Name : _____Marking Scheme_____

Class   Index Number

METHODIST GIRLS' SCHOOL

Founded in 1887

PRELIMINARY EXAMINATION 2022
Secondary 4

Tuesday	**MATHEMATICS**	**4048/01**
2 August 2022	**Paper 1**	2 hours

Candidates answer on the Question Paper.

INSTRUCTIONS TO CANDIDATES

Write your class, name and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For p, use either your calculator value or 3.142, unless the question requires the answer in terms of p.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80**.**

80
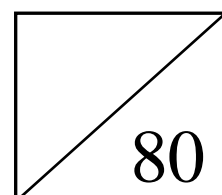
Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of a triangle} = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1. Arrange the following numbers from the smallest to the largest.

$$\frac{2\pi}{3} \quad , \quad -4 \quad , \quad 2.\dot{3} \quad , \quad 227\% \quad , \quad -\sqrt{17}$$

$2.09...$, -4 , $2.33...$, 2.27 , $-4.12...$

Must give the original nos.

[B1]

Answer .... [1]

$$-\sqrt{17} \quad , \quad -4 \quad , \quad \frac{2\pi}{3} \quad , \quad 227\% \quad , \quad 2.\dot{3}$$

2. The marked price of a washing machine in a shop is \$*m*.
   During the Great Singapore Sale, it was sold at a discount of *d* %.

   (a) Express the selling price as a single fraction in terms of *m* and *d*.

   [B1]

   Answer \$ .... [1]

   $$\frac{m\,(100-d)}{100}$$

   (b) The shopkeeper made a profit of 25% from the sale of the washing machine.
   Express the cost price as a single fraction in terms of *m* and *d*.

   S.P. —— 125% —— \$ $\dfrac{m\,(100-d)}{100}$

   C.P. —— 100% —— \$ $\dfrac{m\,(100-d)}{100} \times \dfrac{100}{125}$

   $\qquad\qquad\qquad = \$ \dfrac{m\,(100-d)}{125}$

   [B1]

   Answer \$ .... [1]

   $$\frac{m\,(100-d)}{125}$$

3. Factorise completely $4px^2 - 3k + 12kx^2 - p$.

   $4px^2 - 3k + 12kx^2 - p$
   $= 4px^2 - p + 12kx^2 - 3k$
   $= p(4x^2 - 1) + 3k(4x^2 - 1)$ **[M1]**
   $= (p + 3k)\,(4x^2 - 1)$ **[M1]**
   $= (p + 3k)\,(2x + 1)\,(2x - 1)$

   [A1]

   Answer .... [3]

   $$(p + 3k)\,(2x + 1)\,(2x - 1)$$

_____

4. Solve $\dfrac{x-4}{4} - \dfrac{2x-1}{6} = 1$.

$\dfrac{x-4}{4} - \dfrac{2x-1}{6} = 1$

$3(x-4) - 2(2x-1) = 12$ **[M1]**

$3x - 12 - 4x + 2 = 12$

$-x - 10 = 12$

$x = -22$

[A1]

Answer $x =$ ………………………   [2]

                     −22

5. **(a)** Given that $p = 2^4 \times 3^2 \times 11$ and $q = 2 \times 3^2 \times 5 \times 11$.
Find the greatest integer that divides p and q exactly.

HCF $= 2 \times 3^2 \times 11$

       $= 198$

[B1]

Answer ……………………………   [1]

                198

(b) A number has exactly nine factors. Two of the factors are 12 and 18.
List all the factors of the number.

Factors of $12 = 1 , 2 , 3 , 4 , 6 , 12$

Factors of $18 = 1 , 2 , 3 , 6 , 9 , 18$ **[M1]**

LCM of 12 and 18 = 36

[A1]

Answer ……………………………………………………………………   [2]

            1 , 2 , 3 , 4 , 6 , 9 , 12 , 18 , 36

6. $\varepsilon = \{$all real numbers$\}$

$A = \{$all prime numbers$\}$

$B = \{$all rational numbers$\}$

$C = \{$all integers$\}$

$D = \{$all negative numbers$\}$

$E = \{1, 2, 3, 4, 5, 6, 7\}$

(a) List all the elements contained in the set $A \cap E$.

[B1]

Answer ....$\{2 \;,\; 3 \;,\; 5 \;,\; 7\}$.................... [1]

(b) Explain why $C \cap B' = \phi$.

Answer ....B' are irrational numbers which are non-terminating and non-recurring........

.........decimal numbers that cannot be expressed as a fraction like the integers....**[B1]**

.........in C. Therefore, $C \cap B' = \phi$...

....

.... [1]

(c) Which of the following statement(s) is/are correct?

Statement 1: $C \subset B$

Statement 2: $A \cup C = A$

Statement 3: $A \cap D = \phi$

Statement 4: $C \cap D = \phi$

[B1] + [B1]

Answer Statement(s) number ....1 and 3..............is/are correct [2]

7. Given that $\sqrt{\dfrac{h^2k-3}{5w}} = 2h$, express h in terms of k and w.

$$\sqrt{\frac{h^2k-3}{5w}} = 2h$$

$$\frac{h^2k-3}{5w} = 4h^2 \qquad \textbf{\color{red}[M1] Squaring both sides}$$

$$h^2k-3 = 20wh^2$$

$$h^2k-20wh^2 = 3$$

$$h^2\,(k-20w) = 3 \qquad \textbf{\color{red}[M1] Factorise by taking out the common factor }h^2$$

$$h^2 = \frac{3}{k-20w}$$

$$h = \pm\sqrt{\frac{3}{k-20w}}$$

[A1]

$$h = \pm\sqrt{\frac{3}{k-20w}}$$

Answer …………………………………… [3]

8. Given that n is a positive integer, explain whether $(7n+3)^2 - 4^2$ is divisible by 7. Show your working clearly.

Answer [2]

$(7n+3)^2 - 4^2$

$= 49n^2 + 42n + 9 - 16$

$= 49n^2 + 42n - 7$

$= 7\,(7n^2 + 6n - 1) \qquad$ **[M1]**

$(7n+3)^2 - 4^2$

$= (7n+3+4)\,(7n+3-4)$

$= (7n+7)\,(7n-1)$

$= 7\,(n+1)\,(7n-1)$

Since 7 is a factor, $(7n+3)^2 - 4^2$ is divisible by 7. **[A1]**

9. The chart below shows the monthly sales of an electrical company from January to June in 2022.
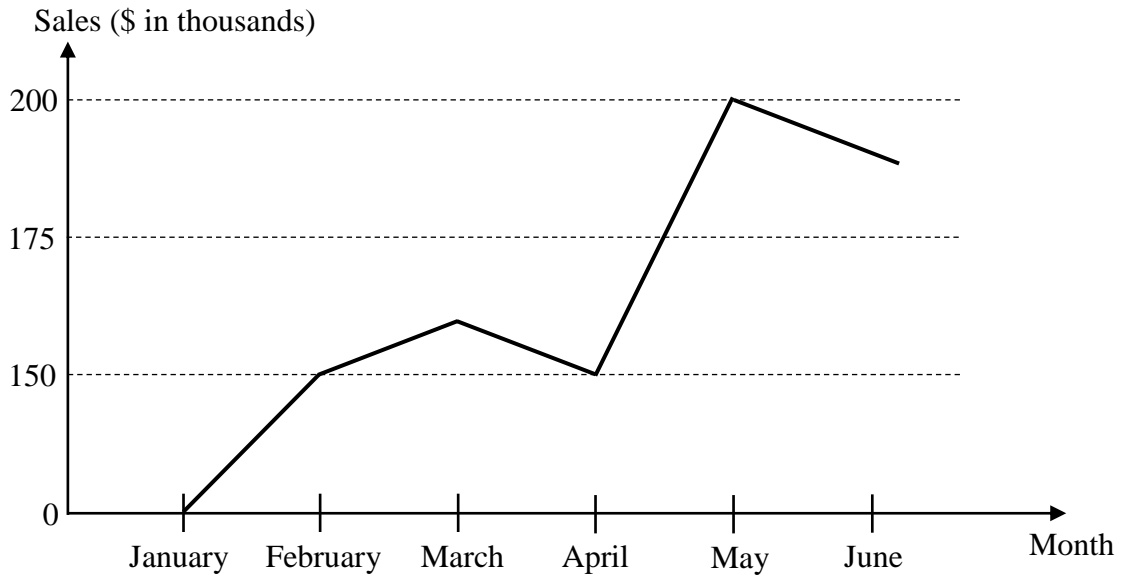
(a) State one feature of the graph that may be misleading.

[B1]

Answer The scale / intervals on the vertical axis is not consistent.

.... [1]

(b) Explain how this may lead to a misinterpretation of the graph.

[B1]

Answer It may mislead the reader to think that the ==increase in sales== from April to May

($50 000) appears to be twice of that from January to February ($150 000).

Actually the amount of increase is only one-third.

or

It may mislead the reader to think that the ==amount of sales== in May ($200 000) [1]

appears to be three time of that in April ($150 000). Actually it is only $1\frac{1}{3}$

times.

___

10. In the diagram, the line L cuts the y-axis at A (0, 15) and the x-axis at B (h, 0).
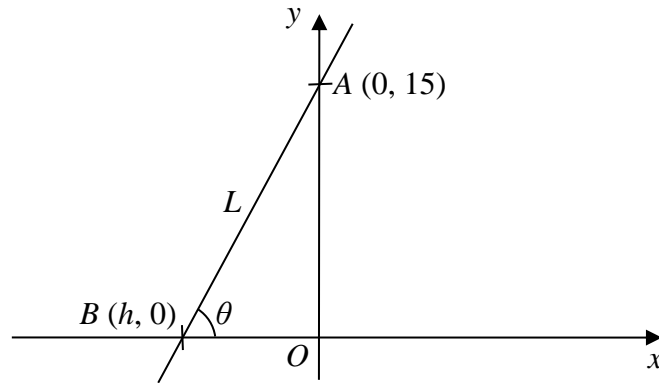
Given that $\tan\theta = \dfrac{5}{2}$, where θ is the angle made by the line L and the horizontal x-axis, find

(a) the value of h,

$$\tan\theta \;=\; \frac{15}{OB} \;=\; \frac{5}{2}$$
$$OB \;=\; \frac{15 \times 2}{5} \;=\; 6 \; units$$

[B1]

Answer $\quad h \;=\; $ …………-6………… [1]

(b) the equation of the line L.

[B1]

$$y \;=\; \frac{5}{2}x \;+\; 15$$

Answer …………….………… [1]

(c) The coordinates of point C is (-6 , 21). Calculate the length of AC.

Length of AC

$= \sqrt{[0-(-6)]^2 + (15-21)^2}$ **[M1]**

$= \sqrt{72}$

$= 8.49$ units (3sf)

[A1]

Answer …………8.49……… units [2]

(d) $ABCD$ is a trapezium. Find the coordinates of a possible point D.

[B1]

Answer (………0……… , ……20……..) [1]

Any value of $y > 15$

_____

11. (a) Solve the inequality $-5 \le \frac{1}{2}(3+5x) < 7+x$.

$-5 \le \frac{1}{2}(3+5x)$ $\frac{1}{2}(3+5x) < 7+x$

$-10 \le 3+5x$ $3+5x < 14+2x$

$-13 \le 5x$ $3x < 11$

$-2\frac{3}{5} \le x$ **[M1]** $x < 3\frac{2}{3}$ **[M1]**

[A1]

$-2\frac{3}{5} \le x < 3\frac{2}{3}$

Answer .... [3]

(b) Illustrate the solutions of the inequality on a number line clearly.
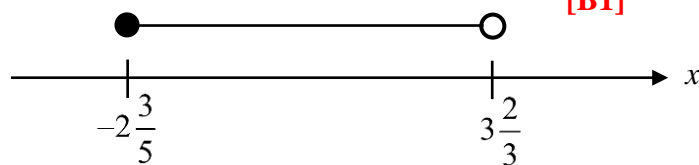
Answer   **[B1]**   [1]

(c) Find the smallest integer value of x, which satisfies the inequality.

[B1]

-2

Answer .... [1]

12. Given that $2^{2n-3} = 2^{2022} - 2^{2021}$, find the value of n.

$2^{2n-3} = 2^{2022} - 2^{2021}$

$2^{2n-3} = 2^{2021}(2-1)$

$2n - 3 = 2021$ **[M1]**

$2n = 2024$

$n = 1012$

[A1]

1012

Answer $n = $ .... [2]

13.

	Vietnam	Singapore
Population	9.8×10^7	5.4×10^6

Use the information from the table to answer the following.

(a) How many more people live in Vietnam than in Singapore?
Give your answer in standard form, to a sensible degree of accuracy.

$$9.8 \times 10^7 - 5.4 \times 10^6 = 92\ 600\ 000$$
$$= 9.3 \times 10^7 \quad (2 \text{ s.f.})$$

[A1]

Answer ....9.3×10^7............... [1]

(b) The land area of Vietnam is 331 690 km^2.
Calculate the average number of people per square kilometre living in Vietnam.

$$\frac{9.8 \times 10^7}{331\ 690}$$
$$= 295.456601...$$
$$= 295 \text{ people/km}^2 \quad (3 \text{ s.f.})$$

[A1]

Answer ....295.............. people/km^2 [1]

14. Select a possible equation from the box to represent each of the sketch graphs below.

$xy = 2$	$y = x^3 - 2$	$y = 2 - x^3$
$y = 2 - x^2$	$y = x^2 - 2$	$x^2 y = 2$

(a)

[B1]

$y = x^2 - 2$

Answer ……………………….. [1]

(b)

[B1]

$xy = 2$

Answer ……………………….. [1]

(c)

[B1]
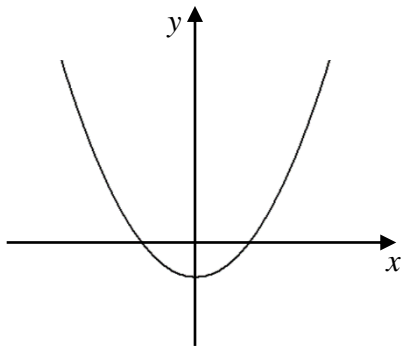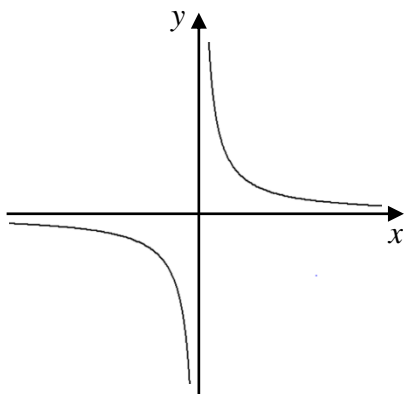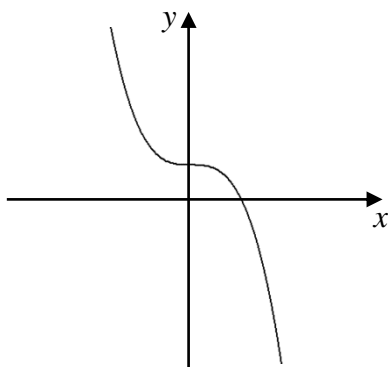
$y = 2 - x^3$

Answer ……………………….. [1]

15. The diagram shows a regular octagon and a regular pentagon.
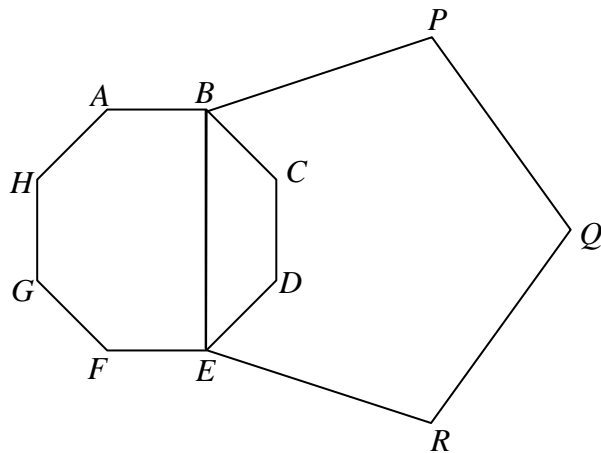
(a) **(i)** Find $\angle PBE$.

$$\angle PBE = \frac{(5-2)\times 180^\circ}{5} = 108^\circ$$

[A1]

Answer $\angle PBE$ = ………..…………….. 108° [1]

(ii) Find $\angle PBC$.

$$\angle ABC = \frac{(8-2)\times 180^\circ}{8} = 135^\circ$$

$\angle ABE = 90°$

$\angle EBC = 135° - 90°$ **[M1]**
$\quad\quad = 45°$

$\angle PBC = 108° - 45°$
$\quad\quad = 63°$

[A1]

Answer $\angle PBC$ = ………..…………….. 63° [2]

(b) Is *BE* parallel to *CD*? Explain your answer with working clearly.

Answer [1]

Since $\angle CBE + \angle BCD$

$= 45° + 135°$

$= 180°$

[A1]

By property of interior angles, *BE* is parallel to *CD*.

16. The heights of 20 students were measured.
The results are shown in the stem-and-leaf diagram below.

```
14 | 6 6 7 8
15 | 1 2 3 5 5 6
16 | 0 1 1 1 4 6 7 7 9
17 |
18 |
19 |
20 |
21 | 5
```

Key 14 | 6 means 146 cm

(a) Find the median height.

$$\text{Median height} = \frac{156+160}{2} = 158 \text{ cm}$$

[A1]

Answer ………………158……… cm [1]

(b) Which is a more appropriate measure of central tendency, median or mean, to represent this distribution? Explain the reason clearly.

Answer …………Median……… is more appropriate because …there is an extreme height… **[B1]**

of 215 cm which will give a value of the mean height that skews the

representation of the central tendency for this distribution. [1]

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17. The diagram shows a parallelogram *ABCD*.
APS, *BRS*, *CRQ* and *DPQ* are straight lines that bisect angles *A*, *B*, *C* and *D* respectively
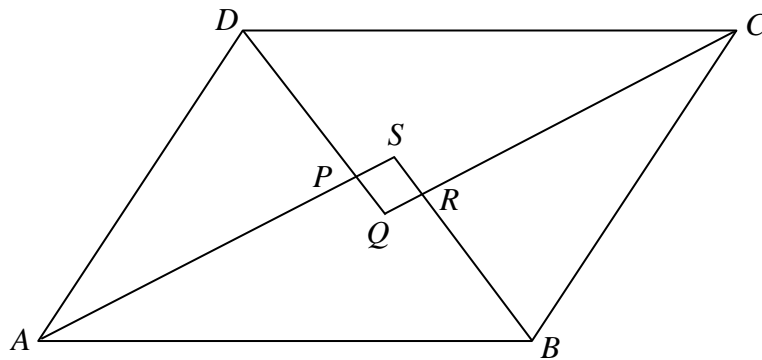
Prove that Δ*PAD* and Δ*RCB* are congruent.

Answer   [3]

$AD = BC$ (opp. sides of parallelogram)

$\angle DAB = \angle DCB$ (opp. angles of parallelogram)

$\angle DAP = \angle BCR$ (since *APS* bisects $\angle A$, *CRQ* bisects $\angle C$)

[M2] 3 correct criteria

[M1] 2 correct criteria

$\angle ADC = \angle ABC$ (opp. angles of parallelogram)

$\angle ADP = \angle CBR$ (since *DRQ* bisects $\angle D$, *BRS* bisects $\angle B$)

$\therefore \Delta PAD \equiv \Delta RCB$ (AAS)   **[A1] with all 3 correct criteria given**

18. The cash price of a motorcycle is $18 000.
Mr Lim made a down payment of 40% on the cash price. He took a loan of the balance amount from a bank that charged a simple interest of r % per annum. He paid 30 equal monthly instalments to the bank.
Mr Lim paid a total of $20 025 for the motorcycle.

(a) Find the value of r.

$$\text{Balance} = \frac{60}{100} \times 18000 = \$10800$$

$$\text{Interest paid} = 20025 - 18000$$
$$= \$2025$$

Let the interest rate be r % per annum

$$2025 = \frac{10800 \times r \times \frac{30}{12}}{100} \qquad \textbf{[M1]}$$

$$r = \frac{2025 \times 100}{10800 \times \frac{30}{12}}$$

$$= 7.5$$

[A1]

Answer $r = $ ……………………….. 7.5 [2]

(b) Calculate each monthly instalment.

Let each monthly instalment be $\$y$.

$$\frac{40}{100} \times 18000 + 30y = 20025 \quad \textbf{[M1]}$$

$$7200 + 30y = 20025$$

$$30y = 12825$$

$$y = 427.50$$

Each monthly instalment

$$= \frac{\frac{60}{100} \times 18000 + 2025}{30}$$

$$= \$427.50$$

[A1]

Answer $ \$ $ ..……………… 427.50 [2]

19. The terms T_1, T_2, T_3, T_4 of a sequence are given as follows:

$$T_1 = \frac{1}{3} = \frac{1}{1 \times 3} = \frac{1}{2 \times 1} - \frac{1}{2 \times 3}$$

$$T_2 = \frac{1}{8} = \frac{1}{2 \times 4} = \frac{1}{2 \times 2} - \frac{1}{2 \times 4}$$

$$T_3 = \frac{1}{15} = \frac{1}{3 \times 5} = \frac{1}{2 \times 3} - \frac{1}{2 \times 5}$$

$$T_4 = \frac{1}{24} = \frac{1}{4 \times 6} = \frac{1}{2 \times 4} - \frac{1}{2 \times 6}$$

(a) **(i)** Write down the next term, T_5, in this sequence $\dfrac{1}{3}$, $\dfrac{1}{8}$, $\dfrac{1}{15}$, $\dfrac{1}{24}$, ...

$\dfrac{1}{35}$ **[B1]**

Answer $T_5 = $ ....…………………… [1]

(ii) Write down the n$^{\text{th}}$ term of this sequence, T_n, in terms of *n*.

Accept $\dfrac{1}{n(n+2)}$

$\dfrac{1}{n^2 + 2n}$ **[B1]**

Answer $T_n = $ ....…………………… [1]

(b) Find the exact value of $T_1 + T_3 + T_5 + \ldots + T_{99}$.

$T_1 + T_3 + T_5 + \ldots + T_{99}$

$$= \frac{1}{2} - \frac{1}{6} + \frac{1}{6} - \frac{1}{10} + \frac{1}{10} - \frac{1}{14} \quad \ldots \quad + \frac{1}{2 \times 99} - \frac{1}{2 \times 101}$$

$$= \frac{1}{2} - \frac{1}{2 \times 101}$$

$$= \frac{50}{101}$$

[M1]

$\dfrac{50}{101}$ **[A1]**

Answer ……..………………… [2]

20. The braking distance, d, of a car is directly proportional to the square of its speed, v.
When the speed is p metres per second, the braking distance is 8 metres.

(a) When the speed is increased by 200%,

    (i) write down an expression, in terms of p, for the new speed of the car,

[B1]

Answer ……………$3p$………… m/s   [1]

    (ii) hence, find the braking distance.

$d = k v^2$ where k is a constant

$k = \dfrac{d}{v^2}$

$k = \dfrac{8}{p^2}$ **[M1]**

When $v_{new} = 3p$,

$d_{new} = k\, v_{new}^2$

$= \dfrac{8}{p^2} (3p)^2$

$= \dfrac{8}{p^2} \times 9p^2$

$= 72$ m

[A1]

Answer ………72………… m [2]

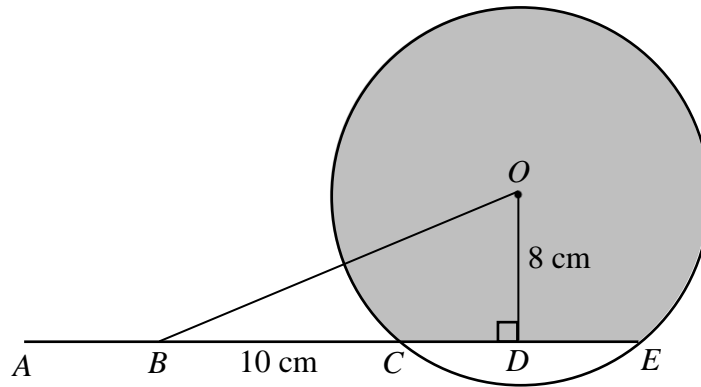**(b)** Find the percentage increase in the braking distance.

Percentage increase

$= \dfrac{72-8}{8} \times 100$ %   **[M1]**

$= 800$ %

[A1]

Answer ……800……… % [2]

21. The diagram shows a circle with centre O. $ABCDE$ is a straight line. C is the mid-point of BE. It is given that $OD = 8$ cm, $BC = 10$ cm, $\angle ODB = 90°$.

(a) Find the exact value of cos $\angle OBA$.

$$CD = \frac{1}{2} \times 10 = 5 \text{ cm}$$

$$OB = \sqrt{8^2 + 15^2} = 17 \text{ cm} \qquad \textbf{[M1]}$$
$$\cos \angle OBA = -\cos \angle OBD$$
$$= -\frac{15}{17}$$

[A1]

Answer ....$-\dfrac{15}{17}$.......... [2]

(b) (i) Show that $\angle DOE = 0.5586$ radian, correct to 4 significant figures.

Answer [2]

$$\tan \angle DOE = \frac{5}{8} \qquad \textbf{[M1]}$$

$$\angle DOE = \tan^{-1}\left(\frac{5}{8}\right)$$
$$= 0.5586 \text{ radian} \quad (4 \text{ s.f.}) \quad (\text{shown}) \qquad \textbf{[A1]}$$

(ii) Hence, find the area of the shaded region.

$$\text{Radius of circle} = \sqrt{8^2 + 5^2}$$
$$= \sqrt{89}$$
$$= 9.4339 \text{ cm}$$

Acute $\angle COE = 2 \times 0.5586 = 1.1172$ rad

Reflex $\angle COE = 2\pi - 1.1172 = 5.1659$ rad

Area of shaded region

$$= \frac{1}{2}\left(\sqrt{89}\right)^2 (5.1659) + \frac{1}{2}\left(\sqrt{89}\right)^2 \sin(1.1172) \qquad \textbf{\color{red}{[M1] + [M1]}}$$

$$= 229.882 + 40.000$$

$$= 270 \text{ cm}^2 \quad \text{(3sf)}$$

Or

Area of shaded region

$$= \frac{1}{2}\left(\sqrt{89}\right)^2 (5.1659) + \frac{1}{2}(8)(10)$$

$$= 270 \text{ cm}^2 \quad \text{(3sf)}$$

Answer ....270.................. cm^2 [3]

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22. The diagram shows the speed-time graph of a motorcycle.
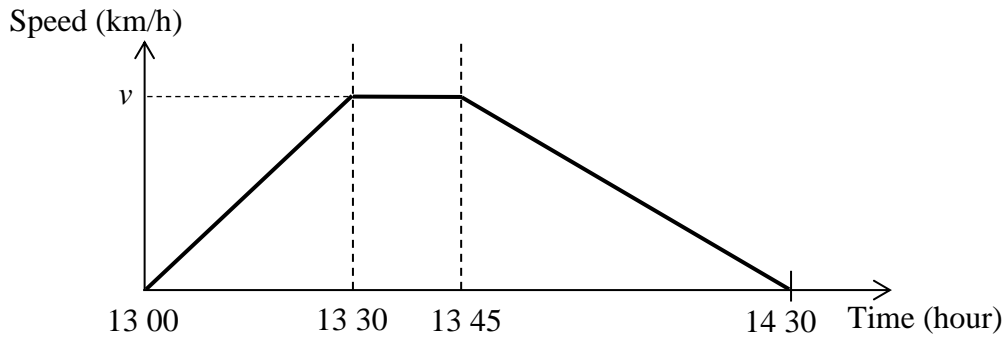
(a) Given that the deceleration of the motorcycle at 14 00 hour is 100 km/h^2, calculate the maximum speed, v, in km/h.

$$\frac{v}{\frac{45}{60}} = 100 \qquad \textbf{[M1]}$$

$$v = 100 \times \frac{45}{60}$$

$$= 75 \text{ km/h} \qquad \textbf{[A1]}$$

Answer v = ...75.............. km/h [2]

(b) Calculate the total distance travelled by the motorcycle from 13 00 hour to 14 30 hour.

Distance $= \frac{1}{2}\left(\frac{30}{60}\right)(75)+\left(\frac{15}{60}\right)(75)+\frac{1}{2}\left(\frac{45}{60}\right)(75)$ or $= \frac{1}{2}(75)\left(1\frac{1}{2}+\frac{15}{60}\right)$ **[M1]**

$= 18.75 + 18.75 + 28.125$ $= 65.625$ km

$= 65.625$ km

   [A1]

Answer   ....65.625............... km [2]

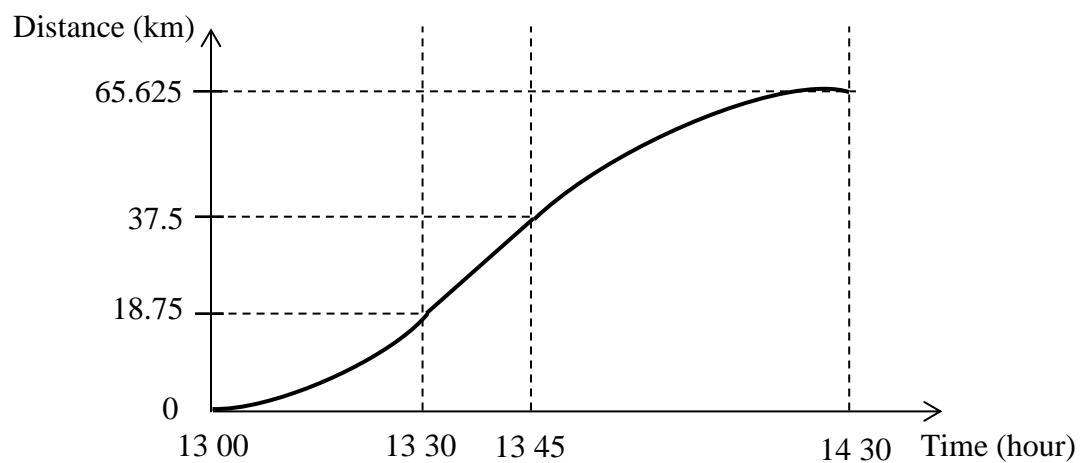**(c)** On the grid in the answer space, sketch the distance-time graph of the motorcycle for the same journey from 1300 to 1430.
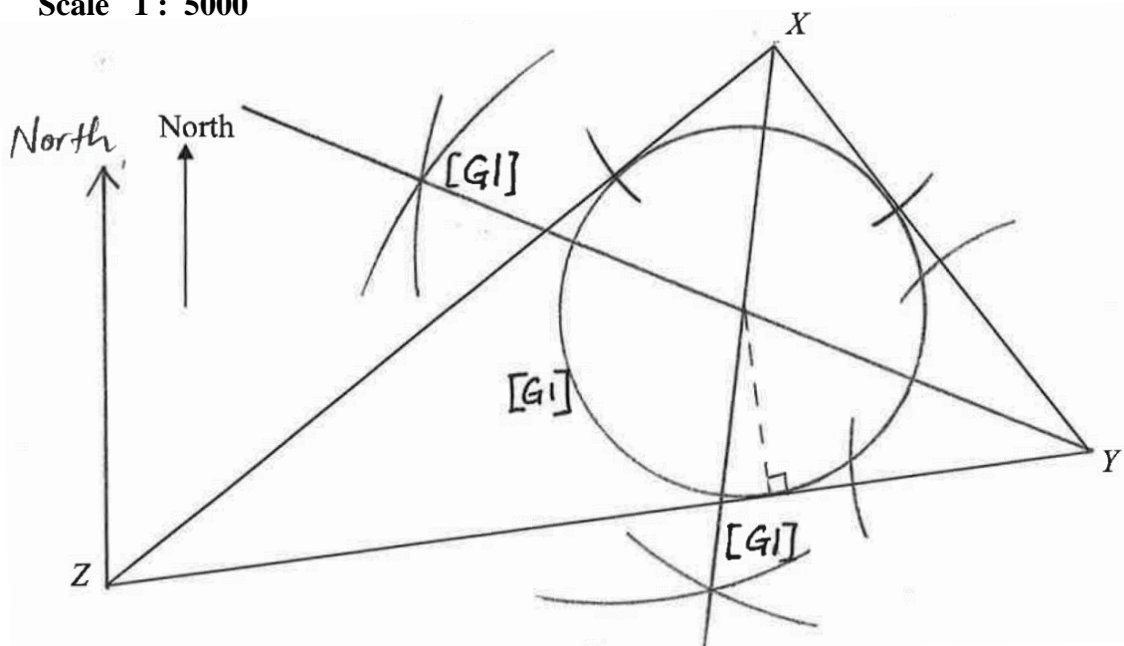
Answer [2]

[B1] **correct shape**

[B1] **correct values on distance axis**

23. The diagram is a scale drawing of a triangular park, *XYZ*.

Scale 1 : 5000

(a) Measure and write down the bearing of *Y* from *Z*.

Answer ....083°............ **[B1]** [1]
(Accept 082° - 084°)

(b) On the diagram, using compasses, construct

    (i) the bisector of angle *ZXY*, [1]

    (ii) the bisector of angle *XYZ*. [1]

(c) A circular playground is to be built inside the triangular park *XYZ*.

    (i) Using compasses, construct a circle touching the three sides of triangle *XYZ*. [1]

    (ii) Find the greatest possible actual area, in m², of the circular playground, correct to the nearest m².

(Accept radius on drawing from 2.3 – 2.5 cm)

Scale : 1 cm : 50000 cm
         1 cm : 50 m
         1 cm² : 2500 m²

Area of circle on drawing $= \pi(2.4)^2 \, \text{cm}^2$

Actual area of circle $= \pi(2.4)^2 \times 2500 \, \text{m}^2$ or $= \pi(2.4 \times 50)^2 \, \text{m}^2$
$= 45239 \, \text{m}^2$ (to nearest m²)

Area on drawing (cm²)	$\pi(2.3)^2$ $=16.619$	$\pi(2.35)^2$ $=17.349$	$\pi(2.4)^2$ $= 18.095$	$\pi(2.45)^2$ $=18.857$	$\pi(2.5)^2$ $=19.634$
Actual area (m²)	41548	43374	45239	47144	49087

Answer ....45239............ **[A1]** m² [2]

End of paper

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