**Calculator Model:**

# KENT RIDGE SECONDARY SCHOOL
# PRELIMINARY EXAMINATION 2024

**ADDITIONAL MATHEMATICS** **4049/02**
**PAPER 2**

**SECONDARY 4 EXPRESS/ 5 NORMAL ACADEMIC**

**Monday 26 August 2024** **2 hour 15 minutes**



**Name:** ______________________________________ ( ) **Class: Sec** ____________

Candidates answer on the Question Paper.

No Additional Materials are required.

**READ THESE INSTRUCTIONS FIRST**

Write your name, index number and class in the spaces at the top of this page.

**Do not open this question paper until you are told to do so.**

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue, correction fluid or correction tape.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an approved scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

**The total number of marks for this paper is 90.**

| For Examiner's Use | |
|---|---|
| **Total** | **90** |

This Question Paper consists of 19 printed pages, including this page.

Setter: Vivian Tan **[Turn over**

## Mathematical Formulae

### 1. ALGEBRA

*Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

*Binomial Theorem*

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where $n$ is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

### 2. TRIGONOMETRY

*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

*Formulae for $\Delta ABC$*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

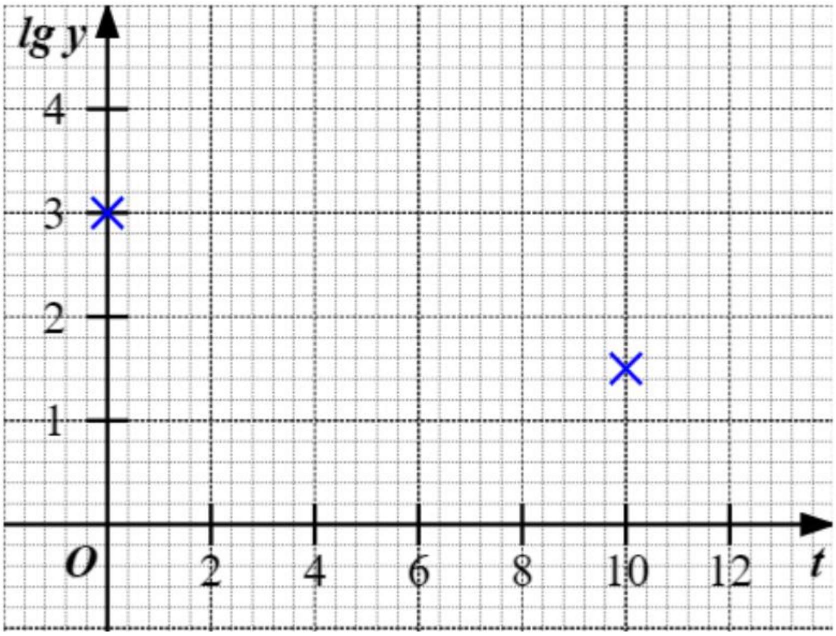
1 Express $\frac{4}{(x^2+1)(x+1)}$ in partial fractions. [5]

2 (a) Radiation intensity, $R$, varies inversely with the square of $d$, the distance from the source of radiation such that $R = \frac{k}{d^2}$, where $k$ is a constant.

Values of $R$ for different values of $d$ have been collected and tabulated.
Explain how a straight-lined graph can be drawn to determine the formula connecting $R$ and $d$. [4]

(b) The number of particles present in a room, $t$ minutes after turning on the air filter is $y$. When corresponding values of lg $y$ and $t$ are plotted on a lg $y$ against $t$ axes, the points form a straight line that passes through (0,3) and (10,1.5) as drawn on the axes on the next page.

(i) Find $y$ in terms of $t$. [4]




(ii) Use the graph to estimate the time taken for the number of particles in the room to be halved. [3]

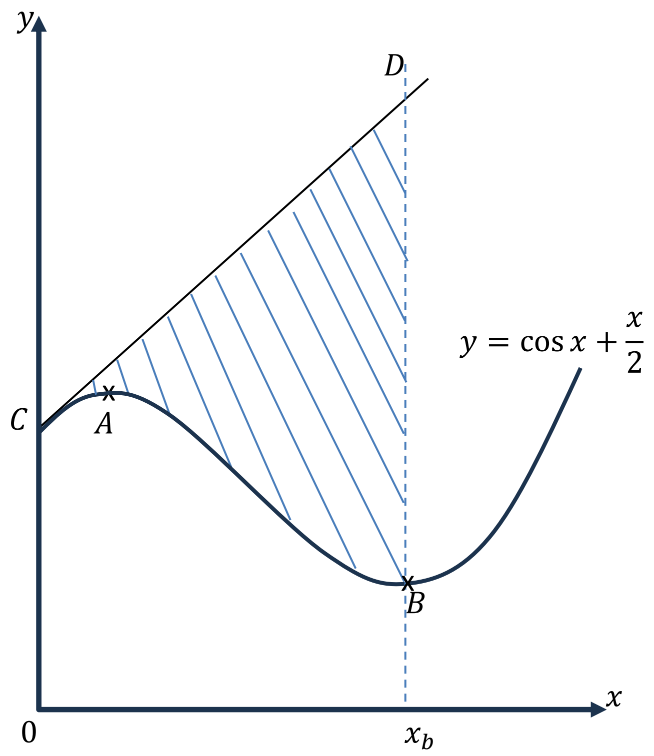
3 (a) The diagram below shows the graph of the curve $y = \cos x + \frac{x}{2}$ for $x \geq 0$ radians.

The tangent to the curve when $x = 0$ at $C$, is drawn to $D$ which is vertically above point $B$, the minimum point of the curve.

Points $A$ and $B$ are the first two stationary points of the curve.

Find $x_b$, the $x$ coordinates of point $B$.

You do not need to show that it is a minimum point. [4]




(b) (i) Find the equation of $CD$. [2]

(ii) Find the area shaded that is bounded by the tangent to the curve $y = \cos x + \frac{x}{2}$ at $x = 0$, the curve and the line $x = x_b$. [5]

4 (a) $2y = 16x + k$ is a tangent to the curve $y = \frac{1}{2x} + 2kx$. Find the value of constant $k$. [4]

(b) Find the range of values of $a$ such that $ax^2 + \sqrt{8}x + (a - 1) < 0$ for all values of $x$. [4]

5 Given $y = e^{2x} \sin 3x$.

(a) Find $\frac{dy}{dx}$. [2]

(b) Find $\frac{d^2y}{dx^2}$. [2]

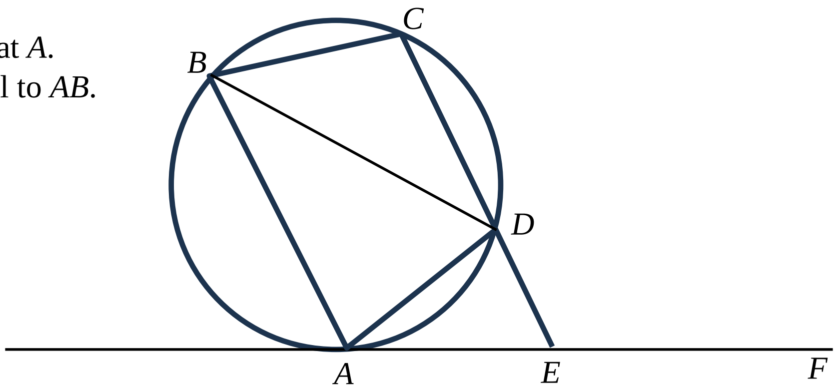
(c) Given that $\frac{dy}{dx} + \frac{d^2y}{dx^2} + ay = be^{2x}\cos 3x$, form 2 equations involving $a$ and $b$ and use them to find the value of $a$ and of $b$. [4]

6 (a) Show that $\frac{d}{dx}\left(\frac{x-2}{\sqrt{3x+1}}\right) = \frac{3x+8}{2\sqrt{(3x+1)^3}}$. [4]

(b) Hence evaluate $\int_0^5 \frac{3x+7}{2\sqrt{(3x+1)^3}}\,dx$. [5]

For continuation of working for question 6 part (b)

7 $AF$ is a tangent to the circle $ABCD$ at $A$.
$E$ is on $AF$ such that $EDC$ is parallel to $AB$.




(a) Prove that triangle $ABD$ and triangle $DAE$ are similar. [3]

(b) Show that if triangle $BCD$ and triangle $DAB$ are similar, $BD$ must be the diameter of the circle. [3]

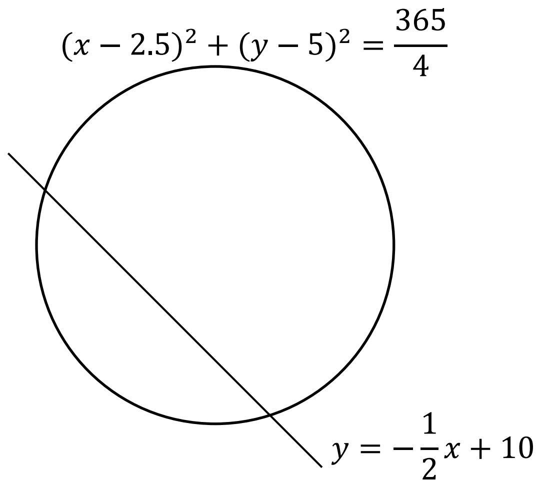
8 (a) Solve $3(3^{x+1}) = 10 - 3^{-x}$. [4]

(b) Given $\log_{100} x + \lg y = 3$, express $y$ in terms of $x$. [4]

9 The chord $AB$ of a circle $C$ has equation $y = -\frac{1}{2}x + 10$, where the $x$ coordinate of $A$ is smaller than the $x$ coordinate of $B$.

The circle $C$ has equation $(x - 2.5)^2 + (y - 5)^2 = \frac{365}{4}$ with centre $E$.

(a) Find the coordinates of $A$. [4]



(b) State the centre of circle $C$, and use it to show that the perpendicular bisector of $AB$ passes through the origin. [4]

(c) The chord $AB$ is extended to cut the $x$-axis at point $D$. Show that the mid-point of $AD$ lies inside circle $C$. [4]

10 A particle starts moving in a straight line when it is 6 metres from a fixed point $O$, such that its velocity, $t$ seconds after the start of the motion is given by $v = 4e^{-2t} + t - 3$ m/s.

(a) Find the initial velocity of the particle. [2]

(b) Show that the minimum velocity is negative, and it happens when $t = \frac{1}{2}\ln 8$. [4]

(c) Using your answer from part (a) and part (b), explain if the particle changes its direction of motion. [2]

(d) Find the displacement of the particle from $O$, 2 seconds after the start of the motion. [4]

**End of Paper**