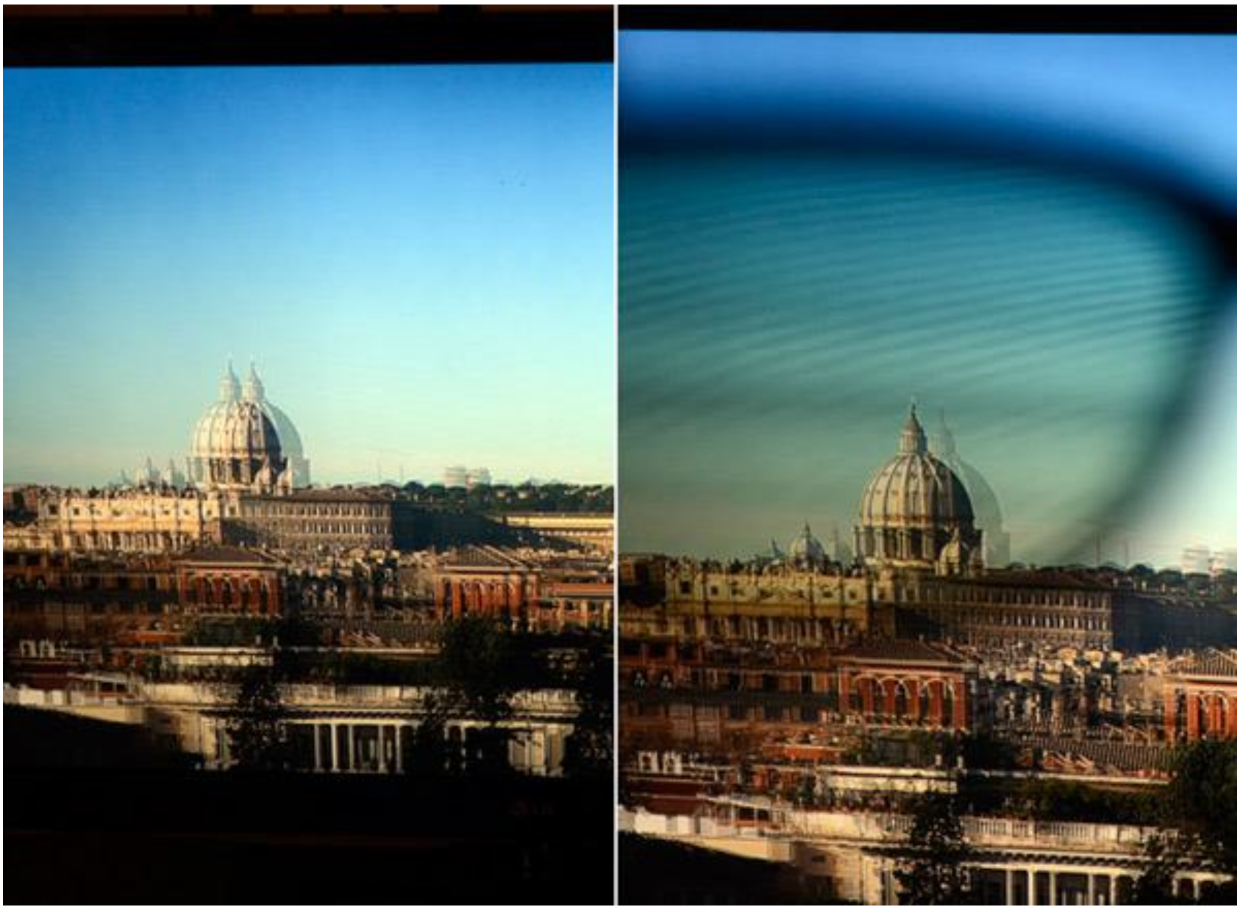
Chapter 9

Wave Motion



Left: a 3D image viewed without glasses on the LG set; right: the same image viewed through 3D glasses shows noticeable ghosting and visible horizontal bands. (Photo taken at closer-than-normal viewing distance.)

Source: Consumer Reports.org

Topic 9: Wave Motion

Syllabus 9749

Content

- Progressive waves
- Transverse and longitudinal waves
- Polarisation
- Determination of frequency and wavelength of sound waves

Learning Outcomes

Candidates should be able to:

(a) show an understanding of and use the terms displacement, amplitude, period, frequency, phase difference, wavelength and speed.

(b) deduce, from the definitions of speed, frequency and wavelength, the equation $v = f\lambda$.

(c) recall and use the equation $v = f\lambda$.

(d) show an understanding that energy is transferred due to a progressive wave.

(e) recall and use the relationship, intensity $\propto$ *(amplitude)*^{2}.

(f) show an understanding of and apply the concept that a wave from a point source and travelling without loss of energy obeys an inverse square law to solve problems.

(g) analyse and interpret graphical representations of transverse and longitudinal waves.

(h) show an understanding that polarisation is a phenomenon associated with transverse waves.

(i) recall and use Malus' law (intensity $\propto \cos^2\theta$) to calculate the amplitude and intensity of a plane polarised electromagnetic wave after transmission through a polarising filter.

(j) determine the frequency of sound using a calibrated oscilloscope.

(k) *determine the wavelength of sound using stationary waves.

* To be covered in Topic 10 Superposition.

Table of Contents

9.1 Introduction

Consider a rope that is held under tension. If we give the left of the rope a small wiggle, a pulse is formed (called the waveform) which can be seen to travel along the length of the rope (Fig. 9.1.1).

When the waveform travels down the rope, **there is a transfer of energy and momentum** from one end of the rope to the other. However each section of the rope only vibrates up and down about an equilibrium position--- **no section of the rope actually travels from one end to the other**.

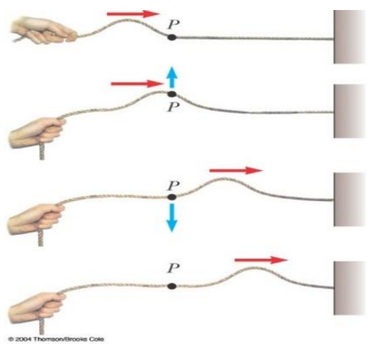


Fig. 9.1.1 Wave pulse travelling down the rope

9.2 Classification of Waves

Waves can be categorised by three broad properties (Fig. 9.2.1):

(i) the type of the wave,

(ii) the mode of vibration of the wave, and

(iii) the motion of the wave.

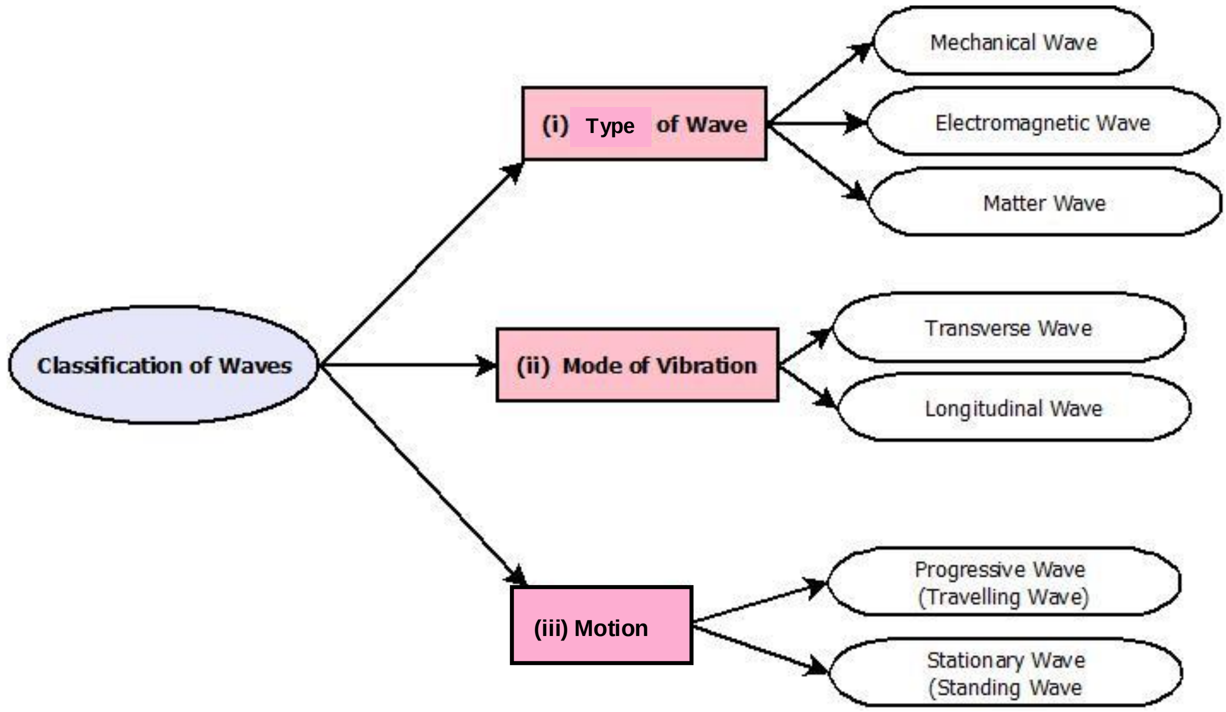


Fig. 9.2.1 Classifications of Waves

For example, a sound wave can be characterised as mechanical in nature, longitudinal in terms of its displacement and can be either progressive or stationary.

Mechanical Waves

Mechanical waves are created when part of a physical medium is disturbed. The disturbance spreads through the medium distributing energy to points far from the original disturbance. All mechanical waves therefore require a medium for propagation.[1] The speed of wave propagation depends on the medium (specifically both its inertial and elastic properties). Sound waves, waves in ropes, water waves and seismic waves are all examples of mechanical waves.

Electromagnetic (EM) waves

Electromagnetic (EM) waves consist of *mutually perpendicular time-varying electric field ($\vec{\mathbf{E}}$) and magnetic field ($\vec{\mathbf{B}}$) oscillating perpendicularly to the direction of wave travel* (Fig. 9.2.2).

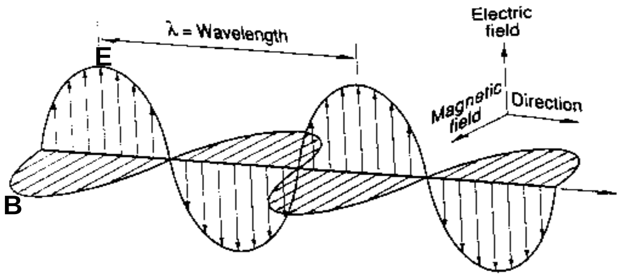


Fig. 9.2.2 The E-field and B-field are mutually perpendicular to each other and both fields are also perpendicular to the direction of wave travel.

Characteristics of EM Waves:

- All EM waves are transverse in nature, i.e. the planes of oscillation of the electric and magnetic fields are perpendicular to the direction of motion.
- All EM waves are self-propagating and hence do not require a medium in which to travel. Unlike mechanical waves they can move through a vacuum but are also able to propagate through materials e.g. light through glass, X-rays through bone and radio waves through walls.
- In vacuum, all EM waves travel at the same speed *c* known as the speed of light:

$$c = 3.00 \times 10^8 \text{ m s}^{-1}$$

The Electromagnetic Spectrum

Common examples of electromagnetic waves include visible light, ultraviolet light, radio wave, microwaves and x-rays. Fig. 9.2.3 shows the electromagnetic spectrum and relative positions of these EM waves on the spectrum.

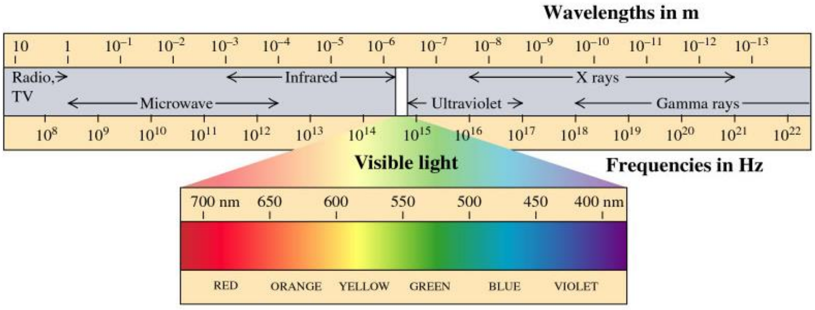


Fig. 9.2.3 The electromagnetic spectrum.

[1] Propagation of mechanical waves is possible due to the restoring forces produced upon deformation; Sound waves, which rely on oscillations of molecules of the medium resulting in pressure variation, obviously cannot be propagated through a vacuum.

The following table shows the names commonly apportioned to the different sections of the EM spectrum and their **corresponding wavelengths and frequencies**.

Type	**Wavelength (λ)**	**Frequency**	**Typical sources, uses and Effects**
Gamma Rays (γ)	~ 10^{-15} m (Size of the nucleus)	~ 10^{20} Hz	From radioactive materials. Used to destroy cancer cells.
X-Rays	~ 10^{-10} m (Size of an atom)	~ 10^{18} Hz	From electrons stopped rapidly in X-ray tube. Causes ionization and fluorescence. Used for X-ray photography.
Ultraviolet (u-v) light	~ 10^{-8} m	~ 10^{16} Hz	From very hot objects, ionizes atoms. Causes fluorescence in some chemicals. Kills germs and causes suntan.
Visible light	~ 400 nm to 700 nm	~ 10^{14} Hz	Only form of radiation visible to the human eyes.
Infrared light	700 nm to ~1 mm	~ 10^{12} Hz	From hot objects. Used for heating. Also used for very short range transmission of information (between laptops, printers, mobile phones.)
Microwaves	~10^{-2} m (Grids of microwave oven)	~ 10^{10} Hz	From electrons oscillating in magnetron. Used for communication (e.g. by cellular mobile phones), radar and cooking.
Radio waves	~ 1 m	~ 10^{8} Hz	From electrons oscillating in aerial. Used for global communication.

- A useful mnemonic to remember for visible light :

Red :	**R**ichard	(700 nm)
Orange :	**O**f	
Yellow:	**Y**ork	
Green:	**G**ave	
Blue:	**B**attle	
Indigo:	**I**n	
Violet:	**V**ain	(400 nm)

The electromagnetic spectrum is a *continuous spectrum* i.e. there are no gaps or *sharp boundaries* between one type of radiation and next. For example, the wavelengths of γ-rays and x-rays overlap. Their key difference is in their mechanism of production.

Transverse and Longitudinal Waves

A **transverse wave** is a wave in which the points of disturbance oscillate about their equilibrium positions ***perpendicular* to the direction of wave travel or energy propagation**.

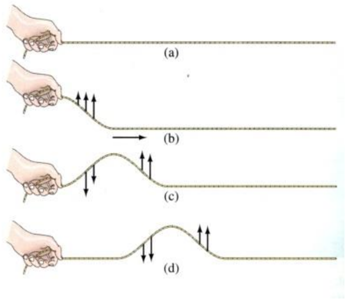


Fig. 9.2.4 Transverse Wave Pulse

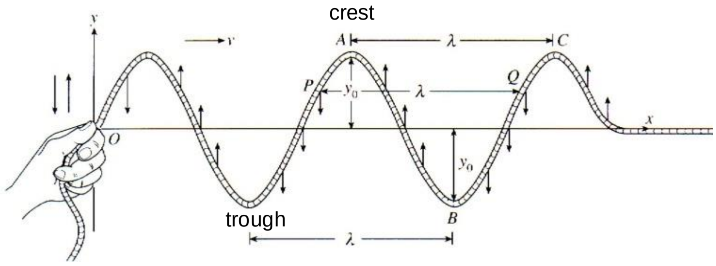


Fig. 9.2.5 Transverse Wave Train

Examples of transverse waves are waves in a rope and electromagnetic waves. Fig. 9.2.4 and Fig. 9.2.5 show a transverse wave pulse and a continuous transverse wave train on a rope respectively.

Fig. 9.2.6 Direction of oscillation of a point compared with direction of travel for a transverse wave.

HTML5 simulation: Transverse Traveling Wave

https://ngsir.netfirms.com/j/Eng/Twave/Twave_js.html

A **longitudinal wave** is a wave in which the points of disturbance oscillate about their equilibrium position along **the direction of wave travel or energy propagation**.

- Fig. 9.2.7 shows a longitudinal wave set up on a slinky. Sound wave is an example of a longitudinal wave.
- In a longitudinal wave, the regions in which the points or particles are closest together are known as regions of "**compression**". The regions in which the point or particles are farthest apart are known as regions of "**rarefaction**". Add in diagrams to show compression and rarefaction.

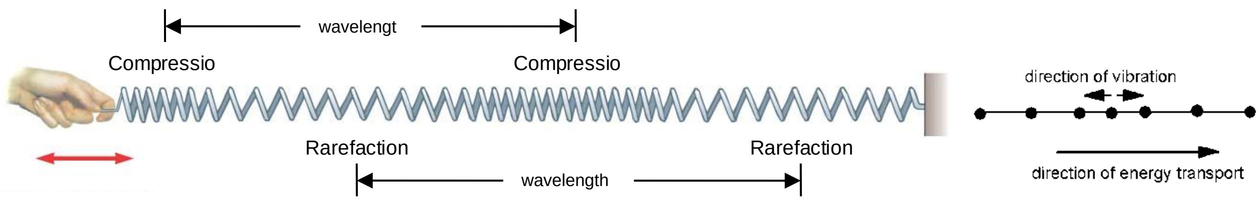


Fig. 9.2.7 The direction of oscillation of a point on longitudinal is parallel to the direction of the wave travel and energy transport

HTML5 simulation: Longitudinal Traveling Wave https://ngsir.netfirms.com/j/Eng/Lwave/Lwave_js.htm

Progressive Waves and Stationary Waves

- Progressive Waves are also known as ***travelling waves***.
    - Progressive waves are waves in which **the wave profile moves away from the source** and **causes energy to be transferred away from the source** to other regions.
    - Fig. 9.2.8 and Fig. 9.2.9 below illustrate a progressive transverse wave pulse and a progressive longitudinal wave pulse respectively.

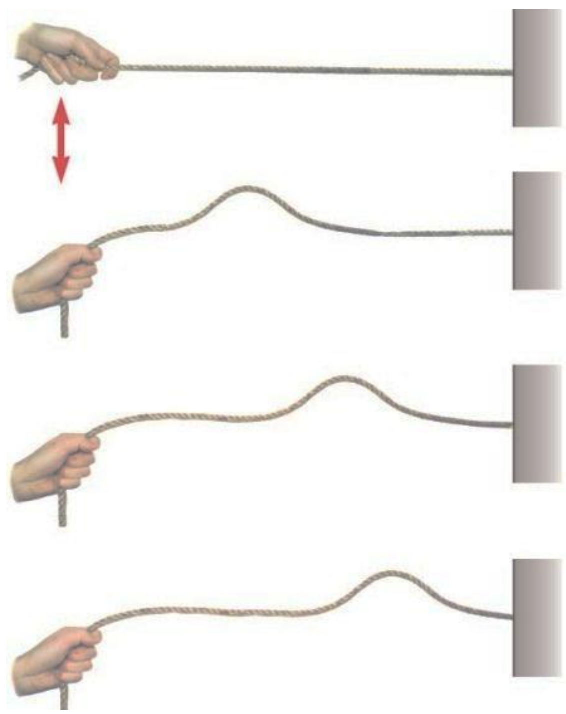


Fig. 9.2.8 Progressive Transverse Wave Pulse

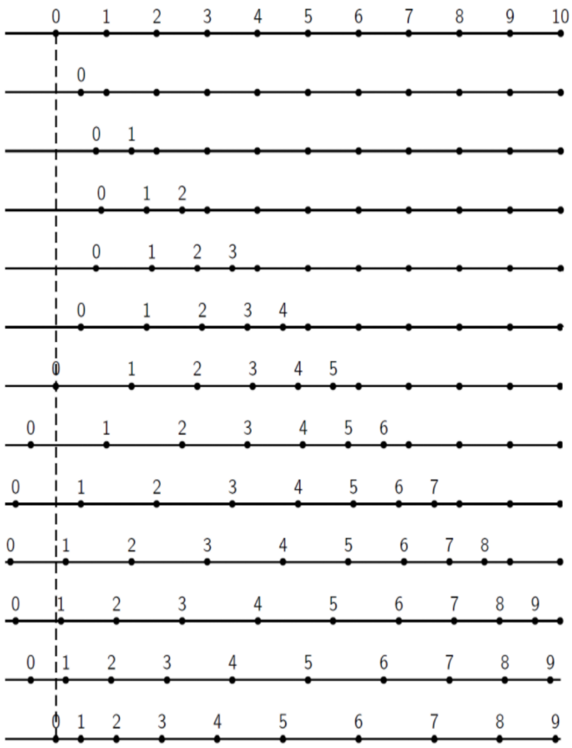


Fig. 9.2.9 Progressive longitudinal wave pulse

- **Stationary waves** are also known as *standing waves*. [2]
    - For a stationary wave, there seems to be no progression of the wave profile in either direction (Fig. 9.2.10) and there are points on the wave which are permanently at rest (known as nodes, N).
    - As there is no progression of the waveform, there is no transfer of energy. Energy is said to be trapped in the waves and does not propagate. The energy is said to be *localised*.

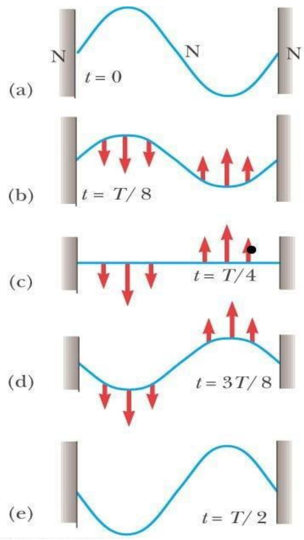


Fig. 9.2.10 A standing wave on a string

HTML5 simulation: Types of Waves

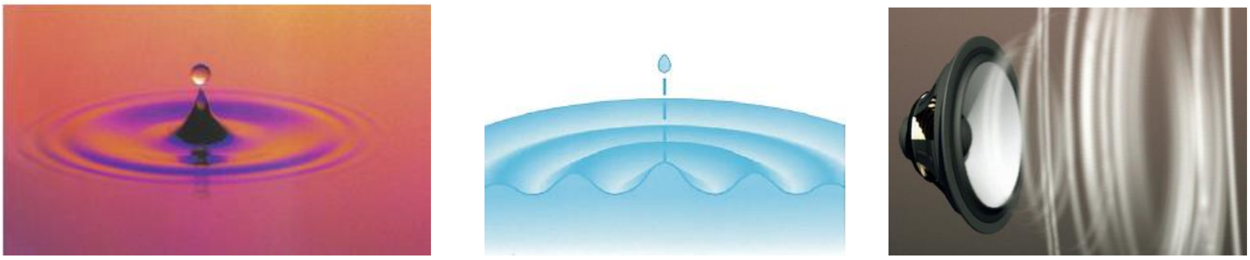
Transverse vs Longitudinal, Traveling vs Standing

https://ngsir.netfirms.com/j/Eng/waves/waves_js.htm

[2] We will study stationary waves in greater depth in the next chapter – Superposition.

9.3 Basic Terminology

The waves that we encounter may be one-dimensional, two-dimensional or three-dimensional. Waves on string and slinky are examples of one-dimensional waves, i.e. the waves are travelling only in one dimension. A water wave that is produced by a raindrop that is striking the surface of a lake is an example of a two-dimensional wave. (Fig. 9.3.1). A sound wave in air is an example of a three-dimensional longitudinal wave. It consists of compressions and rarefactions of air molecules spreading out in all directions



(a) (b) (c)

Fig. 9.3.1 (a) and (b) Two-dimensional water wave (c) Three-dimensional sound waves propagating from a speaker

Sinusoidal Waves

If we sound a tuning fork which produces a "pure tone" i.e. a single frequency and use a microphone and oscilloscope (See Appendix A for review of CRO) to capture the wave form we will observe a near perfect sine profile. So, for a medium that undergo a periodic disturbance of a single frequency, we characterise the waves that emanate from the disturbance and pass through the medium as sinusoidal.

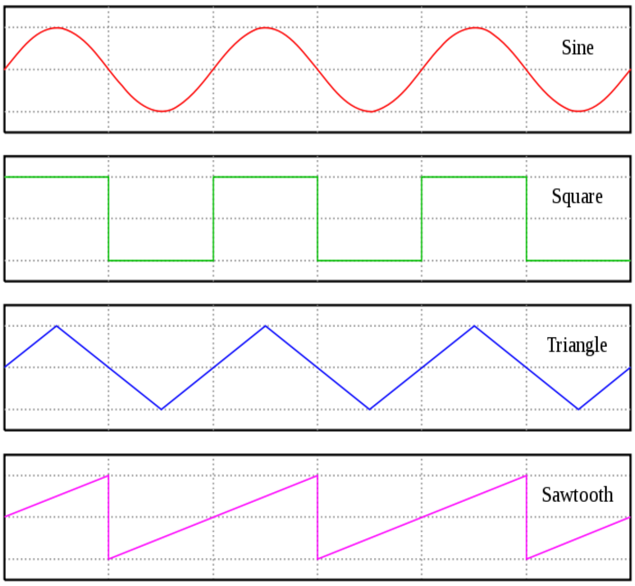


Fig. 9.3.2. Different types of Periodic Waves. All periodic waves can be formed from a combination of sinusoidal waves.

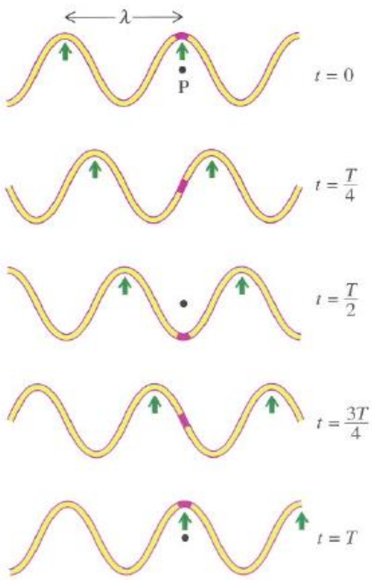


Fig. 9.3.3. As the wave progresses from left to right (see arrow). A point (P) on a wave oscillates simple harmonically.

We will use a transverse sinusoidal wave set up on the string to illustrate some basic terminology used in waves.

Suppose we set the free end of a string in simple harmonic motion. A transverse wave will be set up along the string and propagate down the string.

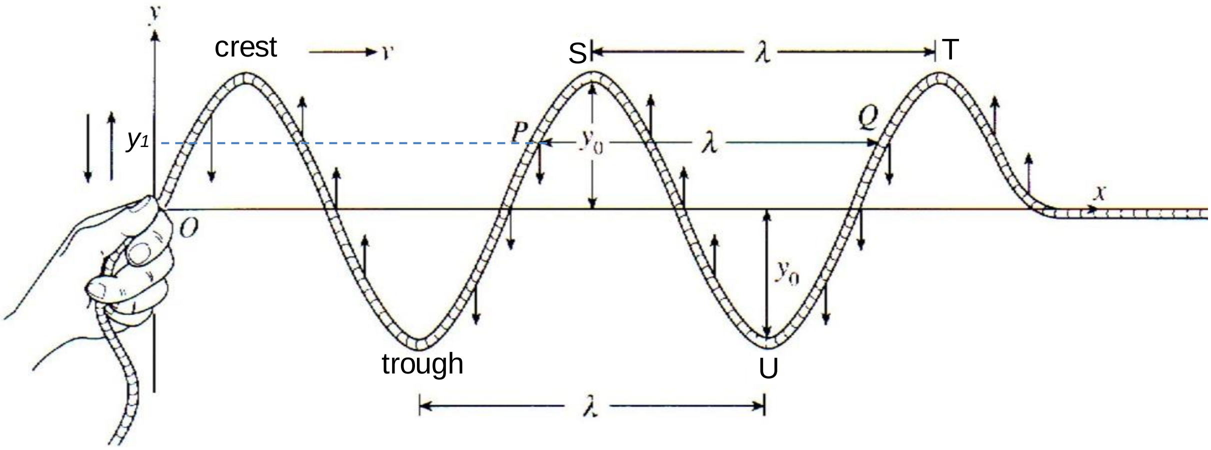


Fig. 9.3.4 The figure shows a snapshot of the wave on the rope at an instant of time.

> **Displacement (y) :**
>
> The displacement of a particular point on the wave refers to the **distance and the direction** of the point from its equilibrium position.

Before the wave is setup, the rope lies along the x-axis. When the wave is setup in the rope, some points on the rope are being displaced from their equilibrium positions.

- Points P and Q are displaced at a distance of y_1 *from* their equilibrium position in the positive direction (as defined by the *y-axis*). Hence, their displacements are both $y = +y_1$.
- Point U is displaced at a distance of y_o from its equilibrium position in the negative direction. Hence its displacement is $y = -y_o$.

> **Amplitude (A or y_o) :**
>
> The amplitude of a wave is the magnitude of the maximum displacement of a point on the wave.

- Fig. 9.3.4: Points S, T and U are at the amplitudes of the wave.

> **Wavelength (λ)** :
>
> The wavelength of a wave is the distance between two adjacent points which are oscillating in phase.[3]

Note:

- For a transverse wave, the distance between two successive crests or two successive troughs corresponds to a single wavelength.
- For a longitudinal wave, the distance between two successive compressions or rarefactions also corresponds to a single wavelength.

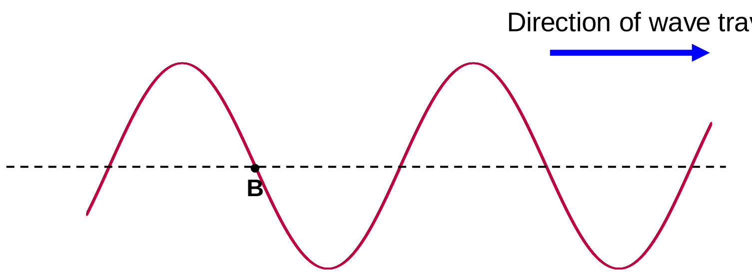
[3] This refers to two points in the same state of oscillation. The concept of phase associated with oscillations will be dealt with further in the chapter.

Example 9.3.1:

Consider the snapshot of the wave shown below. Other than the distance between the 2 successive crests or troughs,

(i) identify another point (B') that pairs with B that will correspond to a single wavelength.

(ii) Choose another pair of points on the wave that correspond to one wavelength.



(iii) What do you notice about the pairs of points that represent one wavelength?

Each pair of points not only have the _______________________________________ but also the _____________________________. They are said to be in the same state of oscillation or ________________________.

Period (*T*) :

Period is the time taken for a point on the wave to *complete one oscillation cycle*.

- Period is also the time taken for the wave to advance by a distance of one wavelength or the time taken for *2 consecutive crests/ compressions or troughs (rarefactions) to pass through a certain point.* (see Fig. 9.3.3)

Frequency (*f*):

Frequency is the *number of oscillations per unit time* made by a point on a wave.

- Frequency is also the number of crests/compressions or troughs/ rarefactions that pass through a point along the path of wave travel in a unit time.
- S.I. unit of frequency is s^{-1} or *Hz (hertz)*.
- The relationship between frequency *f* and period *T* is given by : $f = \frac{1}{T}$

Wavefronts

- A **wavefront** (of a 2-dimensional or 3-dimensional wave) is an imaginary line or curve which joins points on the wave which are oscillating in phase.
    - *All points on a wavefront are at the same part of the cycle* of their vibration.
    - By convention, when we represent waves in a diagram, the *distance between consecutive wavefronts is one wavelength apart*.
- The direction of propagation of the waves perpendicular to the wavefront as indicated by arrows (→) are known as *rays*.

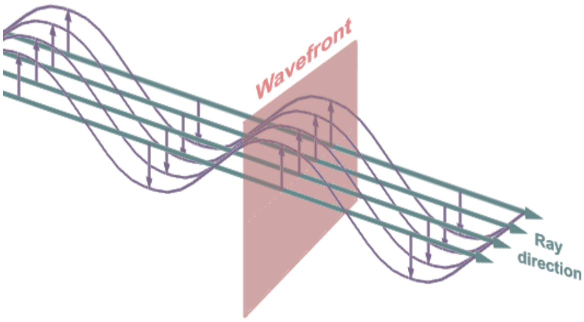
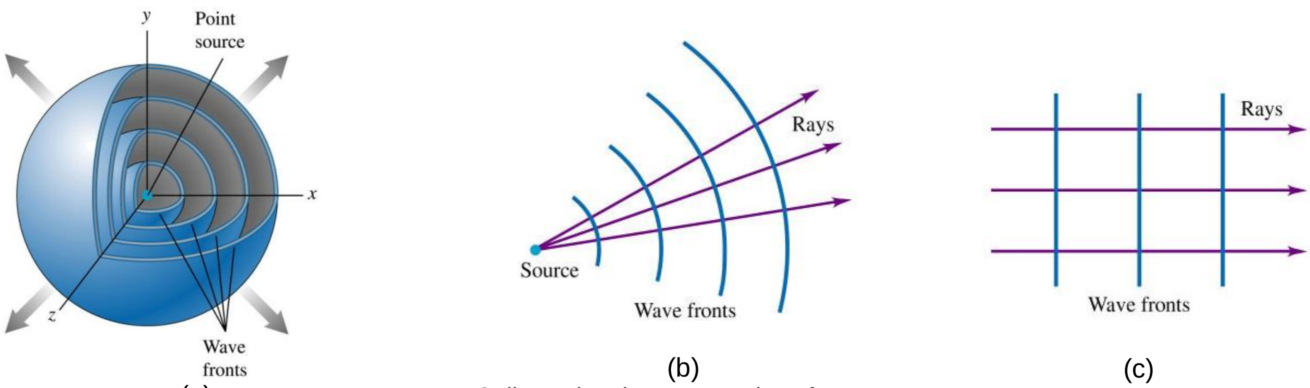


Fig. 9.3.5. Wavefront of a wave.



(a)
3-dimensional waves that spread out uniformly in all directions from a point source.

(b)
2-dimensional representation of waves from a point source.

(c)
2-dimensional representation of waves from a planar source.

Fig. 9.3.6. Wave representations using wave fronts and rays.

<u>**Wave Speed (*v*) :**</u>

The speed of a wave is the distance that the wave profile travels per unit time.

Note:
There are two sets of motion associated with waves.

- One is that of the motion associated with the oscillation of the individual points on the wave, the points always oscillate about the equilibrium position.
- The other is that of the motion associated with the propagation of waveform of the wave. This is the one that defines the wave speed.

Relationship between Frequency, Speed and Wavelength

The time taken for a wave to move a distance of one wavelength λ is one period *T*.

From the definition of wave speed: Speed of the wave, $v = \dfrac{\text{Distance Travelled}}{\text{Time Taken}} = \dfrac{\lambda}{T}$

Now, $f = \dfrac{1}{T} \Rightarrow$ $v = f\lambda$ where *f* is the frequency of the wave.

Example 9.3.2 : Transverse wave

The diagram below shows an instantaneous position of a string as a transverse progressive wave travels along it from left to right.

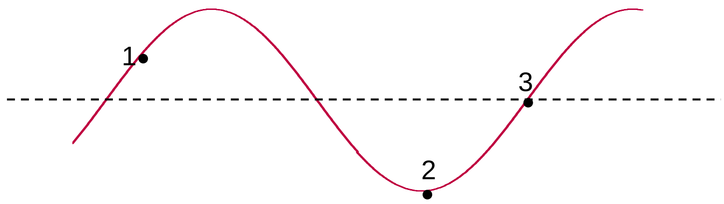


Fig. 9.3.7

(a) Taking upwards to be positive, for each of the points 1, 2, and 3 on the string,

(i) state whether the displacement of each point is positive or negative.

(ii) What are the directions of the velocities of the points?

(iii) Of the three points, which one has the largest speed?

(b) Mark on the string a point A with zero velocity.

(c) What is the direction of acceleration of point A?

Example 9.3.3: Longitudinal Wave

Fig. 9.3.8 shows a 'snapshot' of a horizontal slinky spring when it is (a) at rest and (b) carrying a longitudinal wave moving from left to right. There is no energy dissipation along the slinky spring.

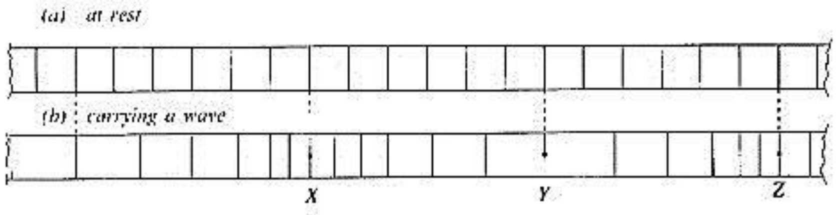


Fig. 9.3.8

State and explain whether each of the following statements is true or false.

Statement A: The distance between X and Y is one wavelength.

False. XY is the distance between the centre of a compression and the centre of a rarefaction. Hence XY corresponds to _____ λ. (_______ corresponds to 1λ)

Statement B: The amplitude of oscillation of Y is greater than the amplitude of oscillation of X.

False. Since no energy is dissipated, the amplitude of oscillation is the same for all points for a 1-D progressive wave. Hence, amplitude of X = amplitude of Y.

Statement C: The displacement of X at this instant is zero.

True. X is at its ________________ position. Hence its displacement at this instant is ___________ (same for Y, Z).

Example 9.3.4: Speed of a Wave

A fisherman notices that wave crests pass the bow of his anchored boat every 6.0 s. He measures the distance between two adjacent crests to be 15.0 m. How fast are the waves travelling?

Example 9.3.5

Calculate the frequency of green light of wavelength 0.60 μm.

9.4 Graphical Representation of Waves

Consider a transverse water wave travelling from left to right as shown in the diagram below.

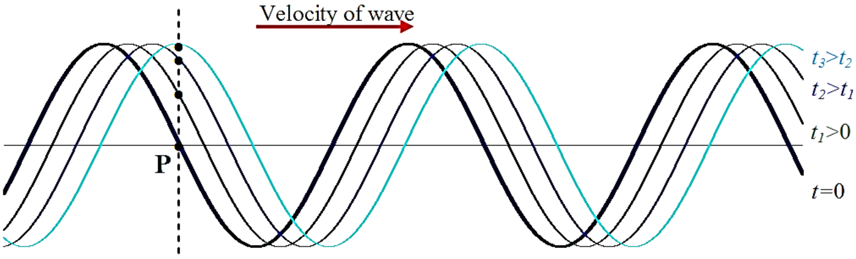


Fig. 9.4.1. A progressive transverse wave moving towards the right.

As the wave moves through this region of space, we note the following:

- At a particular instant of time, the *displacement of each point on the wave varies with the distance it is located from the source*.
- For a particular point on the wave e.g. at **P**, the *displacement of the point also varies with time*.

Therefore, to completely represent a wave, we need two graphs:

1 **Graph of displacement (*y*) varies with position (*x*)** of the points on a wave (at a particular instant of time.)

At a particular instant of time $\Rightarrow$ Graph of y vs. x.

2 **Graph of displacement (*y*) varies with time (*t*) for a particular point on the wave** (at a particular position).

At a particular position $\Rightarrow$ Graph of y vs. t.

Graph of Displacement (*y*) vs. Position (*x*)

Consider a snapshot of a wave on a rope. (Fig. 9.4.2). By superimposing a pair of x and *y* axes on the snapshot, we can describe how the displacement of each point on the rope varies with position.

This graph of displacement vs. position at a particular instant of time (*y vs. x*) gives the **wave profile at an instant of time.** You can imagine this as a snapshot of the wave at a particular instant.

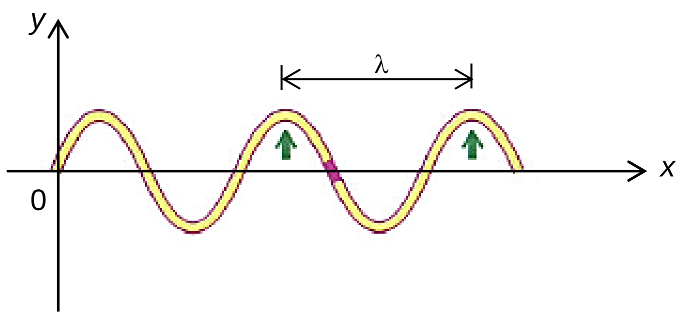


Fig. 9.4.2. Displacement-position graph of a wave on a rope.

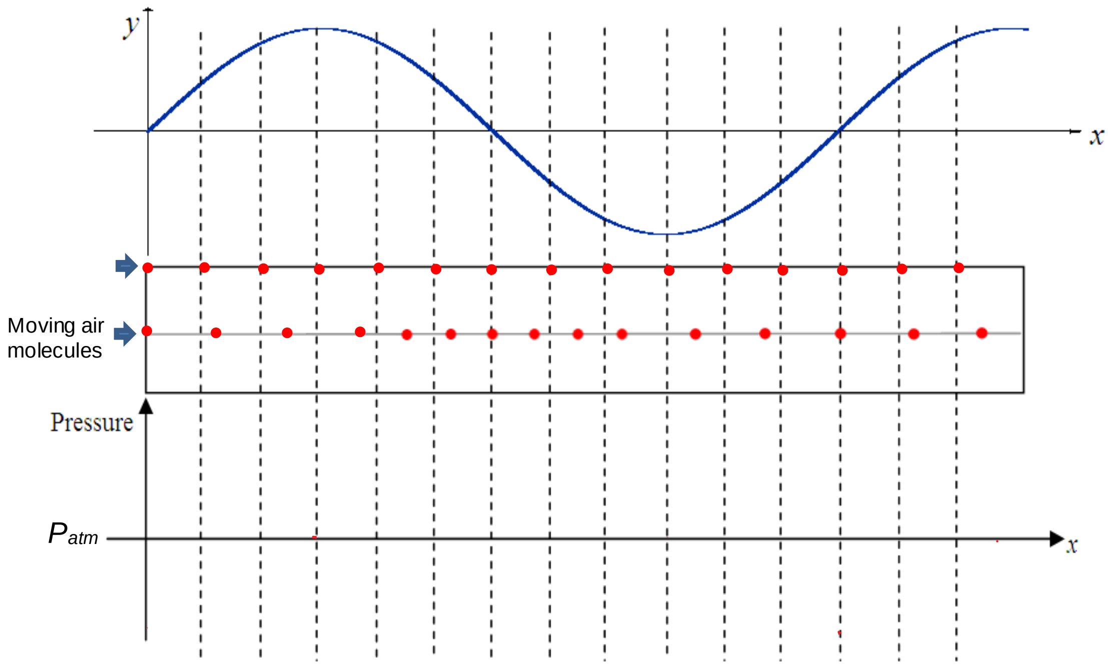
Note :

- +ve *y* : displacement of a point on the wave when it is above its equilibrium position
- -ve *y*: displacement of a point on the wave when it is below its equilibrium position
- *x* axis ($y = 0$) : equilibrium position of points when there is no disturbance

$\Rightarrow$ ***Wavelength (λ) of the wave can also be obtained from the graph.***

Example 9.4.1: Displacement-Position Graph for a Longitudinal Wave

A displacement–position graph can also be drawn for a longitudinal wave. Consider the following displacement-position graph for the air molecules along a line when a sound wave passes through a region of space. Sketch the corresponding pressure variation of this region.



Note: Points of **compression and rarefactions** corresponds to points where **displacements of air molecules are zero.**

Compressions are regions of **high air pressure**, while **rarefactions** are regions of **low air pressure**.

Graph of Displacement (*y*) vs. Time (*t*)

Consider a point P on a rope. As a wave propagates through the rope (Fig. 9.4.3), the displacement of this point varies sinusoidally with respect to time (as shown in Fig. 9.4.4).

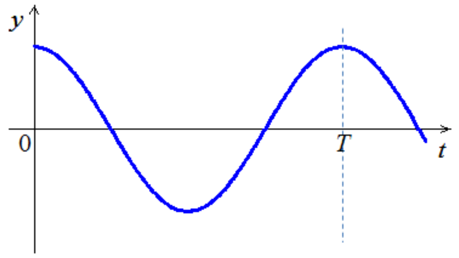


Fig. 9.4.4.

The graph of displacement vs. time for a wave gives us **the variation of the displacement of a particular point on a wave (at a fixed position *x*) with respect to time.** You can imagine it as a motion video of a particle in a wave.

⇒ **The period (*T*) associated with the wave can be obtained from this graph.**

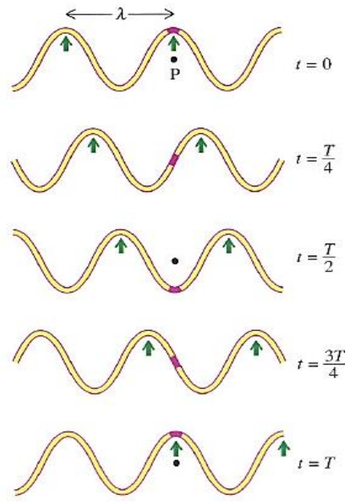


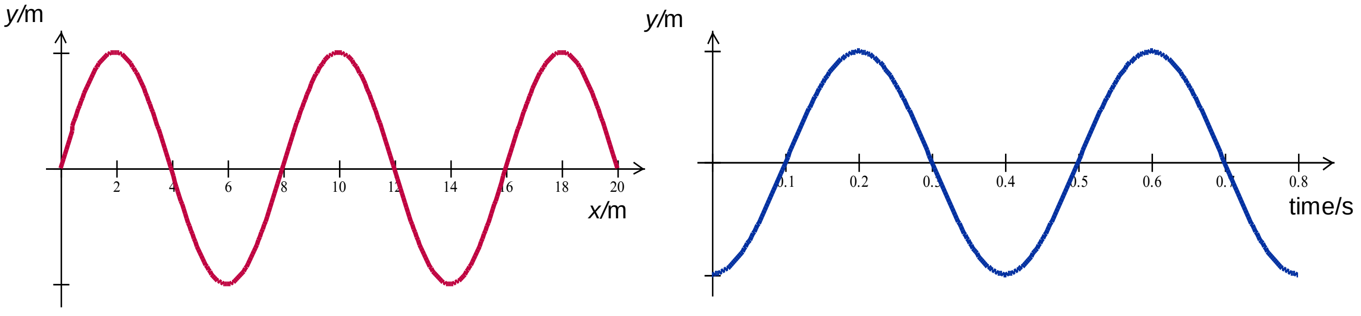
Fig. 9.4.3. A sinusoidal wave travelling on along a rope.

In summary, two graphs are needed to fully depict a wave:

- The graph of **displacement vs. position** (wave profile of a wave at a certain instant of time),
  - ⇒ Wavelength λ may be obtained
- The graph of **displacement vs. time** (Motion of a particular point on a wave w.r.t. time)
  - ⇒ Period *T* of the waves may be obtained.

Example 9.4.2: Determining the speed of a wave

A vibrating bar makes waves in a ripple tank. Fig. 9.4.5(a) shows the displacement of the wave as it travels out from the bar. The position of the floating cork in the tank varies with time as shown in Fig. 9.4.5(b).



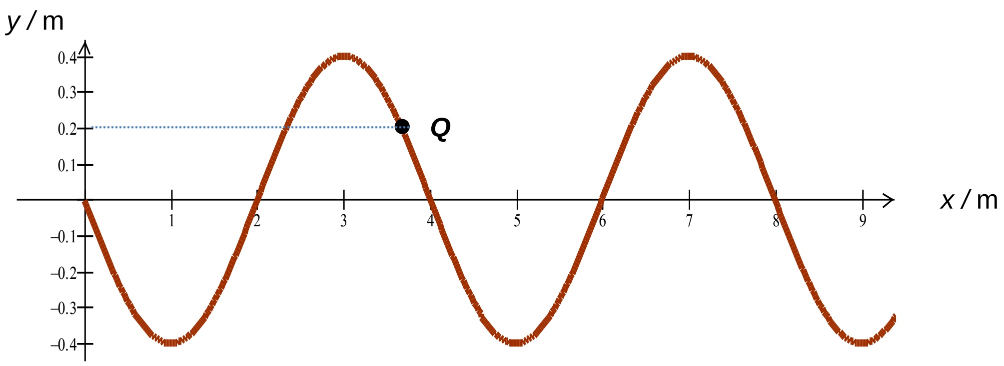
(a) **(b)**

Fig. 9.4.5

From the above two graphs, determine the speed of the wave.

Example 9.4.3

The following graph shows a displacement vs. position graph of a progressive transverse wave at t = 0. For the point **Q** on the wave, sketch the corresponding displacement vs. time graph, given that the wave is moving in the positive x direction at a speed of 1.0 m s^{-1}.



9.5 Phase and Phase Difference

Phase of a particle in a wave at any given time indicates the particle's current state of vibration, usually expressed as an angle. (Also refer to Oscillations LN p.15)

Phase difference in waves refers to the difference in phase:

(a) at two different instances of time for a point on a wave
(b) two points on a wave, or
(c) between two waves.

Note the axes for the subsequent explanations of (a) to (c).

(a) Phase difference (Δ∅) for a point on a wave at two different time instances

For a sinusoidal progressive wave, every point on the wave oscillates simple harmonically. Hence, the phase difference, Δ∅ at two different instances of time for a point on a wave that are at an interval of Δt apart from **displacement vs. time graph** is given by:

$$\boxed{\frac{\Delta t}{T} = \frac{\Delta \phi}{2\pi}}$$

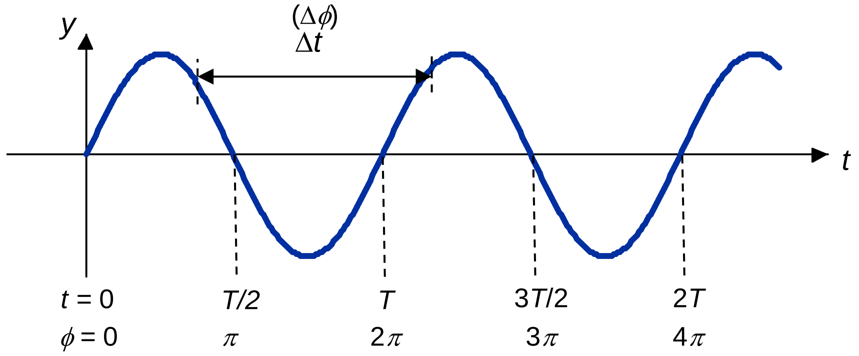


Fig. 9.5.1

(b) Phase difference between two points on a wave

(i) One complete wavelength is represented by a phase angle of 360° or 2π radians. Therefore, the **phase difference** $\Delta\phi$ **between two points on a wave separated by a distance** Δx **from displacement vs. position graph** is given by:

$$\boxed{\frac{\Delta x}{\lambda} = \frac{\Delta \phi}{2\pi}}$$

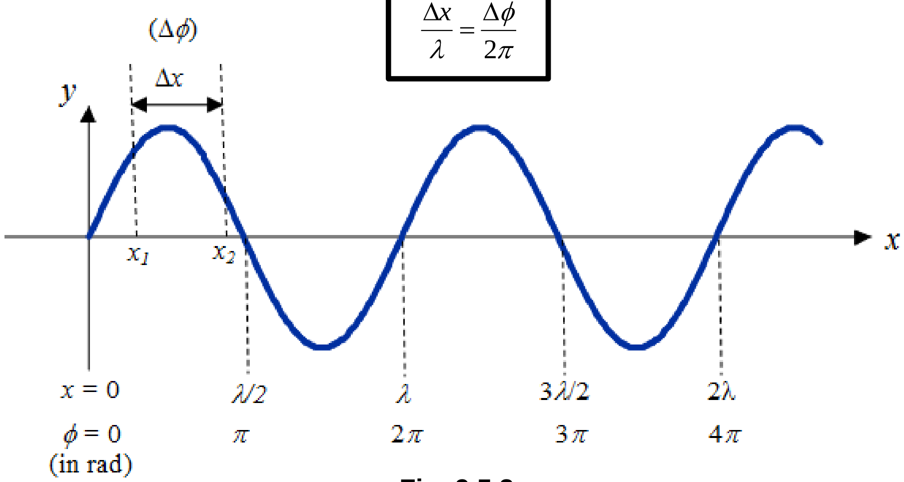


Fig. 9.5.2

(ii) The displacement vs. time graphs of the two points on a wave can also be used to compare the state of oscillation on the wave.

We can plot on the same axes the **displacement vs. time graphs** for the two points on the wave in Fig. 9.5.2 (one graph for each point). If the wave is progressing in the positive *x* direction, we will get the following graphs for the points at x_1 and x_2 as shown in Fig. 9.5.3:

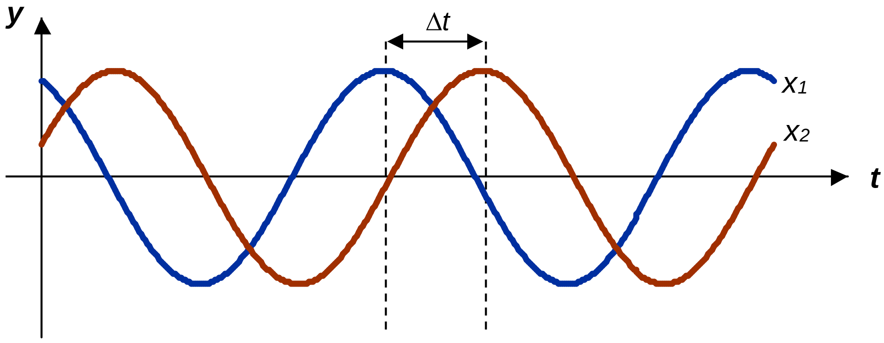


Fig. 9.5.3

From the graphs above, we can clearly see that, the oscillation of the point at x_2 always lags behind the oscillation of the point at x_1 by a time interval of Δt, and hence point at x_2 lags behind point at x_1 by a phase angle of $\Delta\phi$ where

$$\boxed{\frac{\Delta t}{T} = \frac{\Delta \phi}{2\pi}}$$

(c) Phase difference between two waves

We can also compare *the phase (angles) between two waves (usually, having the same frequency)* by comparing the oscillations of every point on a wave at a particular instant.

Consider two waves ***P*** and ***Q*** having the same wavelength and velocity moving towards the right as shown in Fig 9.5.4.

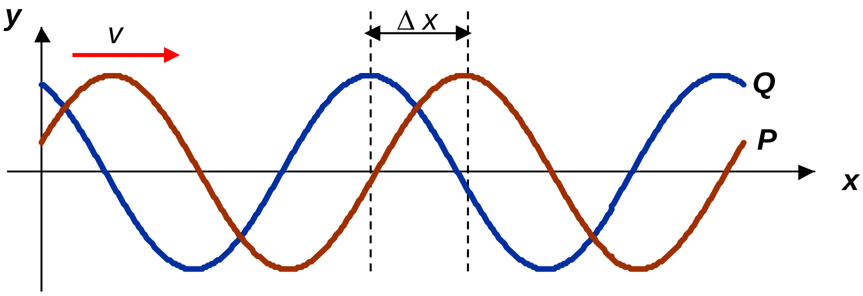


Fig. 9.5.4

Notice that the Wave P always leads Wave Q by a constant distance of Δx. If the wave Q shifts a distance towards the right by Δx then every point on Wave Q will be in the same phase of oscillation on points on wave *P* at this moment.

Therefore, we say that

- *Wave Q lags Wave P by a phase difference of $\Delta\phi$*, and
- the **phase difference between the two waves is $\Delta\phi$** can be found from: $\boxed{\frac{\Delta x}{\lambda} = \frac{\Delta \phi}{2\pi}}$

Note :

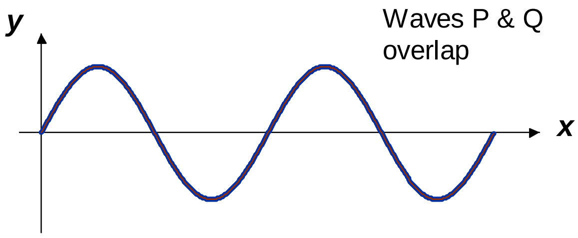
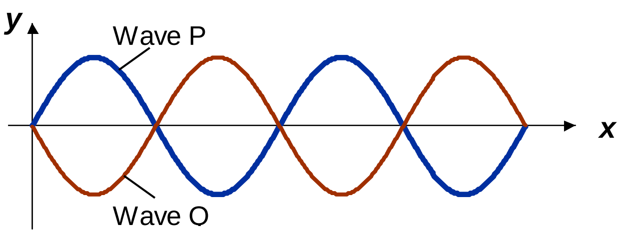


Fig. 9.5.5 (a) Wave P is in phase with wave Q



(b) Wave P is in antiphase with wave Q

1. **The two waves are said to be in phase** if every point on wave **P** is at the same state of oscillation as the corresponding point of the same position in wave **Q** at the same time. The phase difference between the two waves are $\Delta\phi = 0, 2\pi, 4\pi, \ldots$
   - Notice that when the two transverse waves are in phase, their "*crests*" will correspond and their "*troughs*" will also correspond.

2. When the phase difference between the two waves are $\Delta\phi = \pi, 3\pi, 5\pi, \ldots.$, we say that the **two waves are in antiphase.**
   - When the two waves are in antiphase, every particle in Wave P is in antiphase with the corresponding point.
   - Note that the waveforms seem like a reflection of one another.

Example 9.5.1 :

(a) The following graph shows the displacement vs. position of two waves **P** and **Q** at a particular instant of time, what is the phase difference between the two waves **P** and **Q**?

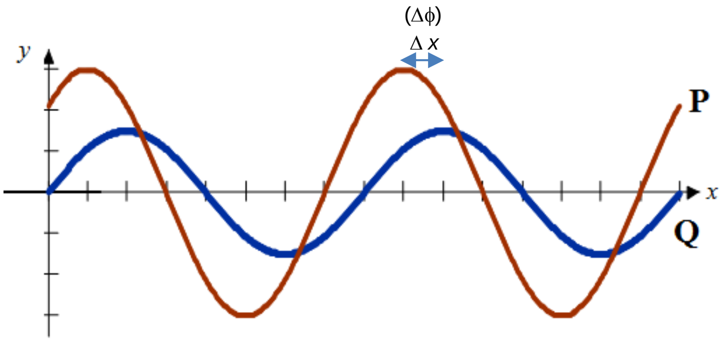


Fig. 9.5.6

(b) Is it possible to compare the phase difference between two waves of different wavelengths? What is the phase difference between the two waves shown below?

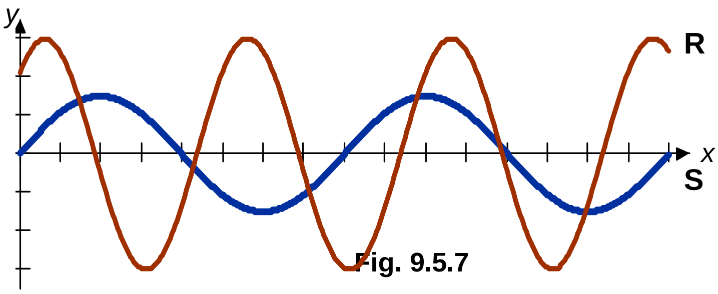


Fig. 9.5.7

Answer:

No. As Δx is not constant for the 2 waves, the phase difference between the two waves keeps changing value.

It is only meaningful to talk about phase difference between 2 waves only if they have the same wavelength or same frequency.

9.6 Energy Transmitted by a Wave

As a wave propagates, it transports energy from one place to another. Consider a sinusoidal wave moving through a string shown in Fig. 9.6.1, the source of energy is some external agent at the end of the string which does work in producing the oscillations.

Fig. 9.6.1

As the external agent performs work on the string, it transfers energy to the string in the form of kinetic energy and (elastic) potential energy to each part of the string.

From Chapter 8 Oscillations, we know that the total energy E of a simple harmonic oscillator is given by $E = \frac{1}{2}m\omega^2 A^2$. Since every part of the string is in simple harmonic motion,

Energy transmitted by a wave to a point is proportional to the square of the amplitude of the oscillator at that point.

$$\text{Energy} \propto (\text{Amplitude})^2$$

Intensity of a Wave

Waves on a string carry energy in one dimension of space (along the direction of the string). But most of the waves that we commonly encounter like the sound waves in air and seismic waves in the body of the earth carry energy along all three dimensions of space. As the wave spreads out in space, the power received per unit area along the path of the wave diminishes as the distance from the source of the wave increases.

- **Power of a Wave:**

  The amount of energy transported per unit time by a wave is called the *power* of a wave.

  The SI unit is joules per second (J s^{-1}) or watts (W).

<u>Wave Intensity, *I* :</u>

The **intensity of a wave** is the rate at which energy is transported by the wave, per unit area, across a surface perpendicular to the direction of propagation i.e. it is the power per unit area.

$$Intensity, I = \frac{Power}{Area}$$

- **S.I. unit of intensity:** W m^{-2}

Since the energy transmitted by a wave is proportional to the square of the amplitude of the wave, so similarly, we have

$$\text{Intensity} \propto (\text{Amplitude})^2$$

Intensity of a Wave from Point Source or Spherical Source

In the special case where the source is **point source** or **spherical source**, and the propagation is *isotropic* (i.e. the same in all directions), then the wavefronts of the waves are spherical in shape. The wave itself is said to be a *spherical wave*.

The energy spreads out over the wavefront as the wave propagates, hence, the intensity I at a particular point a distance r from the source is given by :

$$\text{Intensity, } I = \frac{\text{Power}}{\text{Area}} = \frac{P}{4\pi r^2}$$

where P is the time rate of energy delivered by the wave to a distance of r away from the source.

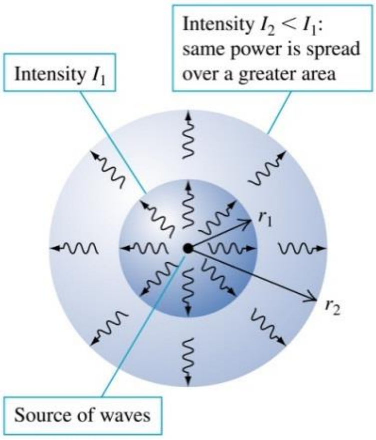


Fig. 9.6.2

Therefore, we see that the **intensity I at any distance r is therefore inversely proportional to r^2.**

$$I \propto \frac{1}{r^2} \qquad \textit{(For constant P)}$$

Note:

- This relationship is called the *inverse-square law for intensity*.
- This relationship is only valid for *point sources / spherical sources* and is based on the assumption that **the power of the wave does not diminish as it spreads out.**

<u>Example 9.6.1 : (N94/I/10 – modified)</u>

A sound wave of amplitude 0.20 mm has an intensity of 3.0 W m^{-2}.

(a) What will be the intensity of a sound wave of the same frequency which has an amplitude of 0.40 mm?

(b) How will the intensity change if the frequency of the source of wave is doubled but the amplitude remains unchanged? Treat the source as a simple harmonic oscillator.

Example 9.6.2 :

If the intensity of an earthquake *P*-wave is 1.0×10^6 W m^{-2} at 100 km from the source, what is the intensity of the wave 400 km from the source?

9.7 Polarisation of Waves

One characteristic that distinguishes *transverse waves* and *longitudinal waves* is *polarisation*[4].

Plane Polarisation

Polarisation is a phenomenon whereby vibrations in a transverse wave are restricted to only one direction in the plane normal to the direction of energy transfer.

To polarise a wave, we often use a *polarising filter*, or *polariser*, that permits only waves with a certain polarisation direction to pass through.

Consider waves set up on a string. A slot as shown in Fig. 9.7.1 can be used to limit the string's plane of propagation to the x-y plane and the oscillation to the y-axis.

Polarising Axis or Transmission Axis

The polarisation direction of the polariser is also known as the ***polarising axis*** or ***transmission axis***.

Only components of the waves parallel to the polarising axis are allowed to pass through.

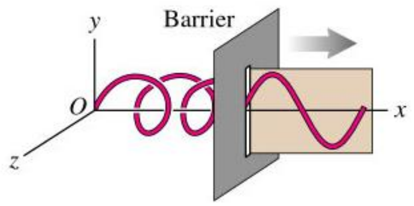


Fig. 9.7.1 A vertical slot is used to polarise the wave on the string so that oscillations are limited to a vertical plane.

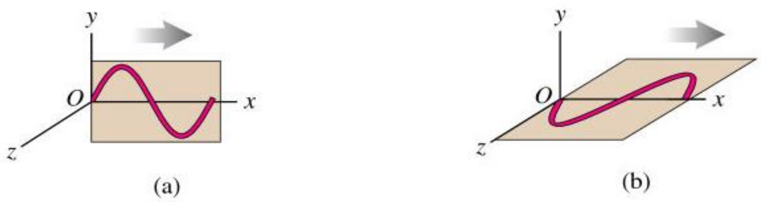


Fig. 9.7.2 A wave on a string polarised to oscillate in the *x-y* plane (a) and *x-z* plane (b). For each case, what is the polarizing axis on the polarizer?

[4] For A-level syllabus, we will consider only plane polarisation, where we limit the oscillations to a plane. If you are interested in knowing more about Circular Polarisation or Elliptical Polarisation, you may refer to Young and Freedman (11th Edition) Chapter 33.5 Pg 1268.

Why Longitudinal Waves cannot be polarised?

Polarisation **does not occur** for longitudinal waves because *oscillations of the points on the wave are along the direction of wave propagation*. Restricting the direction of oscillation will in turn restrict the direction of wave propagation.

Therefore, **longitudinal waves cannot be polarised.**

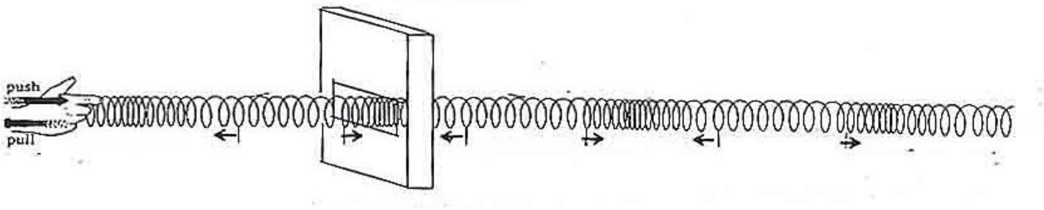


Fig. 9.7.3 Longitudinal wave on a slinky cannot be polarised as the direction of oscillation of the elements on the slinky in parallel to the direction of propagation of the longitudinal wave. Limiting the direction of oscillation will therefore also limit the motion of the wave.

In summary,

1. Only transverse waves can be polarised. Longitudinal waves cannot be polarised.
2. Only the component of the wave that is parallel to the **polarising axis** of the polariser will pass through the polarisers.

9.7.1 Polarisation of Light

Light waves are electromagnetic (EM) waves which consist of *mutually perpendicular time varying electric field (*$\vec{\mathbf{E}}$*) and magnetic field (*$\vec{\mathbf{B}}$*) oscillating perpendicularly to the direction of wave travel*. (See Fig 9.2.2 on p.2)

EM wave is commonly represented by *just the electric field* ($\vec{\mathbf{E}}$). The arrows indicate the plane of oscillation of the **E**-field.

- Fig. 9.7.4 shows a plane polarised EM wave travelling out of the paper towards you.
- Fig. 9.7.5 shows an *unpolarised* EM wave travelling out of the paper. Note the multiple planes of oscillation in the unpolarised EM wave.

Fig. 9.7.4 Polarised EM Wave

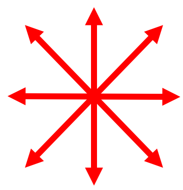


Fig. 9.7.5 Unpolarised EM Wave

When unpolarised light is incident on an ideal polariser, the intensity of the transmitted light is exactly half that of the incident unpolarised light, no matter how the polarising axis is oriented.

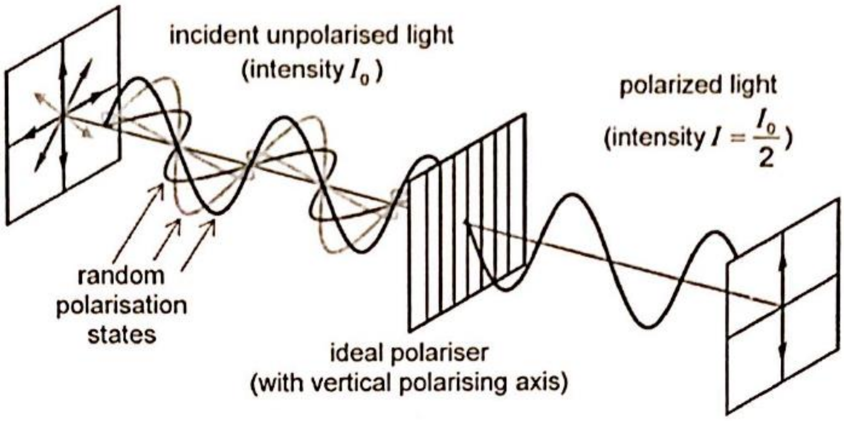


Fig. 9.7.6 Intensity of plane polarised light

Since incident light is a random mixture of all polarisation states, the components of the electric field vectors that are perpendicular or parallel to the polarising axis is roughly equal. By only transmitting the component that is parallel to the polarising axis, **only half the incident intensity** is transmitted.

An ideal polariser passes 100% of the incident light that is plane-polarised along its polarisation direction and 0% of the light that is 90° to its polarisation direction. Since unpolarised light is a mix of all possible polarisations, an ideal polariser transmits only 50% of the incident unpolarised light intensity.

Light Passing Through Two Polarisers

By using two polarisers, we can also control the intensity of light.

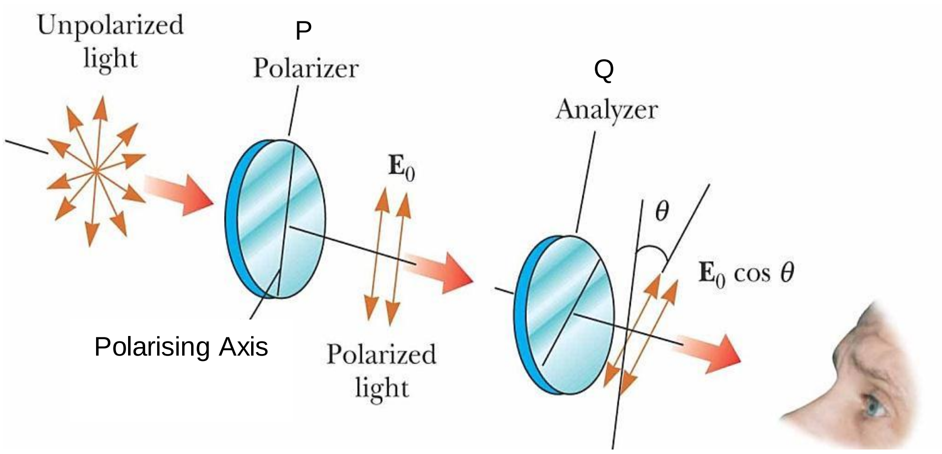


Fig. 9.7.7 Using two polarisers to control intensity of light.

Fig. 9.7.7 shows the typical setup used to control the intensity of light. An unpolarised light beam is incident on two polarisers **P** and **Q** with their polarising axes at an angle of θ with each other.

- In such a setup, the second polariser is often known as an **analyser**.
- After passing through the first polariser **P**, the light is polarised along the polarising axis of polariser P. Suppose the light then has an amplitude of $\mathbf{E_o}$ and an associated maximum intensity of I_o.
- If this polarised light then falls on a second polariser Q, *then the component of* $\mathbf{E_o}$ *perpendicular to the analyser axis is completely absorbed, while only the component parallel to polarising axis is allowed to transmit through Q.*
- Since the plane of oscillation of the electric field of this light is oriented at an angle of θ to the polarising axis of Q, the resultant amplitude of vibration of the light after passing through the polariser Q is given by $\mathbf{E = E_o \cos\theta}$.

- Since the intensity of a wave is directly proportional to the square of the amplitude of vibration, therefore,

$$\frac{\text{Intensity of Light after passing through the analyser Q}}{\text{Intensity of Light after passing through the polariser P}} = \frac{I}{I_o} = \left(\frac{E_o \cos\theta}{E_o}\right)^2$$

$$\Rightarrow \qquad I = I_o \cos^2\theta$$

Hence the intensity of light that passes through two polarisers that are aligned with an angle between their polarisation directions is

$$\boxed{I = I_o \cos^2\theta}$$

where I_o is the maximum intensity of light transmitted when the two polarisers are aligned along the same direction of polarisation. The above equation is known as **Malus' law** after its discoverer Etienne Louis Malus, who first observed this effect experimentally in 1809.

Malus's law applies only if the incident light passing through the analyzer is already *linearly polarized.*

Malus' law states that the intensity of a beam of plane-polarised light after passing through a polariser varies with the square of the cosine of the angle through which the polariser is rotated from the position that gives maximum intensity.

Based on Malus' law, when 2 polarisers are aligned at 90° with respect to each other, zero intensity of light passes through the second polariser.

Web Simulation on Polarization: https://javalab.org/en/polarization_en/

Some applications of polarised light are:

1. Polarised sunglasses – Glare is usually due to light reflected off a horizontal ground (e.g. snow surface, car roof). As light gets reflected off a horizontal surface, it will be polarized horizontally. Polarised lenses contain a special filter which block off the reflected light from the ground, which reduces the glare.

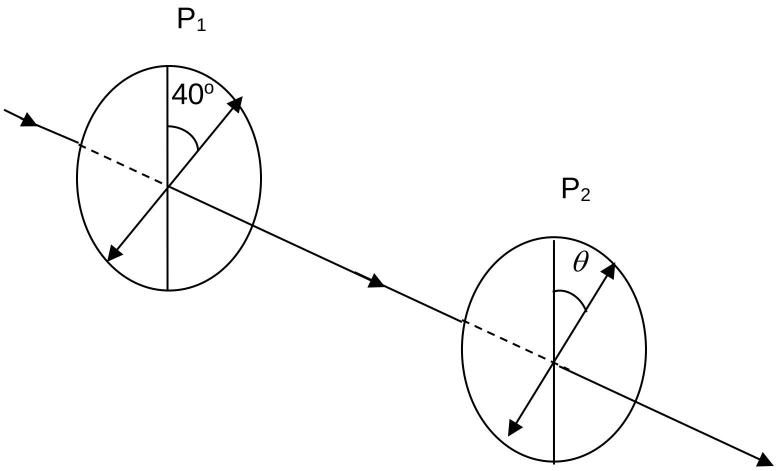
2. Stress Analysis - Certain plastic materials under stress will affect the plane of polarization of light through them. This can be used to study stress in various structures.

3. 3-D films – Two images are projected through polarising filters onto a screen. The viewer will have to wear a pair of glasses which has 2 different polarising filters for each lens, so that the movie appears to be three dimensional. (see Appendix B 3-D films and Polarisation)

Example 9.7

The figure below shows a beam of initially unpolarised light passing through two polarizing filters P_1 and P_2. The polarising axis of P_1 is fixed at 40° with respect to the vertical axis.

The polarizing axis of P_2 is then rotated clockwise from its vertical axis. At what values of θ will intensity minima of the emergent light occur?



For intensity minima to occur (no emergent light), the two polarizing axes must be

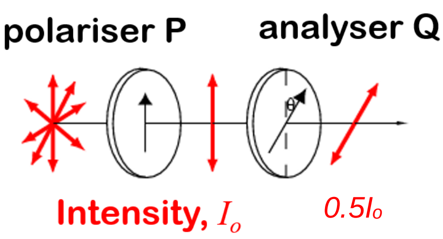
____ to each other.

Rotate P_2 through _______________________ with respect to P_1 , therefore θ =

Rotate P_2 through ________________________ with respect to P_1 , therefore θ =

Example 9.8

Two polarising sheets have their polarizing directions parallel so that the intensity I_o of the transmitted light is a maximum. Through what angle must either sheet be turned if the intensity is to drop by half?



Appendices

Appendix A: Function Generator and Cathode Ray Oscilloscope (C.R.O)

Function Generator

A function generator is an instrument that produces alternating (a.c.) voltage of different frequencies.

The standard voltage signals it can produce are:

- sine waves,
- triangular waves
- rectangular waves

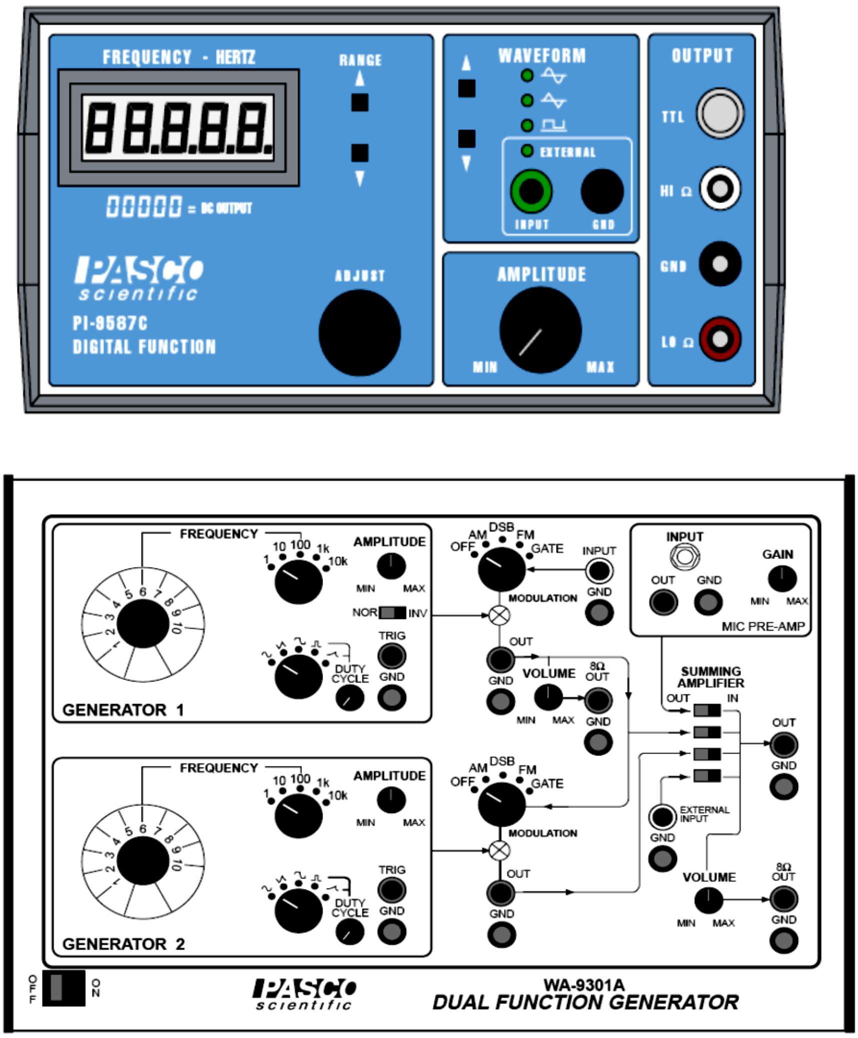


Fig. 1. Examples of function generators. The top one is a single output generator. The bottom generator is able to produce two outputs at the same time.

As not all function generators have a precise digital display of the frequency of the waveform or the amplitude of the voltage wave produced. It will sometimes be necessary to connect the function generator to a CRO in order to determine the precise frequency and amplitude of the voltage wave.

To generate the sound waves, the function generator is often connected to a speaker so that the speaker can convert the voltage signals to vibrations of the membrane of the speakers to produce sound waves.

Interactive Online Virtual Oscilloscope and Signal Generator: https://eleceng.dit.ie/dsp/elab/

Cathode Ray Oscilloscope

The other commonly used equipment in sound experiments is the **cathode ray oscilloscope** (CRO). The oscilloscope is basically a graph displaying device that traces the variation of the input voltage signal with respect to time.

Using the oscilloscope, you can measure the amplitude, period, and frequency of any voltage signal. Therefore, a CRO is actually a more complex voltmeter. Hence, to measure the frequency of a sound wave, we need to first connect a microphone to the scope. The microphone will then convert the vibrational energy of the air molecules to voltage signals which can be measured by the CRO.

Most scopes can display two signals on the screen at one time, enabling you to observe their time relationships. These scopes are known as dual-channel oscilloscopes. A typical front panel view of such a scope is shown in Fig. 2.

Fig. 2. A typical dual channel oscilloscope. Numbers below screen indicate the values for each division on the vertical (voltage) and horizontal (time) scales and can be varied using the vertical and horizontal controls on the scope. Although this may be a typical dual-channel oscilloscope, it does not look like the ones we have in the lab!

Actual CROs vary depending on model and manufacturer, but most have certain common features. The main controls will be discussed below. You will need to refer to the user manual for complete details of the scope you are using.

Vertical Controls:

In the vertical section of the scope (Fig. 2), there are identical controls for each of the two channels (CH1 and CH2).

- **Position Control**. The position control lets you move a displayed waveform up or down vertically on the screen.
- **Volts/Div control.** The Volts/Div (volts per division) control is also known as the **Y-sensitivity / Y-gain control**. It adjusts the number of volts represented by each vertical division (usually 1 cm) on the screen. For this scope, the Volts/Div setting for each channel is displayed at the bottom of the screen. For CH1, it is 500 mV/div and for CH2, it is 200 mV/div.

Horizontal Controls:

In the horizontal section, the controls apply to both channels.

- **Position Control**. The position control lets you move a displayed waveforms left or right horizontally on the screen.

- **Sec/Div Control.** The Sec/Div (seconds per division) control is also known as the **Timebase** control, it adjusts the time represented by each horizontal division or main time base. Similarly, for this scope, the Sec/Div setting is displayed at the bottom of the screen.

Appendix B: 3-D films and Polarization

3-D films such as Avatar (by 20th Century Fox), G-Force (by Walt Disney Pictures) and Cloudy With A Chance of Meatballs (by Sony Pictures Animation) have been a hit with the cinema these days. In fact, 3-D films have existed in some forms since 1950s. There are various techniques to produce 3-D films. Current methods include interference filter technology, for example Dolby 3D and polarization systems, for example RealD. We will look at the latter in detail.

For 3-D films which are shown using polarization systems, the viewers will have to wear 3-D glasses to view the polarized image on the screen. The image will be blurry when viewed without the special glasses. There are two types of polarization systems: linear polarization and circular polarization.

For the linear polarization system, two images are superimposed onto the same screen through two polarizing filters which are at 90 degree angles of each other (orthogonal). The silver screen is specially constructed such that it is non-depolarizing to compensate for light loss and to preserve the polarization. The viewer wears a pair of glasses which is made of a pair of orthogonal polarizing filters. Each filter only allows light from the image which is polarized in the same plane to pass through and blocks the orthogonally polarized light, each eye will only see one of the images. The 3-D effect is then achieved.

Fig. A1. Two pairs of 3-D glasses showing the polarizing effect.

Similarly, for a circular polarization system, two images polarized differently are superimposed onto the same screen. However, unlike linear polarization system, the two images pass through circular polarizing filter of opposite handedness. The viewer wears a pair of glasses made of a pair of circular polarizer mounted in reverse handedness. Hence, each eye will only see one of the images and the 3-D effect is achieved. RealD Cinema is a digital stereoscopic projection technology which uses similar techniques as a circular polarization system; except that it only needs one projector. A circularly polarizing liquid crystal filter which can switch polarity is placed in front of the projector lens, so that the left and right images are displayed alternately.

Information adapted from:
http://entertainment.howstuffworks.com/digital-3d1.htm
http://en.wikipedia.org/wiki/RealD_Cinema
http://en.wikipedia.org/wiki/3-D_film

Appendix C: Polarisation by Absorption[5]

Most light sources e.g. filament lamp, sun etc. produce light that is unpolarised. However, there are ways to polarise light that is initially unpolarised, or to change the direction of polarisation of polarised light. The most common method to produce polarised light waves is by using a polariser which is known as a ***polaroid***. (E.H. Land, 1928).

When light is incident on a polaroid sheet, ideally,

- if the light is polarised with **its electric field E parallel to the polarising axis of the polaroid, it will be completely transmitted,**
- if the light is polarised with **its electric field E perpendicular to the polarising axis of the polaroid, it will be completely absorbed.**
- if the light is **linearly polarised at some angle θ relative to the polarising axis**, the light will be partially absorbed and partially transmitted - **only the component of the electric field E parallel to the polarising axis will be transmitted.**

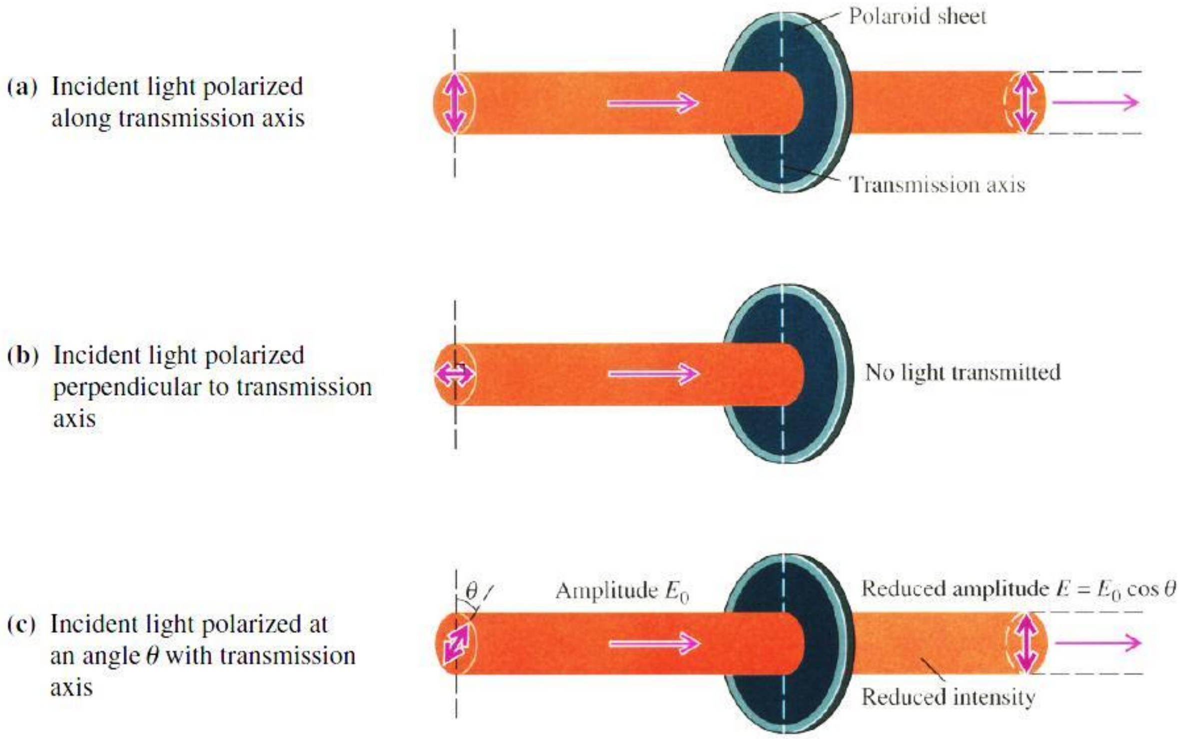


Fig. A2. The effect of a Polaroid sheet on initially polarized light depends on the direction of the initial polarisation relative to the direction of the sheet's transmission axis. (Source: Coletta, p748, Fig. 26.36.)

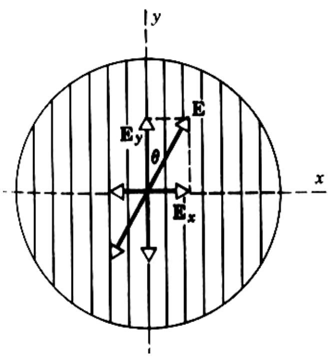


Fig. A3. The figure on the left shows what happens when polarised EM wave falls on a polariser with a vertical polaring axis. The electric field E of the EM wave makes an angle θ with the polarisation axis. The vertical component of E has magnitude $E_y = E\cos\theta$ and is transmitted. The horizontal component $E_x = E\sin\theta$ is absorbed. As a result of polarization, the amplitude of light that passes through has decreased. Hence, the intensity of the light is reduced.

[5] It is also possible to polarise light through reflection. You may read more about it from our e-textbook "Physics Fundamentals" by Coletta, p750.

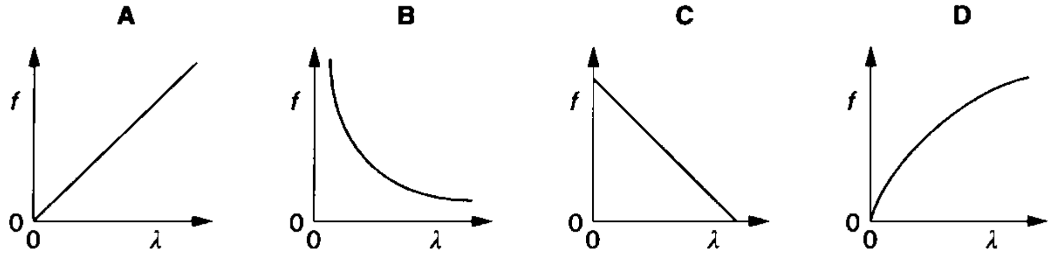
Tutorial 9 Wave Motion

Self-Review Questions

S1 N99/I/10.

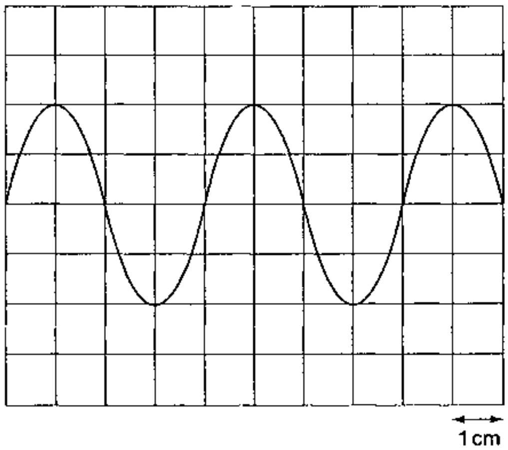
A sound wave of frequency f and wavelength λ travels through air. It may be assumed that its speed is independent of the frequency.

Which graph correctly shows the variation of f with λ ?



S2 N05/I/17

The diagram shows the trace produced by a sound wave on a cathode-ray oscilloscope. The time base is calibrated at 2.00 ms cm^{-1}.

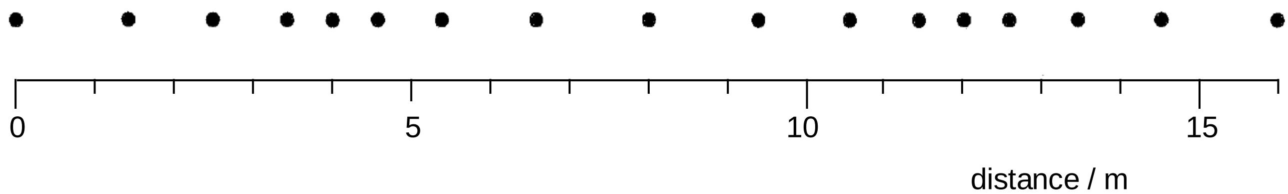


What is the frequency of the sound wave?

A 62.5 Hz **B** 125 Hz **C** 250 Hz **D** 500 Hz

S3 N20/I/18

The diagram illustrates the position of particles along a longitudinal wave at an instant in time.

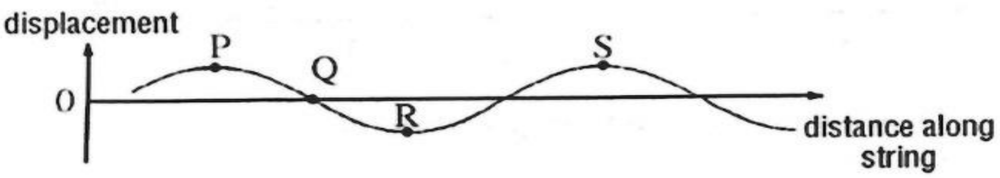


The wave travels at a speed of 12 m s^{-1}.

What is the wave frequency?

A 0.33 Hz **B** 0.67 Hz **C** 1.5 Hz **D** 3.0 Hz

S4 The graph shows a segment of a transverse wave travelling on a string at a particular point in time.



Which statement about the motion of the elements of the string is correct?

A The speed of the element at P is a maximum.

B The displacement of the element at Q is always zero.

C The energy of the element at S is a maximum.

D The acceleration of the element at S is a maximum.

S5 **J87/I/11**

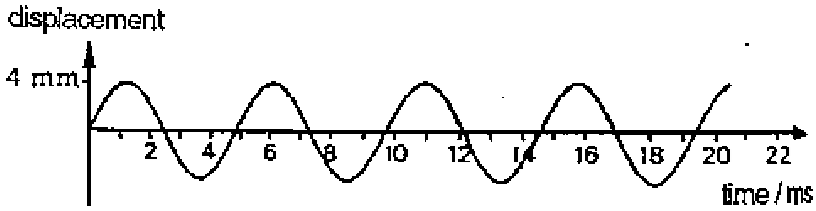
Visible light has wavelengths between 400 nm and 700 nm. What is the maximum frequency of visible light?

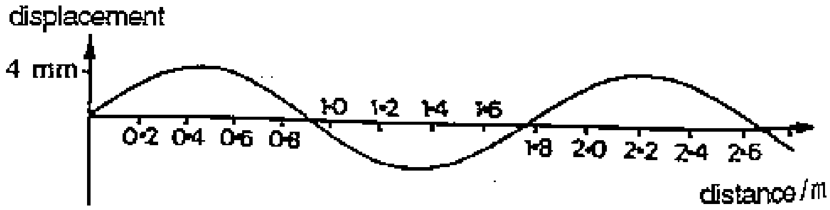
S6 Which of the following gives three regions of the electromagnetic spectrum in order of increasing wavelength?

A γ-rays, microwaves, visible radiation
B radio waves, ultraviolet, X-rays
C ultraviolet, infra-red, microwaves
D visible radiation, γ-rays, radio waves

S7 **N90/III/3**

The figure below shows two graphs which refer to the same wave. What is the speed of the wave?





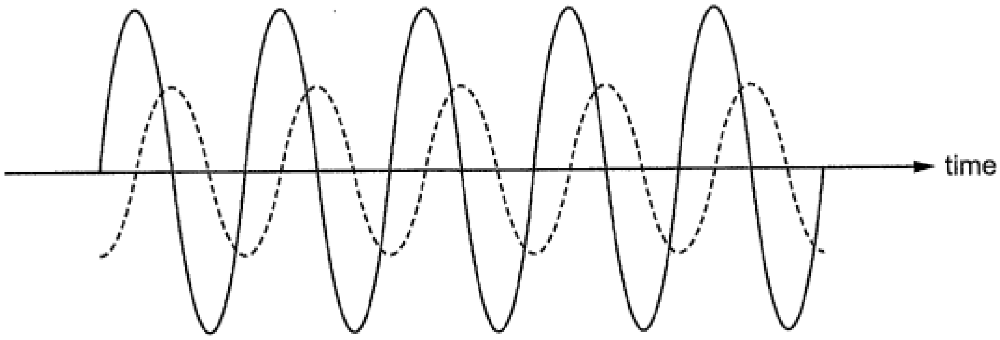
S8 J79/II/13; N82/II/10; N90/I/13; J99/I/11

A sound wave of frequency 400 Hz is travelling in a gas at a speed of 320 m s^{-1}. What is the phase difference between 2 points 0.2 m apart in the direction of travel?

A $\frac{\pi}{4}$ rad **B** $\frac{\pi}{2}$ rad **C** $\frac{2\pi}{5}$ rad **D** $\frac{4\pi}{5}$ rad

S9 N17/I/14, N20/I/17

Two waves of the same frequency are displayed on the screen.



What is the phase difference between the waves?

A $\frac{1}{4}$ rad **B** $\frac{\pi}{4}$ rad **C** $\frac{\pi}{2}$ rad **D** $\frac{3\pi}{4}$ rad

S10 (Young and Freedman)

By measurement, you determine that sound waves are spreading out equally in all directions from a point source and that the intensity is 0.026 W m^{-2} at a distance of 4.3 m from the source.

(a) What is the intensity at a distance of 3.1 m from the source?

(b) How much sound energy does the source emit in one hour if its power output remains constant?

S11 N05/I/16

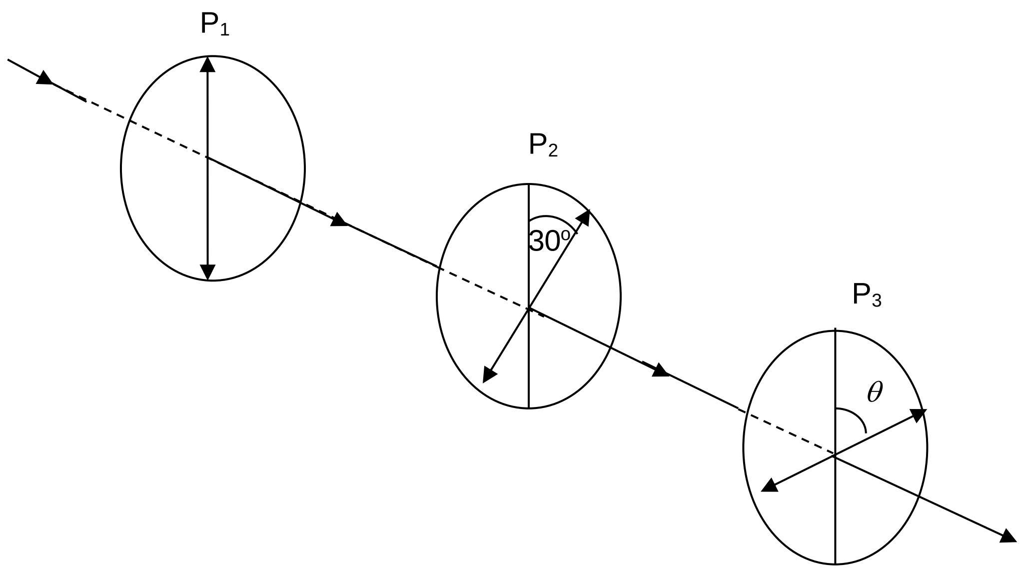
The intensity of a wave depends on the amplitude. The intensity is also proportional to the square of the frequency. A wave has frequency 3.0 Hz, amplitude 1.5 cm and intensity I.

What is the intensity of a similar wave of frequency 6.0 Hz and amplitude 0.5 cm?

A $\frac{4}{9}I$ **B** $\frac{4}{3}I$ **C** $\frac{9}{4}I$ **D** 36 I

S12 J82/II/13

Figure below shows a beam of initially unpolarised light passing through three polaroids P_1, P_2 and P_3. The polarising axis of each polaroid is shown by an arrow. Polaroids P_1 and P_2 are fixed, with their polarising axes at 30° to one another, and P_3 can be set with its polarising axis at a variable angle θ to that of P_1.



For what values of θ do intensity minima of the emergent light occur?

A 30°, 120°, 210°, 300°
B 90°, 120°, 270°, 300°
C 60°, 240°
D 90°, 270°
E 120°, 300°

S13 Two ideal polarisers are aligned with a 30° angle between their polarization directions. If unpolarised light of intensity I_o is incident upon them, what is the intensity of the transmitted light?

Numerical Answers for Self-Review Questions

S1 B **S2** B **S3** C **S4** D **S5** 7.5×10^{14} Hz **S6** C **S7** 372 m s^{-1} **S8** B **S9** C
S10 (a) 0.0500 W m^{-2}, (b) 2.17×10^4 J **S11** A **S12** E **S13** 0.375 I_0

Discussion Questions

D1 N87/I/8

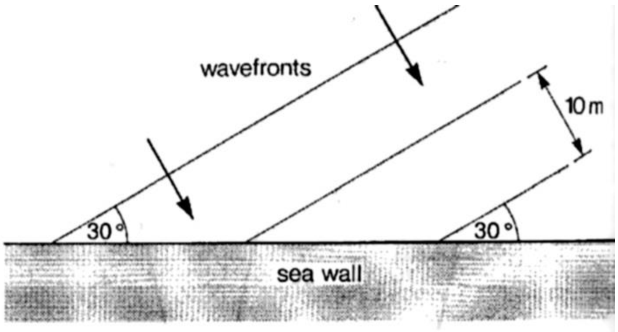
Transverse progressive sinusoidal waves of wavelength λ are passing vertically along a horizontal rope. **P** and **Q** are points on the rope $\frac{5\lambda}{4}$ apart and the waves are travelling from P to Q.

Which one of the following correctly describes Q at the instant when P is displaced upwards but is moving downwards?

	Displacement of Q	Movement of Q
A	upwards	downwards
B	upwards	upwards
C	downwards	upwards
D	downwards	downwards
E	downwards	stationary

D2 J85/I/8; N95/I/10.

Parallel water waves of wavelength 10 m strike a straight wall. The wavefronts make an angle of 30° with the wall as shown.



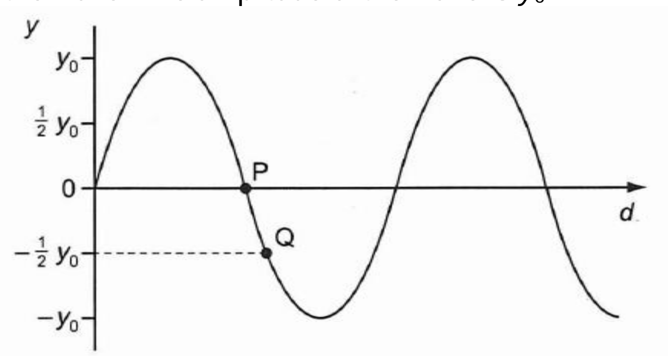
What is the difference in phase at any instant between the waves at two points 5 m apart along the wall?

A 45° **B** 55°

C 90° **D** 180°

D3 N09/I/23

The diagram shows the variation with distance d along a sinusoidal wave of displacement y of particles in the wave. The amplitude of the wave is y_0.



What is the phase angle between the two particles P and Q in the wave?

A 30° **B** 45° **C** 90° **D** 180°

D4 **N13/I/19**

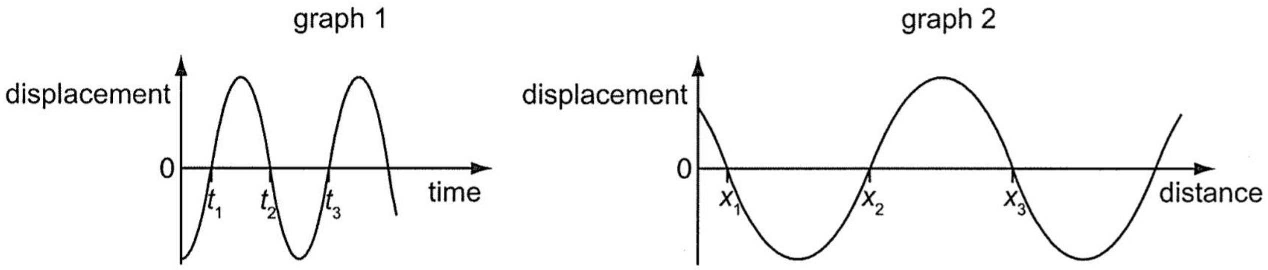
As the intensity of a single frequency sound wave travelling through the air is increased, how do the maximum speed of vibration of the air molecules and the speed of wave travel change?

	Maximum speed of vibration of air molecules	Speed of wave travel
A	increase	increase
B	increase	no change
C	no change	increase
D	no change	no change

D5 **N14/I/21**

Graph 1 shows how the displacement of one particular point of a wave varies with time.

Graph 2 shows how the displacement of the same wave varies with distance along the wave at one particular time.



Which expression gives the speed of the wave?

A $\dfrac{x_1}{t_1}$ **B** $\dfrac{x_2}{t_2 - t_1}$ **C** $\dfrac{x_2 - x_1}{t_2}$ **D** $\dfrac{x_3 - x_2}{t_2 - t_1}$

D6(a) The following graphs refer to the same wave. The graph in Fig. D6 (a) shows the displacement vs. position at t = 0. The graph in Fig. D6 (b) shows the displacement vs. time graph of the point **Y** labelled on Fig. D6 (a). What direction is the wave progressing?

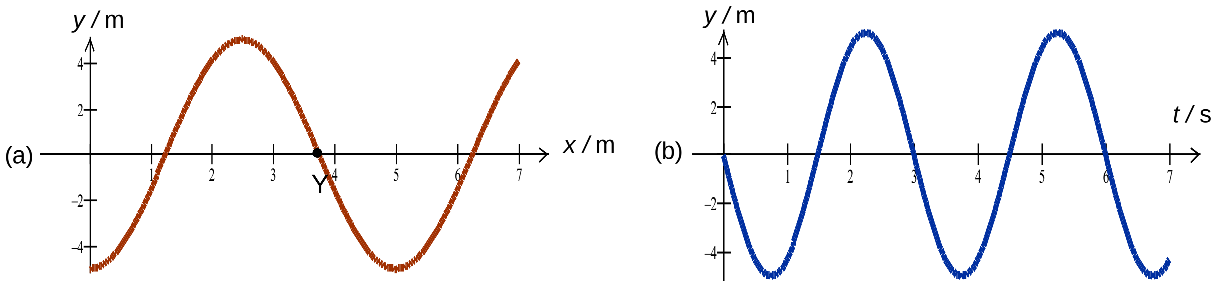


Fig. D6

D6 (b) N21/I/16 Mod

A beam of laser light of frequency 5.0×10^{14} Hz is travelling through vacuum.
What is the phase difference between points on the wave that are 1.5 μm apart?

D7 J91/III/2 (part - modified)

Fig. D6 is a full-scale diagram showing the rest position and the actual positions of a series of particles through which a sinusoidal longitudinal wave is passing from left to right.

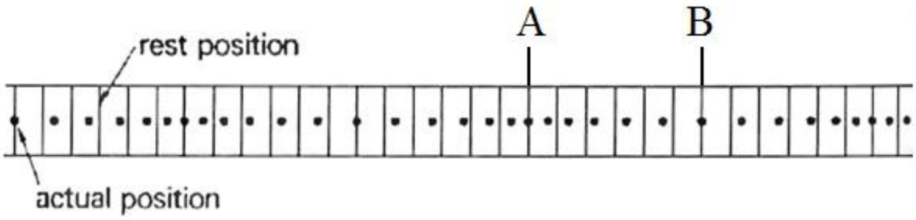


Fig. D6

(a) Describe the movement of a single particle. Describe how the compressions and rarefactions move.

(b) Sketch the displacement vs. position graph for points along A and B for this longitudinal wave at this instant.

(c) Consider the particle at A. Sketch its displacement vs. time graph taking t = 0 to be the instant shown. Label clearly on your graph the period T.

D8 Longitudinal Wave - Pressure and Displacement Graph (Hutchings)

The graph shows how particle displacement varies with distance for a sound wave at one instant in time. The speed of the wave is 340 m s^{-1}.

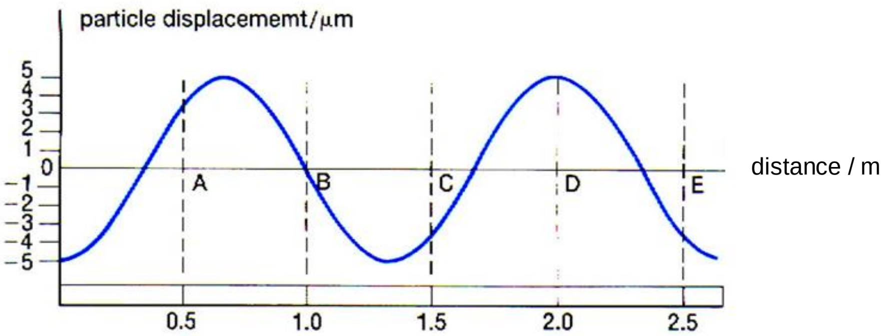


Fig. D8

(a) From the graph, determine

(i) the amplitude,

(ii) the wavelength,

(iii) the frequency,

(iv) the phase difference between the vibration of the particle at A and that of the particle at B.

(b) Which points on the graph correspond to compressions and rarefactions?
Indicate them clearly.

(c) Sketch the corresponding pressure vs. position graph for the sound wave at this instant.

D9 N97/I/10

A plane wave of amplitude *A* is incident on a surface of area *S* placed so that it is perpendicular to the direction of travel of the wave. The energy per unit time intercepted by the surface is *E*.

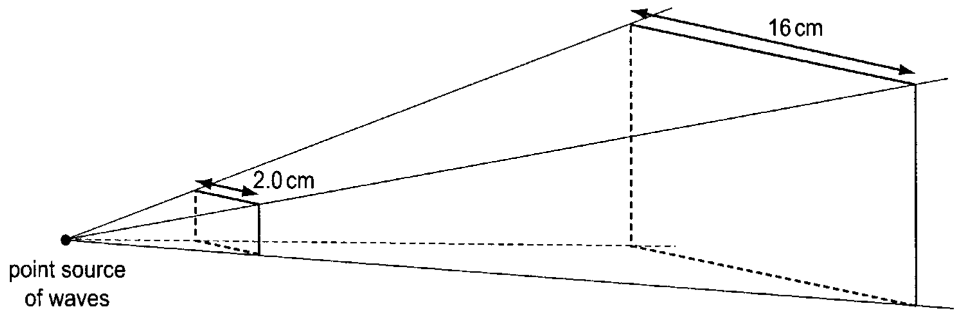
The amplitude of the wave is increased to 2*A* and the area of the surface is reduced to *S/2.*

How much energy per unit time is intercepted by this smaller surface?

A *4E* **B** *2E* **C** *E* **D** *E/2*

D10 N08/I/20

Waves from a point source pass through an area that is 2.0cm wide, as shown.



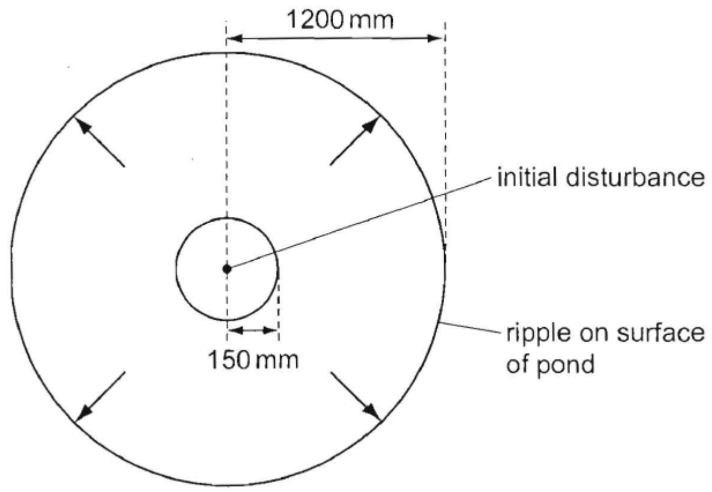
Within this area, the intensity of the waves is *I* and their amplitude is *A*. The waves reach a second area of width 16 cm. What will be the intensity and amplitude of the waves when they reach the second area?

	intensity	amplitude
A	$\frac{I}{8}$	$\frac{A}{4}$
B	$\frac{I}{64}$	$\frac{A}{4}$
C	$\frac{I}{64}$	$\frac{A}{8}$
D	$\frac{I}{256}$	$\frac{A}{16}$

D11 N10/I/20

Ripples on the surface of a pond spread out in circles from the point of an initial disturbance. Assume that the energy of the wave is spread over the entire circumference of the ripple.

For one such ripple, the amplitude of the ripple at a distance of 150 mm from the disturbance is 2.0 mm.



[Please turn over

What will be the amplitude of the ripple at a distance of 1200 mm from the disturbance? (Assume that no energy is lost in the propagation of the ripple.)

A 0.031 mm **B** 0.13 mm **C** 0.25 mm **D** 0.71 mm

D12 N17/I/15

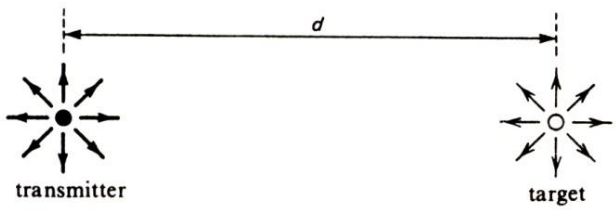
A wave of frequency *f* and wavelength λ has intensity *I*. The wave travels through a boundary into a new medium where the speed and amplitude are both halved.

What will be the intensity, frequency and wavelength in the new medium?

	intensity	frequency	wavelength
A	$0.25I$	$0.50f$	λ
B	$0.25I$	f	0.50λ
C	$0.50I$	$0.50f$	λ
D	$0.50I$	f	0.50λ

D13 J88/III/8

A radar transmitter produces pulses of microwaves each with a mean power *P* which are emitted uniformly in all directions. A small spherical target of effective area *S* is placed at a distance *d* from the transmitter. The target reflects a small fraction *k* of the energy incident on it uniformly in all directions as shown in the following figure.



(a) Show that the mean intensity I_r if the reflected pulse when it is received back at the transmitter is given by $I_r = \dfrac{PkS}{16\pi^2 d^4}$

(b) Given that the mean power *P* is 2 MW and the pulse duration is 3 μs, calculate:

(i) the energy in each emitted pulse,

(ii) the mean intensity of the emitted pulse at a range of 50 km,

(iii) the mean intensity of the reflected pulse when received back at the transmitter if the range of 50 km and the product $kS = 1\ \text{m}^2$.

(c) Briefly discuss the effect on your answer to (b)(iii) if the pulses were emitted in an almost parallel beam.

D14 N19/II/2

(a) **(i)** State what is meant by a *transverse* wave. Give an example. [2]

(ii) State what is meant by *plane polarisation* of a wave. [1]

(b) Two polarising filters are held in line with one another and at different angles to an incoming polarised beam of light of intensity I_0, as shown in Fig. 2.1.

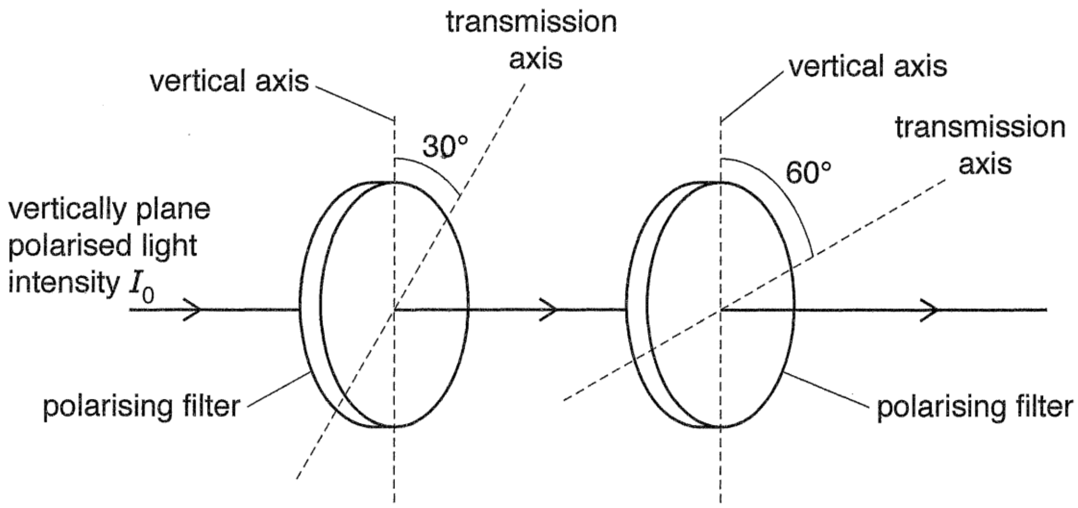


Fig. 2.1

The incoming light is vertically plane polarised. The transmission axis of the first filter is at an angle of 30° to the vertical. The transmission axis of the second filter is at an angle of 60° to the vertical.

(i) Calculate, in terms of I_0, the intensity I_T of the beam emerging from the second filter. [3]

(ii) Use your answer in **(b)(i)** to calculate the ratio

$$\frac{\text{amplitude of beam emerging from second filter}}{\text{amplitude of beam incident on first filter}}$$ [2]

(iii) The second polarising filter is now removed. The first filter is then placed so its transmission axis is parallel to the plane of polarisation of the incoming light. The transmission axis is then rotated through 360°.

Complete Fig. 2.2 to show the variation with the angle of rotation of the intensity of the transmitted light.

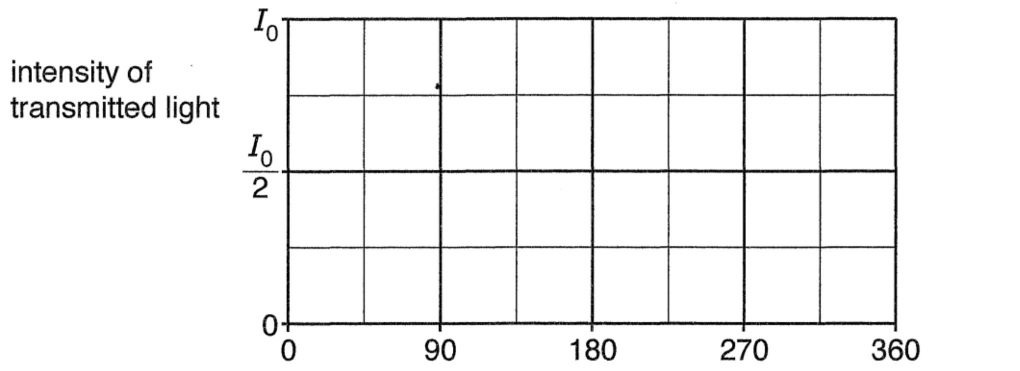


Fig. 2.2 **angle of rotation /°** [2]

(c) **(i)** State why a sound wave **cannot** be polarised. [1]

(ii) A microphone and an oscilloscope are used to measure the period of a sound wave of frequency 740Hz.

The distance between two adjacent peaks of the waveform displayed on the oscilloscope screen is 6.8cm.

Calculate the time-base setting on the oscilloscope in ms cm^{-1}. [2]

Numerical Answers for Discussion Questions

D6 (b) π rad
D8 (a)(i) 5 μm (ii) 1.33 m (iii) 255 Hz (iv) 2.36 rad
D13 (b)(i) 6 J, (ii) 6.37×10^{-5} W m^{-2}, (iii) 2.03×10^{-15} W m^{-2}
D14 (b)(i) 0.75 I_o ; 0.563 I_o , (b)(ii) 0.75, (c)(ii) 0.20 ms cm^{-1}