Name: _____ (   )   Date: _____

Logarithm and Exponential Equations

1 XMS 2020 AM P2   [4]

  (a) Using the substitution $y = 5^x$, solve the equation $5^{1+2x} - 18(5^x - 1) - 2 = 0$.

  (b) Without using the calculator, evaluate x if $\dfrac{1}{\log_9 x} - \dfrac{1}{3\log_3 x} + \log_x 27 = \dfrac{7}{6}$.   [4]

  XMS 2022 AM P1

2 Solve $3^{2x-1} = 4^{2-x}$ and show that $x = \dfrac{\lg 48}{2\lg 6}$.

                                                                                                            [3]

3 XMS 2022 AM P2

  Solve the equation $\lg(x+3) = 1 - \lg(x-2)$.   [3]

4 XMS 2022 AM P2

  Given that $\log_2 p = a$, $\log_q 8 = b$ and $\dfrac{p}{q} = 2^c$, express c in terms of a and b.   [3]

5 FMS 2022 AM P2

  (a)   Solve the simultaneous equations

  $$e\sqrt{e^x} = e^{2y},$$
  $$\log_4(x+2) = 1 + \log_2 y.$$ [8]

  (b)   Solve the equation $2(100^y) - 10^y = 6$.   [4]

6 PHS 2022 AM P1

  (a) Given that $3^{x+1} \times 2^{2x+1} = 2^{x+2}$, evaluate 6^x.   [4]

  (b) Express y in terms of x if $\log_2 y = \log_8 x - \log_2 4$.   [4]

7 It is given that $\log_2\left(4-x^2\right) - \log_{\sqrt{2}}(x-1) = 1$.

  (a)   Explain clearly why $1 < x < 2$.   [4]

  (b)   Hence, solve the equation and show that it has only one solution.   [5]

Logarithm and Exponential Word Problem

1 <u>XMS 2020 AM P1</u>

The population, P, of a type of insect in a forest after n years is modelled by the

formula $P = \dfrac{3500}{1+4e^{-kn}}$, where k is a constant.

(i)   Find the initial population of the insect. [1]

(ii) A group of scientists concluded that the population of the insect increased by
      50% after 30 months. Find the value of k. [4]

(iii) Another group of scientists estimated that the population of the insects will be
      more than 4000 in the future. Do you agree with them? Explain your answer. [3]

2 <u>FMS 2022 AM P1</u>

It was found that the percentage of carbon-14, P, contained in the bones of an animal n years
after it has died is given by $P = 2^{-kn}$, where k is a positive constant. The percentage of
carbon-14 contained in the bones after the animal has been dead for 7200 years was 50%.

How long, to the nearest year, was the animal dead if the percentage of carbon-14 found in
the bones was only 34%? [4]

3 <u>JSS 2022 AM P1</u>

A metal cube is heated to a temperature of 212°C before being dropped into a liquid.
As the cube cools, its temperature T°C, t minutes after it enters the liquid is given by
$T = P + 180e^{-kt}$, where P and k are constants.
It is recorded that when $t = 5$, $T = 185$.

(a) Find the value of P and of k. [4]

(b) By sketching the graph of T against t, explain why T cannot be 30. [3]

4 <u>MGS 2022 AM P1</u>

The mass, m grams, of a radioactive substance remaining, t days after being measured is
given by $m = 10e^{-0.01t} + 0.2$.

(a) Find the initial mass. [1]

(b) Sketch the graph $m = 10e^{-0.01t} + 0.2$ for $t \geq 0$. [2]

(c) Find the least number of days it takes before the amount of substance is reduced to
    5% of its initial mass. [3]

5 NASS 2023 AM P1

The spread of bird flu in a certain duck farm is given by $B(t) = \dfrac{300}{1+e^{5-t}}$, where t is the number of days since the flu first appeared, and $B(t)$ is the total number of ducks which have caught the flu to date.

(a) Estimate the initial number of ducks infected with the flu.
     Give your answer correct to the nearest integer.   [1]

(b) If the day that the flu first appeared is Day 0, find out on which day would the number of infected ducks first reach 100.   [2]

(c) Explain clearly why the number of infected ducks will never exceed 300.   [2]
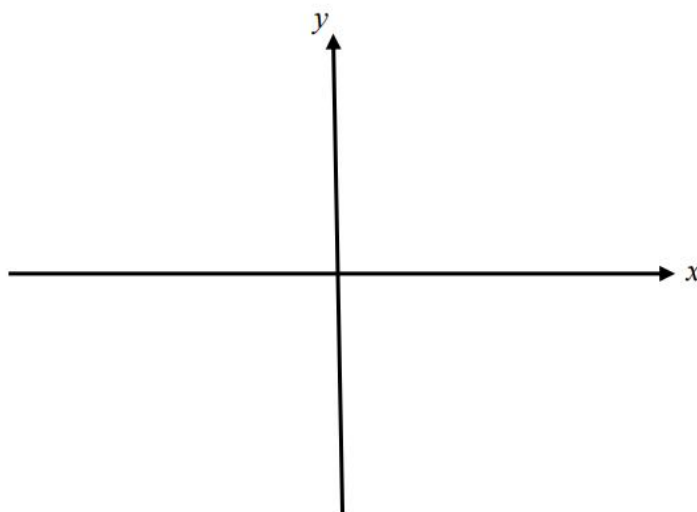
Logarithm and Exponential Graphs

1 XMS 2022 AM P1

In order to obtain a graphical solution of the equation $x = \ln\left(\dfrac{7-x}{5}\right)^2$, a suitable

straight line can be drawn on the same axes as the graph $y = 5e^{\frac{x}{2}} - 4$. Find the
equation of this line. [2]

2 FMSS 2022 AM P2

(i) Sketch the graph of $y = \ln(1+3x)$. [2]

(ii) A student commented that the number of solutions to the equation $e^{\frac{6-3x}{4}} - 3x = 1$ can
be predicted by using the graph in **(i)**.

Explain clearly whether you agree with this student's comment. [3]

3 2020 AM O Level P2

In order to obtain a graphical solution of the equation $x = \ln\left\{\left(\dfrac{2x+7}{3}\right)^2\right\}$ a suitable straight line can be

drawn on the same set of axes as the graph of $y = 3e^{\frac{x}{2}} + 4$. Make $e^{\frac{x}{2}}$ the subject of $x = \ln\left\{\left(\dfrac{2x+7}{3}\right)^2\right\}$

and hence find the equation of this line. [3]