $3x+1-\dfrac{8}{3-x}=3x-7$ $-\dfrac{8}{3-x}=-8$ $3-x=1 \Rightarrow x=2$ Therefore, from the graph, $x\le 2$ or $x>3$

Solution to Q5

(a) First term of AP: $a = 500$

Common difference of AP: $d = 10$

Formulation of problem: $\frac{n}{2}\left[2(500)+10(n-1)\right] > 10000$

Using GC (table of values to be shown), $n = 18$

Date of 18th month: 1 June 2024

(b) Formulation of problem:

Month n	Start of month	End of month
1	X	$1.005x$
2	$1.005x + x$	$1.005^2 x + 1.005x$
3	$1.005^2 x + 1.005x + x$	$1.005^3 x + 1.005^2 x + 1.005x$
$\vdots$	$\vdots$	$\vdots$

At the end of Nth month, account has

$$\begin{aligned} 1.005^N x + 1.005^{N-1} x + \ldots + 1.005x &= x(1.005^N + 1.005^{N-1} + \ldots + 1.005) \\ &= x\left[\frac{1.005(1.005^N - 1)}{1.005 - 1}\right] \\ &= 201x(1.005^N - 1) \end{aligned}$$

Thus we have $N = 60$ at the end of 31 December 2027

$201x(1.005^{60} - 1) \geq 50,000 \quad \Rightarrow \quad x \geq \$713.0747 \quad \Rightarrow \quad$ Least $x = \$714$

Solution to Q6
(a) $A=\frac{1}{2}(PQ)(PR)\sin\angle QPR$ $=\frac{1}{4}(x+1)(4-x)^2$ $=\frac{1}{4}(x+1)(16-8x+x^2)$ $=\frac{1}{4}(16x-8x^2+x^3+16-8x+x^2)$ $=\frac{1}{4}(x^3-7x^2+8x+16)$ (Shown) Or let N be the foot of perpendicular from Q to PR. $QN=PQ\sin 30^\circ=\frac{1}{2}(x+1)$ $A=\frac{1}{2}(QN)(PR)$ $=\frac{1}{4}(x+1)(4-x)^2$ $=\frac{1}{4}(x+1)(16-8x+x^2)$ $=\frac{1}{4}(16x-8x^2+x^3+16-8x+x^2)$ $=\frac{1}{4}(x^3-7x^2+8x+16)$ (Shown)
(b) $\frac{\mathrm{d}A}{\mathrm{d}x}=\frac{1}{4}(3x^2-14x+8)$ At stationary values, $\frac{\mathrm{d}A}{\mathrm{d}x}=0$ $\Rightarrow\ \frac{1}{4}(3x^2-14x+8)=0$ $\Rightarrow\ x=\frac{2}{3}$ or $x=4$ (rejected since it is given that $x<4$) $\left.\frac{\mathrm{d}^2A}{\mathrm{d}x^2}\right\|_{x=\frac{2}{3}}=\left.\frac{1}{4}(6x-14)\right\|_{x=\frac{2}{3}}=-2.5<0$ (maximum) To find QR: When $x=\frac{2}{3}$, $PQ=\frac{5}{3}$ and $PR=\frac{100}{9}$ Using cosine rule, $QR^2=\left(\frac{5}{3}\right)^2+\left(\frac{100}{9}\right)^2-2\left(\frac{5}{3}\right)\left(\frac{100}{9}\right)\cos 30^\circ$ $\therefore\ QR\approx 9.70$ (3 s.f.)

Or $QN = \frac{1}{2}\left(\frac{2}{3}+1\right) = \frac{5}{6}$

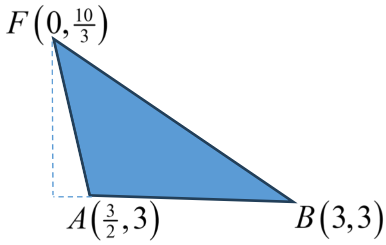
$PN = PQ\cos 30^\circ = \frac{\sqrt{3}}{2}\left(\frac{5}{3}\right) = \frac{5\sqrt{3}}{6}$

$RN = PR - PN = \frac{100}{9} - \frac{5\sqrt{3}}{6}$

$QR = \sqrt{QN^2 + RN^2} = \sqrt{\left(\frac{5}{6}\right)^2 + \left(\frac{100}{9} - \frac{5\sqrt{3}}{6}\right)^2} = 9.70$

Solution to Q7
(a) $\frac{\mathrm{d}x}{\mathrm{d}t}=-\frac{1}{t^2}$, $\frac{\mathrm{d}y}{\mathrm{d}t}=-\frac{8}{t^3}-1$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}t}\div\frac{\mathrm{d}x}{\mathrm{d}t}$ $=\left(-\frac{8}{t^3}-1\right)\left(-t^2\right)$ $=\frac{8}{t}+t^2$ For stationary point, $\frac{\mathrm{d}y}{\mathrm{d}x}=0\Rightarrow\frac{8}{t}+t^2=0$ $\Rightarrow\quad t^3=-8$ $\Rightarrow\quad t=-2$ When $t=-2$, $x=\frac{1}{-2}+2=\frac{3}{2}$, $y=\frac{4}{(-2)^2}+2=3$ Coordinates of A is $\left(\frac{3}{2},3\right)$ Equation of tangent is $y=3$
(b) When $y=3$, $\frac{4}{t^2}-t=3$ $\Rightarrow\quad 4-t^3=3t^2$ $\Rightarrow\quad t^3+3t^2-4=0$ Using GC, $t=1$ or $t=-2$ (Reject $\because t=-2$ is pt A)   [If GC is not allowed, note that t = -2 must be one of the solution since one of the intersection point between the tangent and C is at A $\Rightarrow\quad (t+2)(t^2+t-2)=0$ $\Rightarrow\quad (t+2)^2(t-1)=0$ $\Rightarrow\quad t=-2$ or $t=1$] $\therefore$ coordinates of B is (3, 3).   Gradient of normal at B $=-\frac{1}{\frac{8}{1}+1^2}=-\frac{1}{9}$ Equation of normal is $y-3=-\frac{1}{9}(x-3)$, i.e. $y=-\frac{1}{9}x+\frac{10}{3}$

(c) The point F has coordinates $\left(0, \dfrac{10}{3}\right)$.



Height of triangle $= \dfrac{10}{3} - 3 = \dfrac{1}{3}$

AB has length $3 - \dfrac{3}{2} = \dfrac{3}{2}$

Area of Triangle $= \dfrac{1}{2}\left(\dfrac{1}{3}\right)\left(\dfrac{3}{2}\right) = \dfrac{1}{4}$ unit^2

Solution to Q8

(a)

Let $z = x + y\text{i}$

$(x + y\text{i} + \text{i})^* = 2\text{i}(x + y\text{i}) + 4$

$x - \text{i}(y+1) = 2\text{i}x - 2y + 4$

Comparing real and imaginary part

$x = 4 - 2y \;---(1)$

$-\left(y+1\right) = 2x \;---(2)$

solving:

$-y - 1 = 2(4 - 2y)$

$3y = 9$

$y = 3$

Solving, $x = -2,\ y = 3$

(b)(i)

$|w| = \sqrt{\left(\sqrt{3}\right)^2 + 1} = 2$

$\arg w = -\tan^{-1}\left(\dfrac{1}{\sqrt{3}}\right) = -\dfrac{\pi}{6}$

$w = \sqrt{3} - \text{i} = 2e^{-\frac{\pi}{6}i}$

$\left(\dfrac{z}{w^6}\right)^* = \dfrac{1}{16}e^{\frac{2}{3}\pi i}$

$\dfrac{z}{w^6} = \dfrac{1}{16}e^{-\frac{2}{3}\pi i}$

$z = \dfrac{1}{16}e^{-\frac{2}{3}\pi i}\left(2e^{-\frac{\pi}{6}i}\right)^6$

$= 4e^{-\frac{2}{3}\pi i - \pi i}$

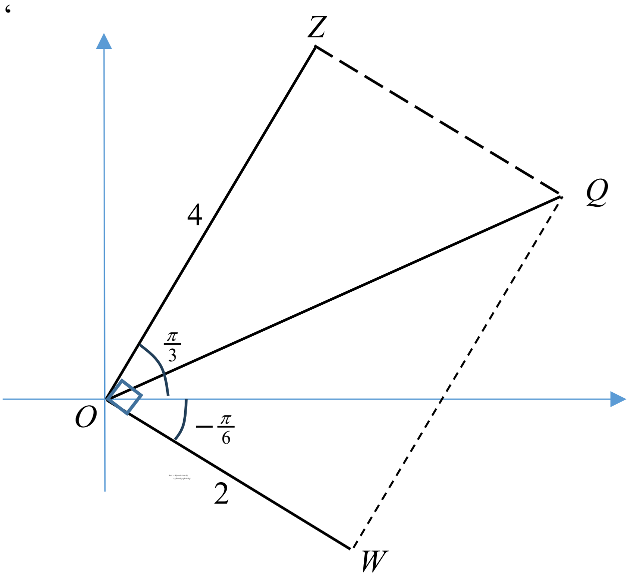
$= 4e^{\frac{1}{3}\pi i}$

$= 4\left(\cos\left(\dfrac{1}{3}\pi\right) + i\sin\left(\dfrac{1}{3}\pi\right)\right)$

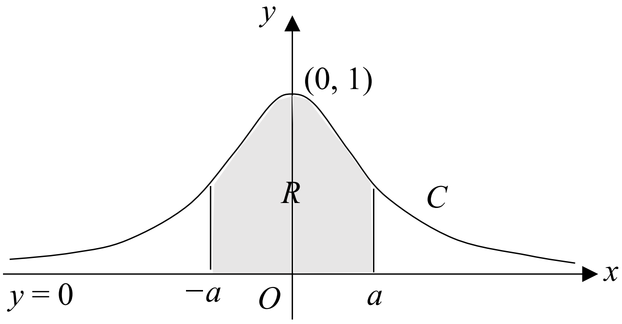
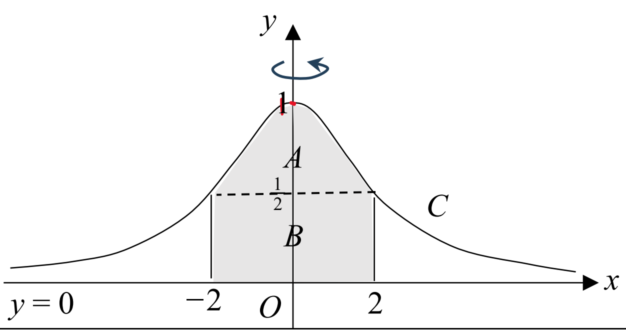
$= 4\left(\dfrac{1}{2} + \dfrac{\sqrt{3}}{2}\text{i}\right)$

$= 2 + 2\sqrt{3}\text{i}$

(b)(ii)
‘



OZQW is a rectangle.

Solution to Q9
(a) 
(b) Area of $R = 2\int_0^a y\,\mathrm{d}x$ $= 2\int_0^{\tan^{-1}\left(\frac{a}{2}\right)} \cos^2\theta\left(2\sec^2\theta\right)\mathrm{d}\theta$ $= \int_0^{\tan^{-1}\left(\frac{a}{2}\right)} 4\,\mathrm{d}\theta$ $= \left[4\theta\right]_0^{\tan^{-1}\left(\frac{a}{2}\right)}$ $= 4\tan^{-1}\left(\frac{a}{2}\right)$ $x = 2\tan\theta$ When $x = a,\ \theta = \tan^{-1}\left(\frac{a}{2}\right)$ When $x = 0,\ \theta = 0$ $4\tan^{-1}\frac{a}{2} = \frac{2\pi}{3}$ $\tan^{-1}\left(\frac{a}{2}\right) = \frac{\pi}{6}$ $\Rightarrow a = \frac{2\sqrt{3}}{3}$
(c) $x = 2\tan\theta$, $y = \cos^2\theta$ $\tan^2\theta = \left(\frac{x}{2}\right)^2$ $\tan^2\theta + 1 = \sec^2\theta$ $\frac{x^2}{4} + 1 = \frac{1}{y}$ $y = \frac{1}{\frac{x^2}{4}+1} = \frac{4}{x^2+4}$
(d) 

When $x = 2 \quad y = \frac{1}{2}$

$x^2 = 4\left(\frac{1}{y} - 1\right)$

Volume = Volume of cylinder + $\pi\int_{\frac{1}{2}}^{1} x^2 \, dy$

$= \pi\left(2^2\right)\left(\frac{1}{2}\right) + 4\pi\int_{\frac{1}{2}}^{1}\left(\frac{1}{y} - 1\right) dy$

$= 2\pi + 4\pi\left[\ln|y| - y\right]_{\frac{1}{2}}^{1}$

$= 2\pi + 4\pi\left[-1 - \ln\left(\frac{1}{2}\right) + \frac{1}{2}\right]$

$= 2\pi + 4\pi\left[-\frac{1}{2} + \ln 2\right]$

$= 4\pi \ln 2$ units^3

Solution to Q10
(a) $\frac{dv}{dt} = 8 - kv$, $k > 0$ $\int \frac{1}{8-kv} dv = \int 1 dt$ $-\frac{1}{k}\ln\lvert 8-kv\rvert = t + c$ $-\frac{1}{k}\ln(8-kv) = t + c$ $\ln\lvert 8-kv\rvert = -kt - kc$ $8 - kv = Ae^{-kt}$ *where* $A = \pm e^{-kc}$ $v = \frac{1}{k}\left(8 - Ae^{-kt}\right)$ When t=0, v=0 $0 = \frac{1}{k}\left(8 - Ae^{0}\right) \Rightarrow A = 8$ When t =10, v =5 $5 = \frac{1}{k}\left(8 - 8e^{-10k}\right)$ Using GC, $k = 1.6$ $\therefore v = \frac{1}{1.6}\left(8 - 8e^{-1.6t}\right) = 5\left(1 - e^{-1.6t}\right)$
(b) In the long run, as $t \to \infty, e^{-1.6t} \to 0, v \to 5$ gallons
(c) $\frac{dv}{dt} = 8 - v^2$
(d) Let T h be the time that elapsed. $\frac{dv}{dt} = 8 - v^2 \quad \Rightarrow \quad \int_{4.5}^{3} \frac{1}{8-v^2} dv = \int_0^T 1\, dt$ $\Rightarrow \quad 0.36195 = T$ Time that elapsed $= 0.36195 \times 60 = 22$ min (correct to nearest minutes)

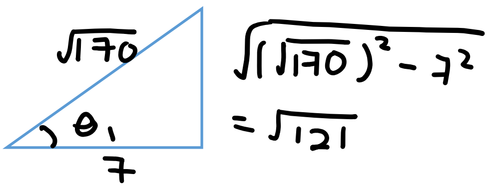
Solution to Q11

(a) $\overrightarrow{AB} = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix}$

$$\cos\theta_1 = \frac{\left|\mathbf{n}\bullet\overrightarrow{AB}\right|}{|\mathbf{n}|\left|\overrightarrow{AB}\right|} = \left|\frac{\begin{pmatrix} -1 \\ 0 \\ 4 \end{pmatrix}\bullet\begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix}}{\sqrt{(-1)^2+4^2}\sqrt{3^2+(-1)^2}}\right|$$

$$\cos\theta_1 = \frac{7}{\sqrt{17}\sqrt{10}}$$

$$\Rightarrow \quad \sin\theta_1 = \sqrt{\frac{121}{170}}$$



$$k = \frac{\sin\theta_1}{\sin\theta_2} = \frac{\sqrt{\frac{121}{170}}}{\frac{\sqrt{2}}{2}} = \sqrt{\frac{121}{85}}$$

(b)

$$\cos\frac{\pi}{4} = \left|\frac{\begin{pmatrix} -1 \\ 0 \\ 4 \end{pmatrix}\bullet\begin{pmatrix} t \\ 0 \\ -1 \end{pmatrix}}{\sqrt{(-1)^2+4^2}\sqrt{t^2+(-1)^2}}\right|$$

$$\frac{1}{\sqrt{2}} = \frac{|-t-4|}{\sqrt{17}\sqrt{t^2+1}}$$

$$\frac{1}{\sqrt{2}} = \frac{|t+4|}{\sqrt{17}\sqrt{t^2+1}}$$

$$\frac{1}{\sqrt{2}} = \frac{t+4}{\sqrt{17}\sqrt{t^2+1}} \quad as \;\; t > 0$$

Squaring both sides, we have

$$\frac{1}{2} = \frac{(t+4)^2}{17\left(t^2+1\right)}$$

$$\Rightarrow \quad 17t^2+17 = 2t^2+16t+32$$

$$\Rightarrow \quad 15t^2-16t-15 = 0$$

$$\Rightarrow \quad (3t-5)(5t+3)=0$$

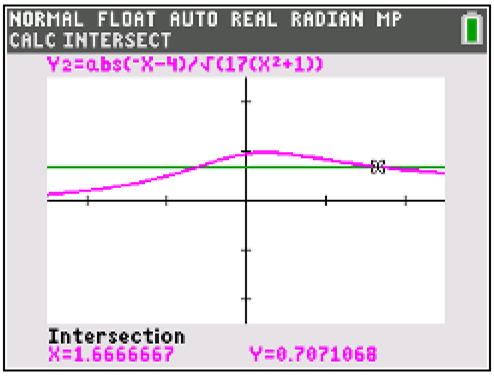
$$\Rightarrow \quad t=\frac{5}{3} \quad \text{or} \quad -\frac{3}{5}\,(\text{rejected})$$

Equation of the line *BC* is $\mathbf{r}=\begin{pmatrix}3\\0\\1\end{pmatrix}+\lambda\begin{pmatrix}\frac{5}{3}\\0\\-1\end{pmatrix},\quad \lambda\in\mathbb{R}$

Method 2

$$\cos\frac{\pi}{4}=\left|\frac{\begin{pmatrix}-1\\0\\4\end{pmatrix}\bullet\begin{pmatrix}t\\0\\-1\end{pmatrix}}{\sqrt{(-1)^2+4^2}\sqrt{t^2+(-1)^2}}\right|=\frac{|-t-4|}{\sqrt{17(t^2+1)}}$$

Using GC



Since t > 0 $\Rightarrow \quad t=\frac{5}{3}$

Equation of the line *BC* is $\mathbf{r}=\begin{pmatrix}3\\0\\1\end{pmatrix}+\lambda\begin{pmatrix}\frac{5}{3}\\0\\-1\end{pmatrix},\quad \lambda\in\mathbb{R}$

(c) Equation of the bottom surface is $\mathbf{r}\bullet\begin{pmatrix}-1\\0\\4\end{pmatrix}=\begin{pmatrix}1\\0\\0\end{pmatrix}\bullet\begin{pmatrix}-1\\0\\4\end{pmatrix}$

i.e. $\mathbf{r}\bullet\begin{pmatrix}-1\\0\\4\end{pmatrix}=-1$

(d)

$$h = \text{Distance of } E \text{ from top surface } = \frac{\left|\overrightarrow{BE}\bullet\begin{pmatrix}-1\\0\\4\end{pmatrix}\right|}{\sqrt{(-1)^2+4^2}}$$

$$= \frac{\left|\left(\begin{pmatrix}1\\0\\0\end{pmatrix}-\begin{pmatrix}3\\0\\1\end{pmatrix}\right)\bullet\begin{pmatrix}-1\\0\\4\end{pmatrix}\right|}{\sqrt{17}}$$

$$= \frac{2\sqrt{17}}{17}$$

Method 2

$$h = \left|\frac{-1}{\sqrt{1+4^2}} - \frac{1}{\sqrt{1+4^2}}\right| = \frac{2}{\sqrt{17}}$$

(e) Since C lies on the line BC,

$$\Rightarrow \overrightarrow{OC} = \begin{pmatrix}3+\frac{5}{3}\lambda\\0\\1-\lambda\end{pmatrix}$$

Since C is the intersection between line BC and bottom surface,

$$\begin{pmatrix}3+\frac{5}{3}\lambda\\0\\1-\lambda\end{pmatrix}\bullet\begin{pmatrix}-1\\0\\4\end{pmatrix} = -1$$

$$\Rightarrow \quad 1-\frac{17}{3}\lambda = -1$$

$$\Rightarrow \quad \lambda = \frac{6}{17}$$

Therefore,

$$\overrightarrow{OC} = \begin{pmatrix}\frac{61}{17}\\0\\\frac{11}{17}\end{pmatrix}$$

Equation of line CD is $\mathbf{r} = \overrightarrow{OC} + \mu\overrightarrow{AB}, \quad \mu \in \mathbb{R}$

i.e. $\mathbf{r} = \begin{pmatrix}\frac{61}{17}\\0\\\frac{11}{17}\end{pmatrix} + \mu\begin{pmatrix}3\\0\\-1\end{pmatrix}, \quad \mu \in \mathbb{R}$

Since point D is on the line $CD \Rightarrow \overrightarrow{OD} = \begin{pmatrix} \frac{61}{17}+3\mu \\ 0 \\ \frac{11}{17}-\mu \end{pmatrix}$

At D, z =0,

$\frac{11}{17}-\mu = 0 \quad \Rightarrow \quad \mu = \frac{11}{17}$

$\overrightarrow{OD} = \begin{pmatrix} \frac{61}{17} \\ 0 \\ \frac{11}{17} \end{pmatrix} + \frac{11}{17}\begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} \frac{94}{17} \\ 0 \\ 0 \end{pmatrix}$

Method 2

Since C lies on the line BC,

$\Rightarrow \overrightarrow{OC} = \begin{pmatrix} 3+\frac{5}{3}\lambda \\ 0 \\ 1-\lambda \end{pmatrix}$

Since C is the intersection between line BC and bottom surface,

$\begin{pmatrix} 3+\frac{5}{3}\lambda \\ 0 \\ 1-\lambda \end{pmatrix} \bullet \begin{pmatrix} -1 \\ 0 \\ 4 \end{pmatrix} = -1$

$\Rightarrow \quad 1-\frac{17}{3}\lambda = -1$

$\Rightarrow \quad \lambda = \frac{6}{17}$

Therefore,

$\overrightarrow{OC} = \begin{pmatrix} \frac{61}{17} \\ 0 \\ \frac{11}{17} \end{pmatrix}$

Equation of line CD is $\mathbf{r} = \overrightarrow{OC} + \mu\overrightarrow{AB}, \quad \mu \in \mathbb{R}$

i.e. $\mathbf{r} = \begin{pmatrix} \frac{61}{17} \\ 0 \\ \frac{11}{17} \end{pmatrix} + \mu\begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix}, \quad \mu \in \mathbb{R}$ --- (1)

At D, line CD intersects with plane given by $\mathbf{r}\bullet\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = 0$ --- (2)

Therefore sub (1) into (2):

$$\left(\begin{pmatrix}\frac{61}{17}\\0\\\frac{11}{17}\end{pmatrix}+\mu\begin{pmatrix}3\\0\\-1\end{pmatrix}\right)\bullet\begin{pmatrix}0\\0\\1\end{pmatrix}=0$$

$$\frac{11}{17}-\mu=0 \quad\Rightarrow\quad \mu=\frac{11}{17}$$

$$\overrightarrow{OD}=\begin{pmatrix}\frac{61}{17}\\0\\\frac{11}{17}\end{pmatrix}+\frac{11}{17}\begin{pmatrix}3\\0\\-1\end{pmatrix}=\begin{pmatrix}\frac{94}{17}\\0\\0\end{pmatrix}$$

Method 3

Since C lies on the line BC,

$$\Rightarrow\overrightarrow{OC}=\begin{pmatrix}3+\frac{5}{3}\lambda\\0\\1-\lambda\end{pmatrix}$$

Since C is the intersection between line BC and bottom surface,

$$\begin{pmatrix}3+\frac{5}{3}\lambda\\0\\1-\lambda\end{pmatrix}\bullet\begin{pmatrix}-1\\0\\4\end{pmatrix}=-1$$

$$\Rightarrow\quad 1-\frac{17}{3}\lambda=-1$$

$$\Rightarrow\quad \lambda=\frac{6}{17}$$

Therefore,

$$\overrightarrow{OC}=\begin{pmatrix}\frac{61}{17}\\0\\\frac{11}{17}\end{pmatrix}$$

Let $\overrightarrow{OD}=\begin{pmatrix}x\\0\\0\end{pmatrix}$ since D is a point on the floor

Since $\overrightarrow{CD}$ parallel to $\overrightarrow{AB}$,

$$\overrightarrow{OD}-\overrightarrow{OC}=k\overrightarrow{AB}$$

$$\begin{pmatrix}x\\0\\0\end{pmatrix}-\begin{pmatrix}\frac{61}{17}\\0\\\frac{11}{17}\end{pmatrix}=k\begin{pmatrix}3\\0\\-1\end{pmatrix}$$

By comparing components,

$x - \frac{61}{17} = 3k$ --- (1)

$\frac{11}{17} = -k$ --- (2)

Solving (1) and (2):

$k = -\frac{11}{17}$, $x = \frac{94}{17}$

$\therefore \overrightarrow{OD} = \begin{pmatrix} \frac{94}{17} \\ 0 \\ 0 \end{pmatrix}$