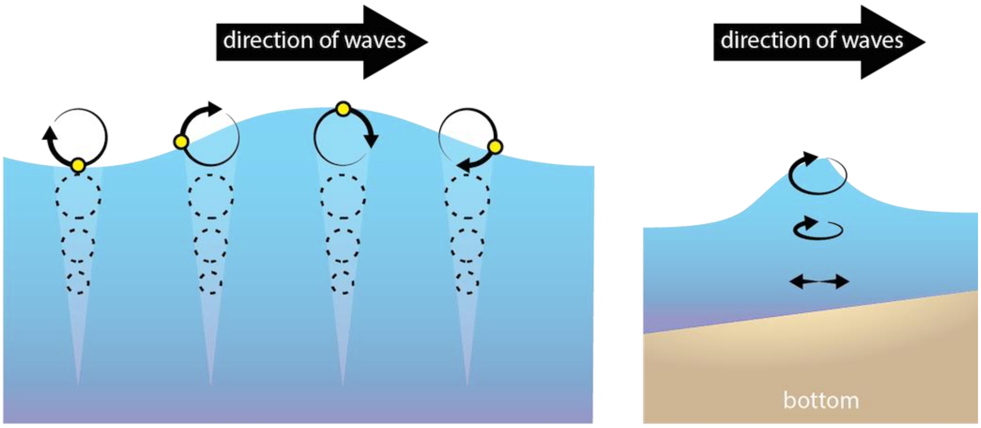
H2 Topic 11 – Waves



Ocean waves exhibit both transverse and longitudinal behaviour. It is observed that particles near the water surface move in a vertical circular motion without significant net displacements in their average positions as a wave propagates. This wave motion "flattens" out as the ocean wave reaches shallower waters, resulting in more of a forward/backward ebbing motion near coasts. This phenomenon also partially explains why tsunamis are devastating: as a tsunami reaches shallower water, its speed of propagation is forced to slow down, its horizontal wavelength is forced to decrease and the kinetic energy is converted to gravitational potential energy – the wave height increases dramatically before smashing onto coastal areas.

Content

- Progressive waves
- Transverse and longitudinal waves
- Polarisation
- Determination of frequency and wavelength of sound waves

Learning Objectives:

Candidates should be able to:

(a) show an understanding of and use the terms displacement, amplitude, period, frequency, phase difference, wavelength and speed

(b) deduce, from the definitions of speed, frequency and wavelength, the equation $v = f\lambda$

(c) recall and use the equation $v = f\lambda$

(d) show an understanding that energy is transferred due to a progressive wave

(e) recall and use the relationship, *intensity* ∝ (*amplitude*)^2

(f) show an understanding of and apply the concept that a wave from a point source and travelling without loss of energy obeys an inverse square law to solve problems

(g) analyse and interpret graphical representations of transverse and longitudinal waves

(h) show an understanding that polarisation is a phenomenon associated with transverse waves

(i) recall and use Malus' law (*intensity* ∝ $\cos^2\theta$) to calculate the amplitude and intensity of a plane polarised electromagnetic wave after transmission through a polarising filter

(j) determine the frequency of sound using a calibrated oscilloscope

(k) determine the wavelength of sound using stationary waves (** to be done in H212 Superposition*)

11.0 Introduction

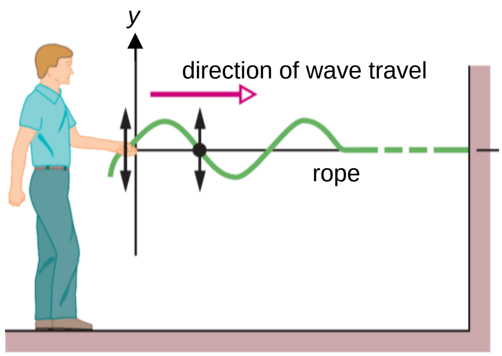
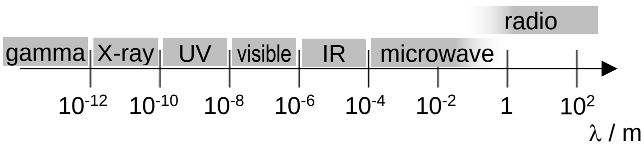
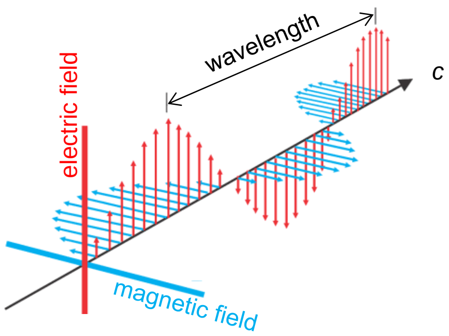
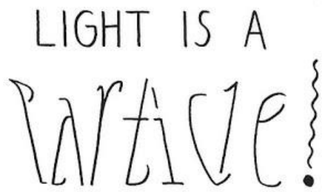
We think of waves as a "bulk phenomenon" – it happens on a larger scale and are made up of adjacent smaller units (such as water molecules, gas molecules, rope segments) individually exhibiting *oscillations*.

> In progressive waves, energy is propagated from one place to another in the direction of wave travel without bulk movement of medium.

Waves can result in energy (but *not* matter) being moved from one point to another OR energy can also be confined to a region in space without transfer. In this topic, we will focus on the former: progressive waves.

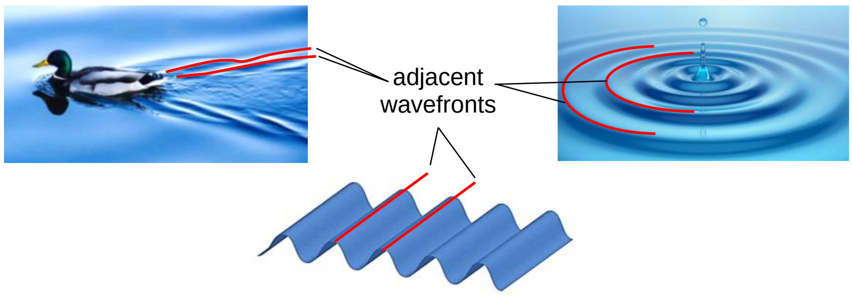
11.1 Types of Waves

In our A-Levels journey we will encounter 3 main types of waves:

Mechanical waves e.g. water waves, sound waves and seismic waves (earthquakes). They obey Newton's laws of motion, and can only exist within a material medium (correspondingly liquid water, air, rock).	
Electromagnetic waves do not need a material medium to exist. They travel through vacuum at a speed of $c = 3.00 \times 10^{8}$ m s^{-1}. Visible light ranges from 400 nm to 700 nm. Take note of the order of magnitude of EM wavelengths: 	
Matter waves is a concept from Quantum Physics in which small particles (e.g. electrons, protons, neutrons or even some molecules) can behave like waves. We will visit it near the end of JC2, in H219.	

Before continuing it is useful to introduce wavefronts.

A wavefront is an imaginary line that joins all the points on a wave that are at the same phase. It helps to visualize how a wave propagates *spatially*.

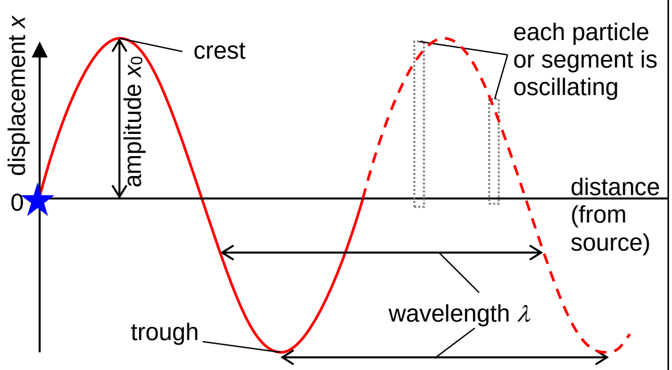


11.2 Quantities of Wave Motion

Let's compare some terms against those introduced under H210 Oscillations:

	describe oscillation	describe waves
displacement *x*	distance in a specified direction from equilibrium position of *oscillating mass*	distance in a specified direction from equilibrium position of *particle or point on the wave*
period *T*	time taken for one complete oscillation of the oscillating mass	time between adjacent wavefronts
frequency *f*	number of complete to-and-fro motions made by the oscillating mass per unit time $f = \dfrac{1}{T}$	number of wavefronts passing a point per unit time

Two types of graphical representations are commonly used to describe waves:

<table>
<tr><th>variation of displacement with distance</th><th>variation of displacement with time</th></tr>
<tr><td>


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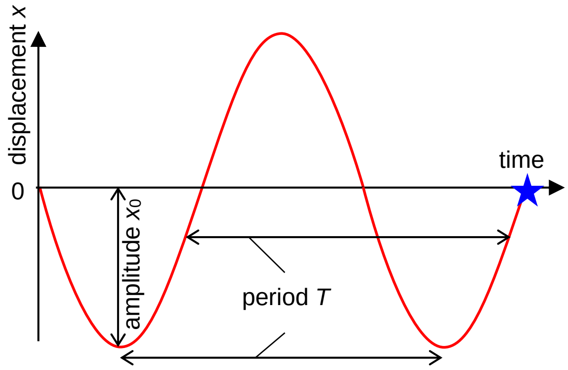
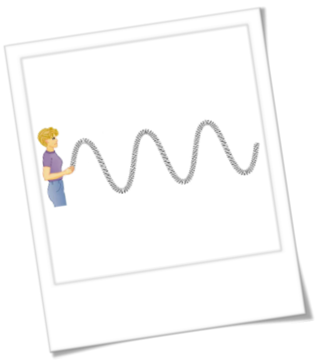
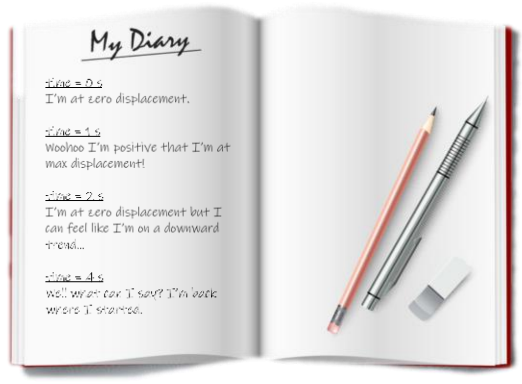


</td></tr>
<tr><td>Shows displacements of all the particles / segments in a wave and how they vary with distance from the source at a particular instant in time.

Since it is a spatial freeze-frame of the wave, it is like a photograph of the oscillations.

</td><td>Shows displacement of a single particle / segment and how it varies with time.

Since it tracks the displacement of 1 single entity only, it is like a log of displacements.


</td></tr>
<tr><td>If the wave is travelling to the right, a particle at the origin ★ is currently moving downwards.</td><td>This particular particle is currently moving upwards.</td></tr>
</table>

11.2.1 Speed and Wavelength

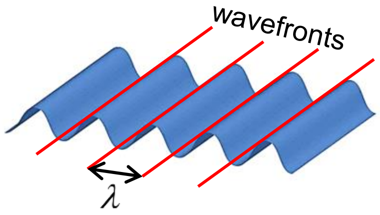
The **speed** of a progressive wave is the speed at which energy is transferred.

$$v = f\lambda$$

v : speed of wave (m s^{-1})
f : frequency of wave (s^{-1})
λ : wavelength of wave (m)

The speed of a progressive wave can also be seen from the speed of the wavefront. **The speed of oscillation of any individual particle or wave segment** $v = \pm\omega\sqrt{x_0^2 - x^2}$ **is different from the speed of wave profile** $v = f\lambda$ **.**

The **wavelength** λ of a wave is the minimum distance between two points with the same phase.

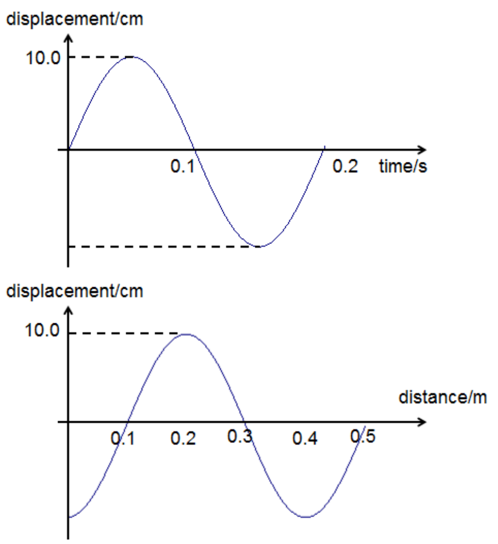


Example 1
Use the definitions of wavelength and frequency to deduce the relationship between λ, f and v, the speed of a wave.

Solution
Frequency is the number of wavefronts passing a point per unit time. wavelength is the minimum distance between two points with the same phase; in other words: wavelength is also the distance moved by a wavefront in the time for a complete oscillation of source

$$v = \frac{\text{distance moved by wavefront in a period}}{\text{one time period}} = \frac{\lambda}{T} = f\lambda$$

Example 2
A wave is represented by the graphs shown.
Find the speed of the wave.



Solution

$$v = f\lambda$$
$$= \left(\frac{1}{T}\right)(\lambda)$$
$$= \left(\frac{1}{0.2}\right)(0.4)$$
$$= 2.00 \text{ m s}^{-1}$$

Example 3

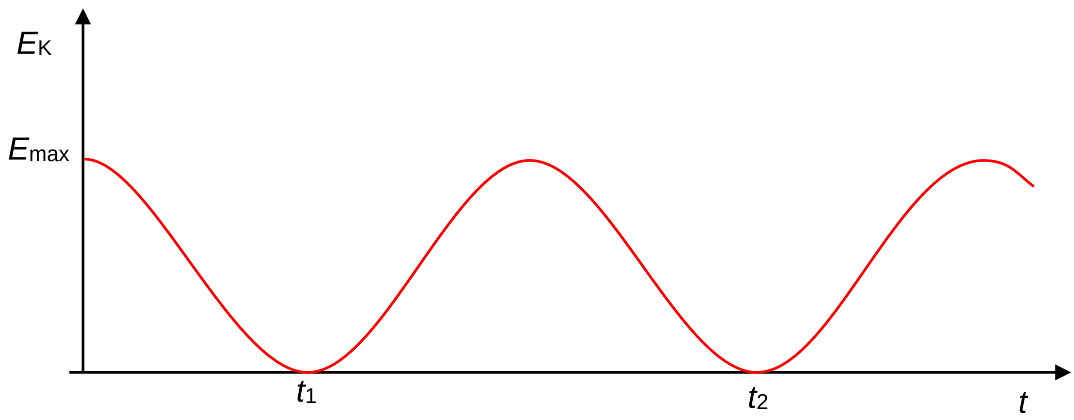
A sound wave travels through a gas. The gas molecules oscillate with simple harmonic motion. Each gas molecule is of mass 5.3×10^{-26} kg, vibrates at a frequency of 835 Hz, and has an vibration amplitude of 60 nm. The variation with time *t* of the vibrational kinetic energy E_K of a molecule is shown below. Find the

(i) period of oscillations,

(ii) time interval $(t_2 - t_1)$, and

(iii) maximum speed v_{max} for one vibrating molecule.

(iv) By reference to the speed of sound in a gas at room temperature, comment on v_{max}.



Solution

(i) period $T = \frac{1}{f} = \frac{1}{835} = 0.00120$ s

(ii) time interval

$$(t_2 - t_1) = \frac{T}{2} = \frac{1}{2f} = 0.000599 \text{ s}$$

(iii) maximum oscillation speed

$$v_{max} = \pm\omega\sqrt{x_0^2 - x^2} = \pm\omega\sqrt{x_0^2 - 0}$$
$$= \omega x_0$$
$$= (2\pi f)x_0$$
$$= (2\pi(835))(610 \times 10^{-9})$$
$$= 0.00320 \text{ ms}^{-1}$$

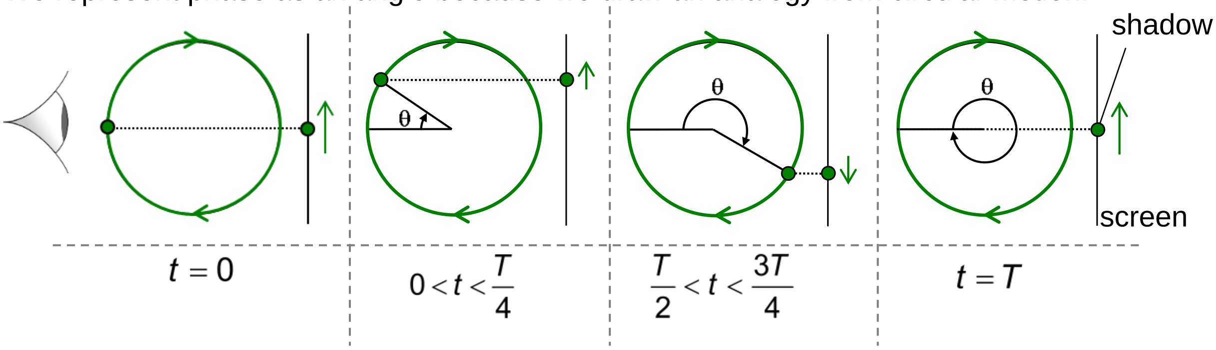
(iv) speed of propagation of sound waves in gas at room temperature is about 330 ms^{-1}, which is about 5 orders of magnitude larger than maximum speed of oscillation of a gas molecule. Propagation of sound energy does not involve transfer of medium (mass), so can be greater than the maximum speed of vibration of individual particles in the medium.

11.2.2 Phase and Phase Difference

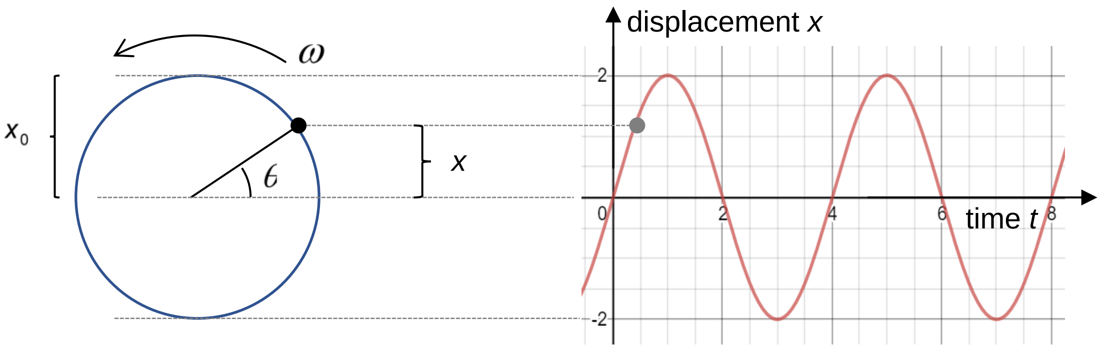
Here we revisit the concept of *phase* first introduced in H210 Oscillations:

phase ϕ	an angular measure (in either degrees or radians) of the fraction of a cycle completed by the oscillating mass
phase difference $\Delta\phi$	measure of how much an oscillation is out of step with another oscillation at the same instant in time

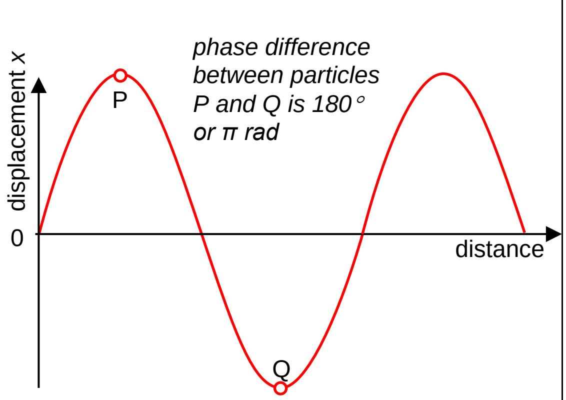
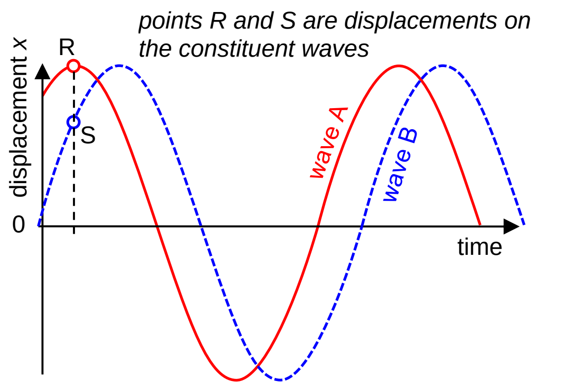
We represent phase as an angle because we draw an analogy from circular motion:



The shadow is oscillating purely vertically and so doesn't *physically* have any measurable angles. Because the vertical shadow oscillation repeats itself, like how the object repeats its circular motion, we apply the idea of a phase *angle* onto oscillations.



We then need to clarify that phase *difference* compares 2 oscillations which can be
(i) on the *same* wave OR
(ii) on 2 different waves:

compares 2 oscillations on *same* wave	compares 2 *different* waves meeting at same point
	

Since phase ϕ is measured by an angle, when we compare phase *difference* $\Delta\phi$, we consider the angle *relative* to the complete cycle of 360° or 2π:

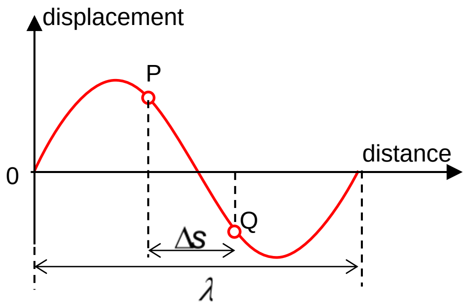
Phase difference $\Delta\phi$:

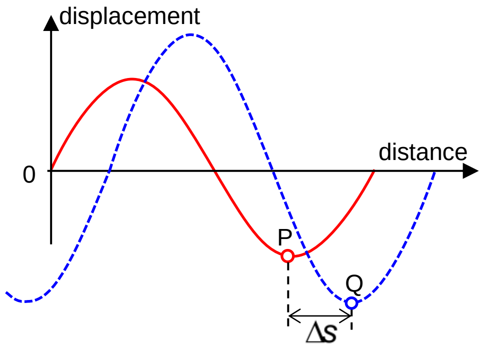
$$\frac{\Delta\phi}{2\pi} = \frac{\Delta t}{T} = \frac{\Delta s}{\lambda}$$

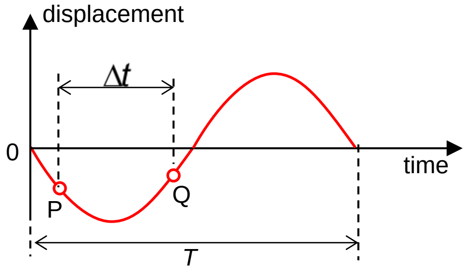
Note that the denominators are all quantities relating to a full cycle.

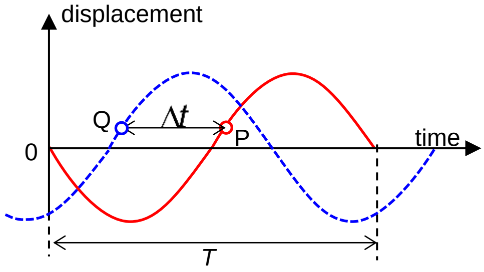
Δt is sometimes called the *time difference* or *time lag*. It refers to the duration of time it takes for a 2nd oscillation to reach the *same phase* as the reference oscillation.
s Δs is usually called the *path difference*. It is the difference in length when comparing between particles that are at the *same phase*.

Sometimes questions or other notes use $\frac{\Delta x}{\lambda}$ in place of $\frac{\Delta s}{\lambda}$. If so, the *x* here is a length along the direction of travel of the energy of the wave, and *not* the displacement in each oscillation!

<table>
<tr><th></th><th>same wave</th><th>different waves</th></tr>
<tr><td>path difference</td><td>

phase difference $\Delta\phi$ between P and Q:
$$\Delta\phi = (2\pi)\frac{\Delta s}{\lambda}$$</td><td>

phase difference $\Delta\phi$ between 2 waves (P and Q are at the same phase on their individual waves):
$$\Delta\phi = (2\pi)\frac{\Delta s}{\lambda}$$</td></tr>
<tr><td>time difference</td><td>

phase difference $\Delta\phi$ between P and Q:
$$\Delta\phi = (2\pi)\frac{\Delta t}{T}$$</td><td>

phase difference $\Delta\phi$ between P and Q:
$$\Delta\phi = (2\pi)\frac{\Delta t}{T}$$</td></tr>
<tr><td></td><td></td><td>Note that a meaningful comparison between 2 waves can be made only if both waves are of the same wavelength and frequency, using particles / segment at the same phase.</td></tr>
</table>

Example 4

A sound wave of frequency 400 Hz travels in air at a speed of 320 m s^{-1}. Find the phase difference between two points on the wave that are 0.20 m apart in the direction of travel.

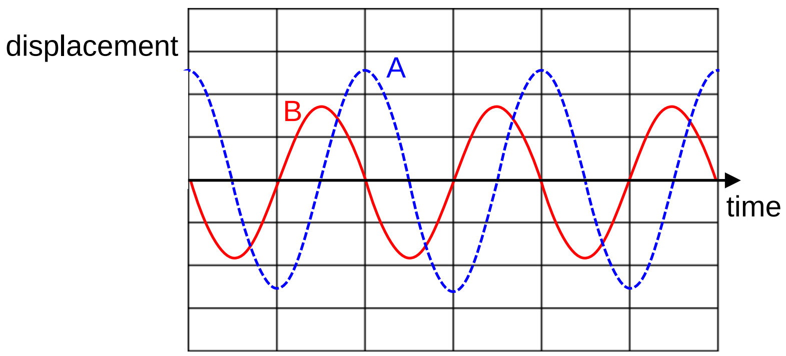
Solution

$$v = f\lambda$$
$$\lambda = \frac{v}{f} = \frac{320}{400} = 0.800 \text{ m}$$

$$\text{phase difference } \Delta\phi = (2\pi)\frac{\Delta s}{\lambda} = (2\pi)\frac{0.2}{0.800} = \frac{\pi}{2} \text{ rad} = 90.0°$$

Example 5

The graphs below show the variation of displacement with time of 2 waves of the same frequency as received by a detector. Find the phase difference between the 2 waves.

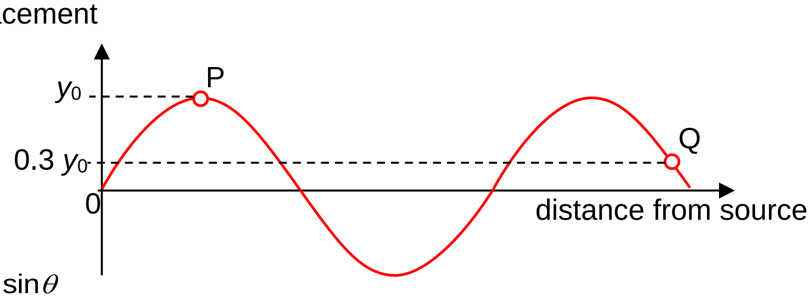


Solution

$$\text{phase difference } \Delta\phi = (2\pi)\frac{\Delta t}{T} = (2\pi)\frac{\frac{1}{4}T}{T} = \frac{\pi}{2} \text{ rad} = 90.0°$$

Example 6

The graphs below show the variation of displacement with distance of a sinusoidal wave at an instant is shown. Find the phase difference of two particles, P and Q, along the wave.



Solution

phase angle of P

$\theta_P = \frac{\pi}{2}$ rad or 90°

wave can be expressed as $y = y_0 \sin\theta$

for point Q $0.3y_0 = y_0 \sin\theta$

phase angle of Q $\theta_Q = \pi - \sin^{-1}(0.3)$ or $\sin^{-1}(0.3)$ (rejected, not in first quadrant)

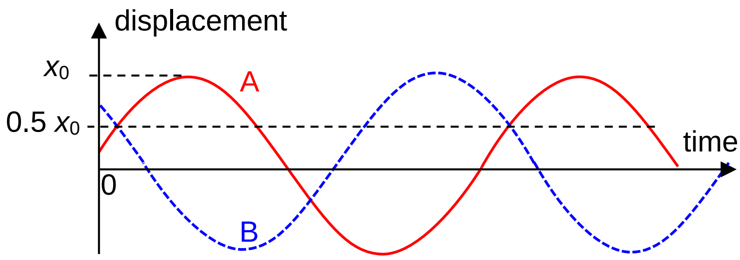
$= 2.84$ rad or 162°

$\Delta\phi = \theta_Q - \theta_P$

$= 1.27$ rad or 72.5°

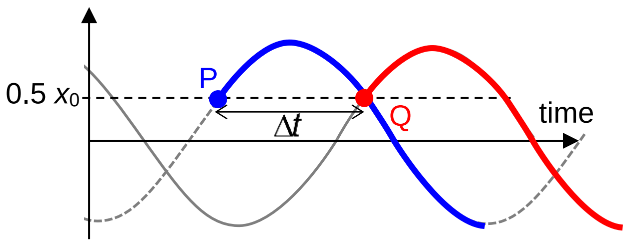
Example 7

2 sinusoidal waves A and B have the same amplitude and frequency. Their variation of displacement with time is shown. Find the phase difference between the 2 waves wave.



Solution

We need to compare the time lag between points on the wave that are in phase:



For point P:

$0.5x_0 = x_0 \sin\theta_P$

$\theta_P = \sin^{-1}(0.5)$

$= \frac{\pi}{6}$ rad or 30.0°

Point Q (red) coincides with the $\left(\pi - \frac{\pi}{6}\right)$ rad or $(180° - 30°)$ phase angle of the blue wave.

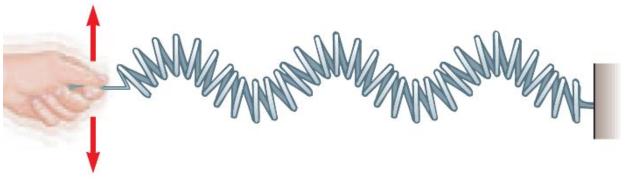
phase difference between P and Q $\Delta\phi = \theta_Q - \theta_P$

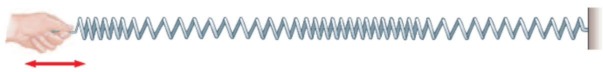
$= \left(\pi - \frac{\pi}{6}\right) - \frac{\pi}{6}$ OR $(180° - 30°) - 30°$

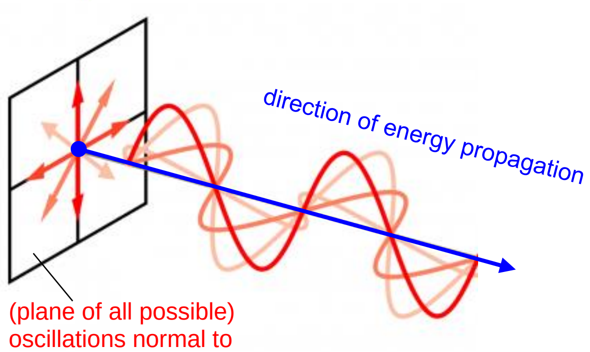
$= \frac{2\pi}{3}$ rad OR 120°

11.3 Transverse Wave vs Longitudinal Wave

A **transverse wave** is one where the oscillations are normal to the direction of energy propagation.	A **longitudinal wave** is one where the oscillations are parallel to the direction of energy propagation.



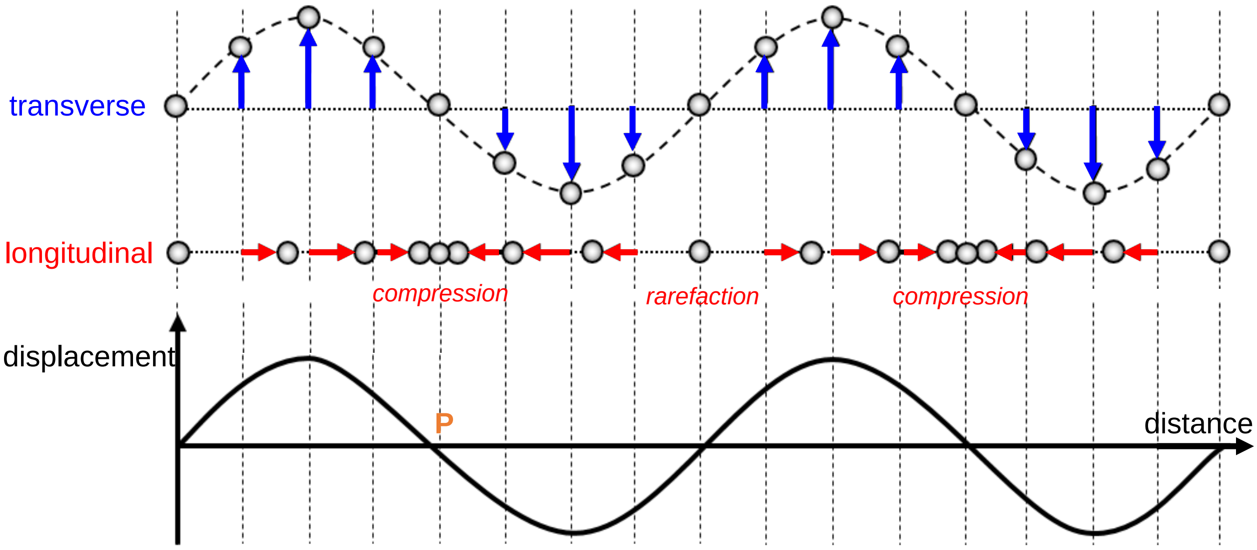




In both illustrations above involving the slinky above, the arm is the source of oscillation of the progressive waves.

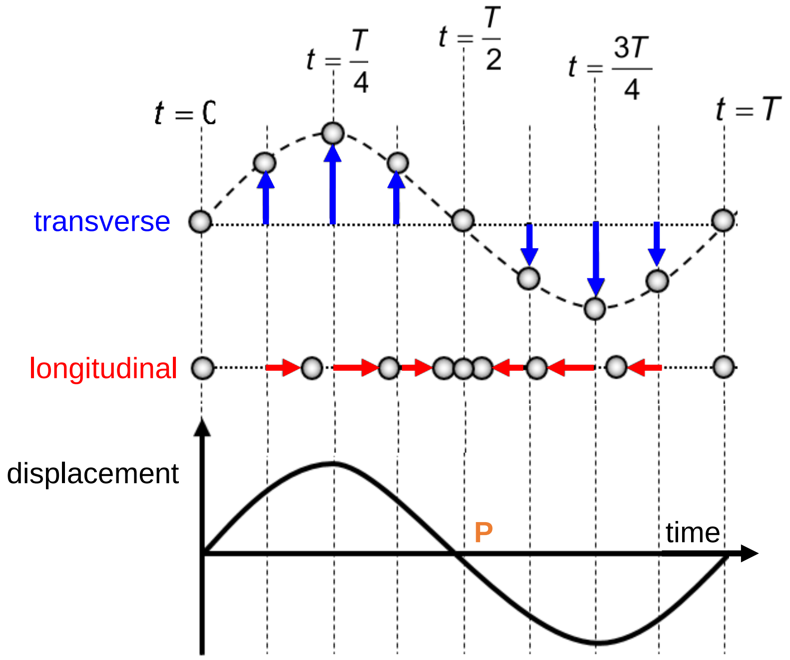
Energy from the arm moves to the right - the *wave front* or *wave profile* is moving to the right even though there is no net movement of the slinky; the *medium* through which the energy is propagated.

11.3.1 Variation of displacement with distance



Conventionally, rightwards is taken to be the positive direction. For the longitudinal wave, the neighbouring particles before point P have positive displacement – they are displaced to the right of their average positions towards P. The reverse is true for points beyond P, so P is a region of compression.

11.3.2 Variation of displacement with time



Note that for a displacement-time graph, we track the displacement of a *single particle* across time.

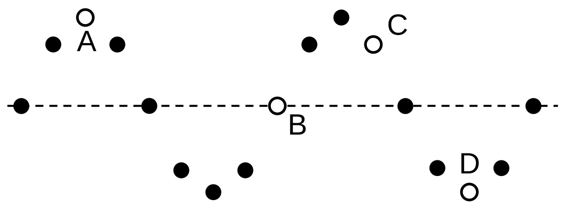
Each of the particle illustrated on the graphs here is akin to a single frame of a continuous video.

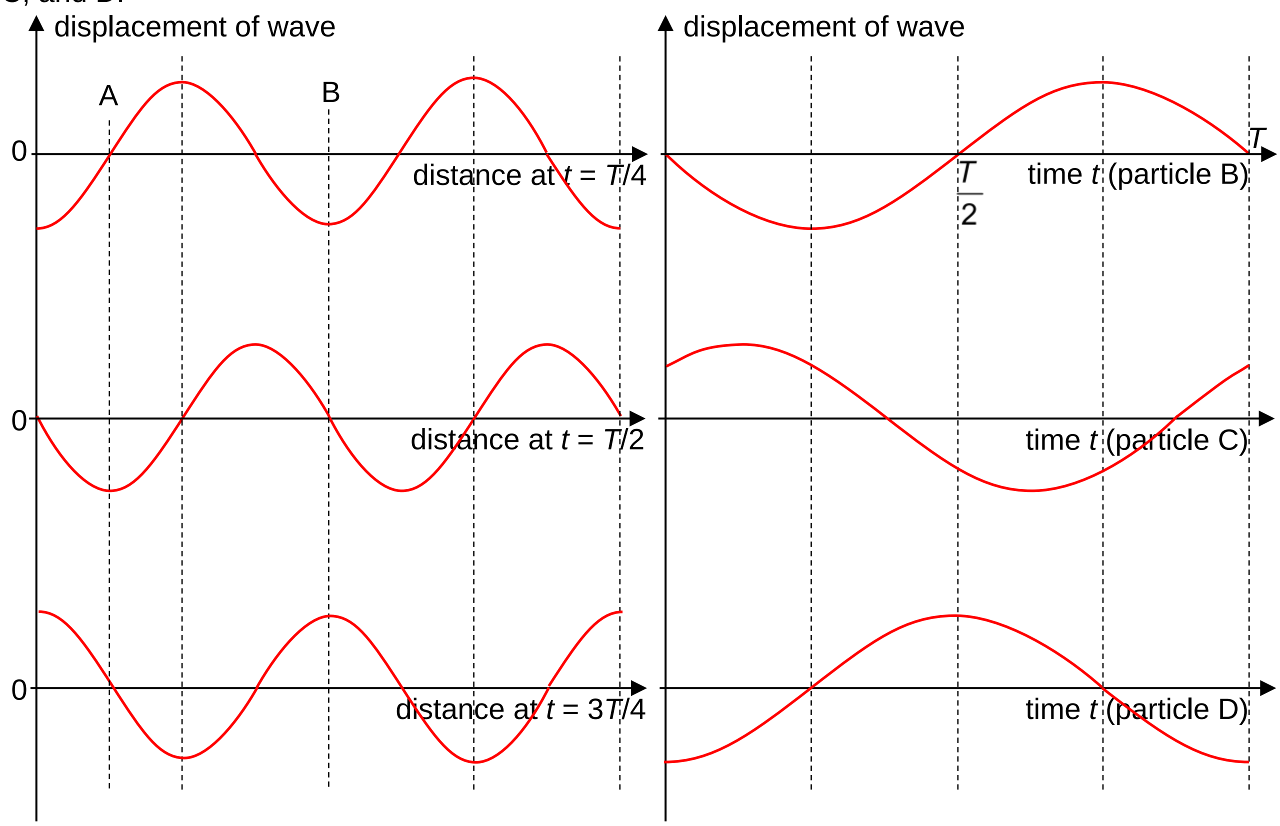
P is *not* a region of compression.

P tells of the time taken for half the period, and also gives directional information, that the particle has negative velocity at the time of P.

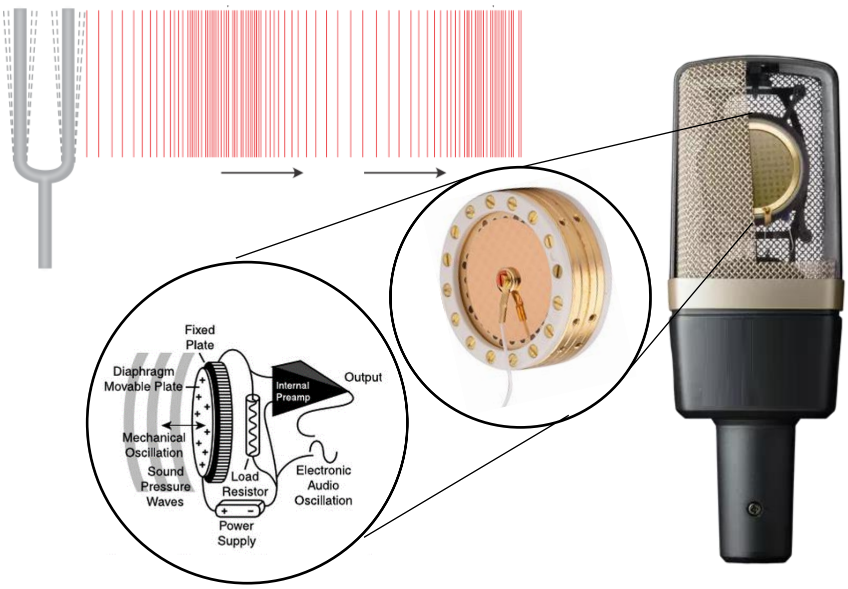
Example 8

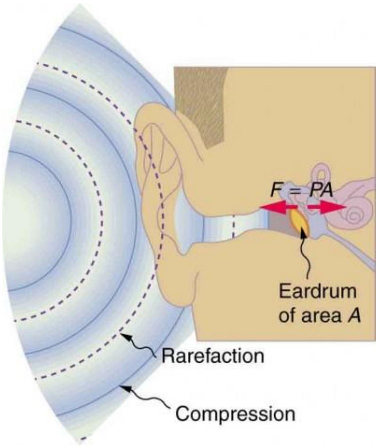
Particles A, B, C and D are particles on a progressive transverse sinusoidal wave moving to the right. A photograph of the wave at time t = 0 s is identical to one when $t = T$ where T is the period of an oscillation. Sketch the (i) variation of displacement with distance for particle A and (ii) variation of displacement with time for particles B C, and D.





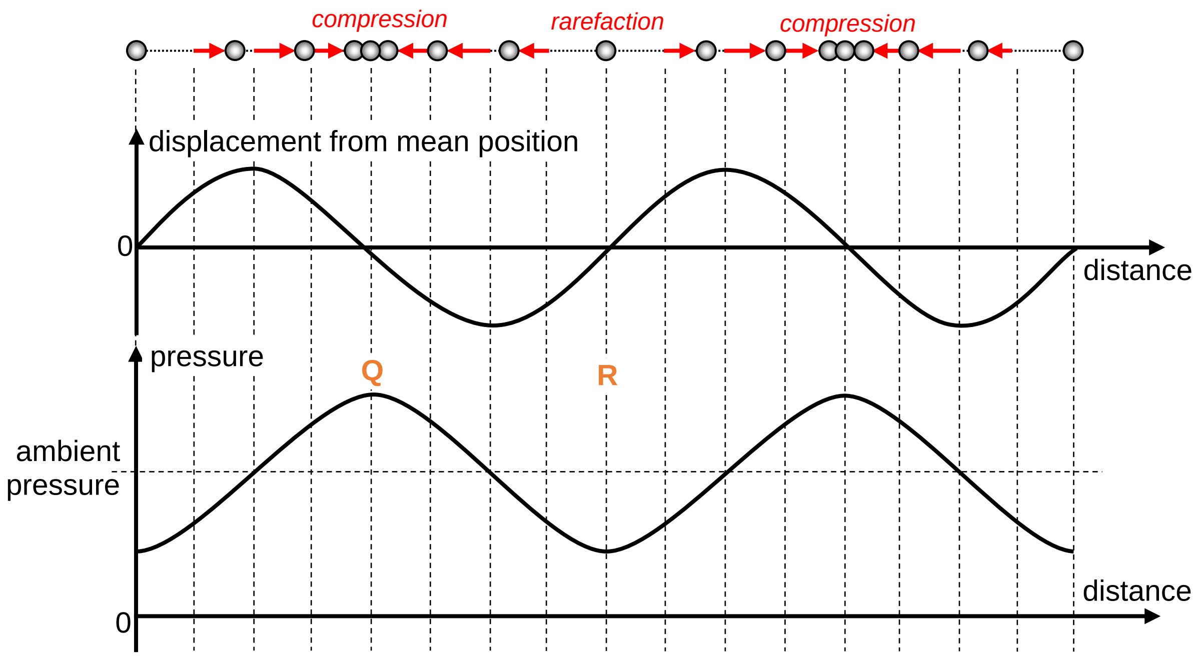
11.3.3 Pressure variations of Sound Wave





Our ears work basically similar to a microphone – the sound waves vibrates a thin sheet of material (our ear drums, or a "diaphragm" in a microphone).

Recall that there is acceleration associated with oscillatory vibrations – so there has to be *force(s)* acting across the *area* of the thin sheet of material. **Sound waves are pressure waves**. We hear sound due to the variations of pressure with time at our eardrums.



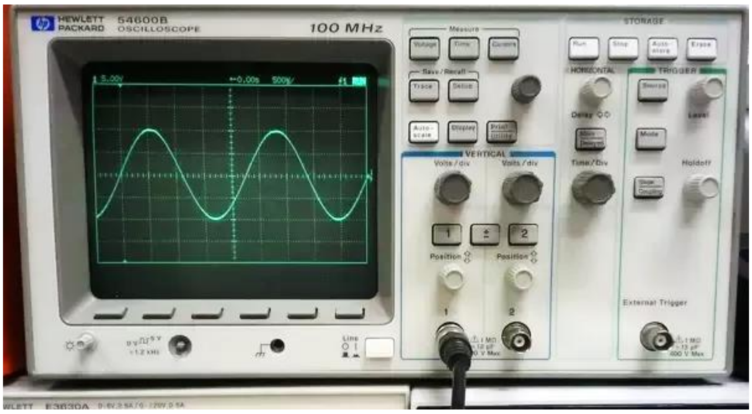
We should take note:

- at regions of maximum pressure (e.g. Q) the particle has no displacement but its "neighbours" have been displaced towards it
- at regions of minimal pressure (e.g. R) the particle has no displacement but its "neighbours" have been displaced away from it
- a phase difference of $\frac{\pi}{2}$ or 90° between the displacement and the pressure-variation graph
- both the displacement and pressure-variation graph can give us the wavelength of the sound
    - the wavelength is the distance between adjacent compressions or rarefactions

11.4 Use a Cathode Ray Oscilloscope (c.r.o.) to measure Frequency of Sound

Regard a cathode ray oscilloscope (c.r.o.) as a voltmeter that can display a rapidly varying voltage. It accepts a changing voltage signal as input – this changing voltage shows up on the *y*-axis of the screen.

The c.r.o. then "sweeps" the vertically changing signal horizontally across the *x*-axis at a steady speed to display the wave form.

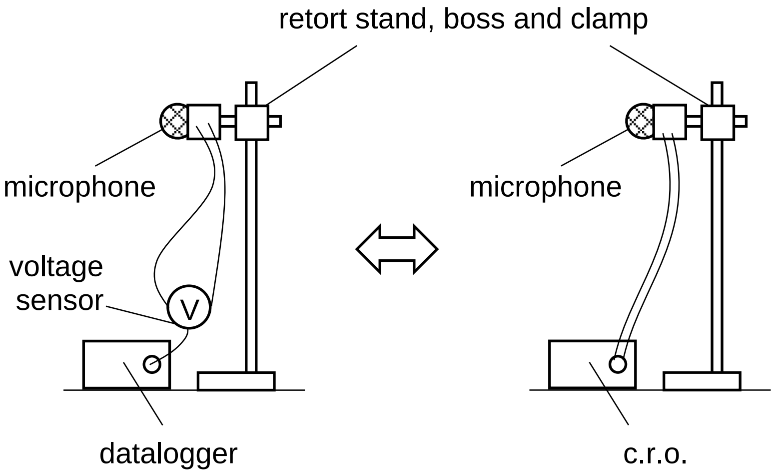


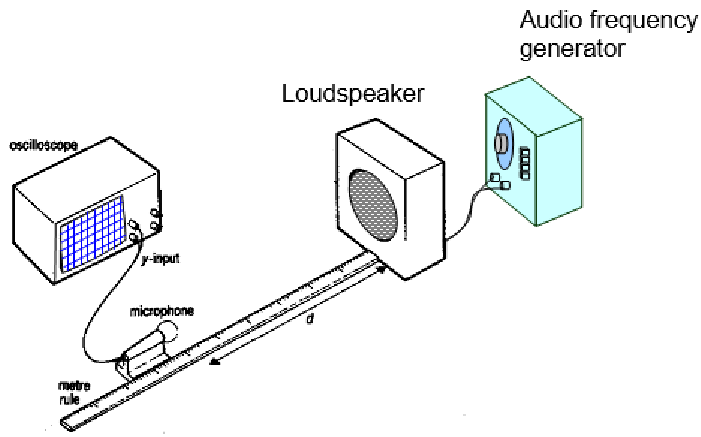
To measure the frequency of sound waves:

(a) Use a microphone to convert the sound wave into a varying voltage which is of same frequency as the sound wave.
(b) Connect the microphone to the input of a c.r.o.
(c) Adjust the time-base and input sensitivity to display complete waveforms on the c.r.o. screen.
(d) Read the period of the waveforms T.
(e) Frequency is given by $f = \dfrac{1}{T}$

Nowadays, the function of a CRO can be easily replicated by a voltage sensor connected to a datalogger.

In both cases, the measurement we get is from the variation of voltage with time graph.



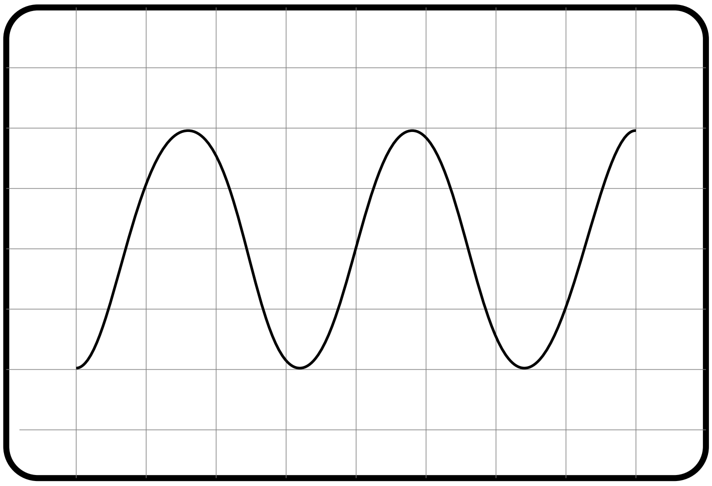


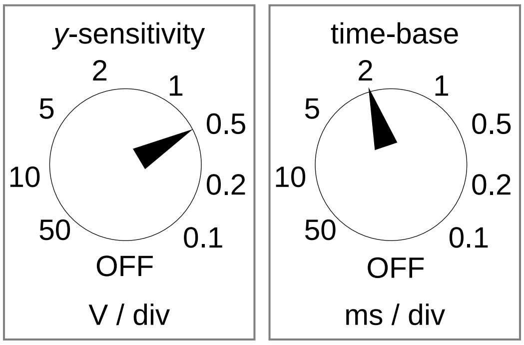
Example 9

A sinusoidal sound wave of unknown frequency is fed into a c.ro.. The waveform on the c.r.o. screen is as shown.

(a) Determine the frequency of the sound.

(b) Sketch the trace produced by a sound wave of twice the frequency and the same loudness displayed on the c.r.o. screen with the same settings.



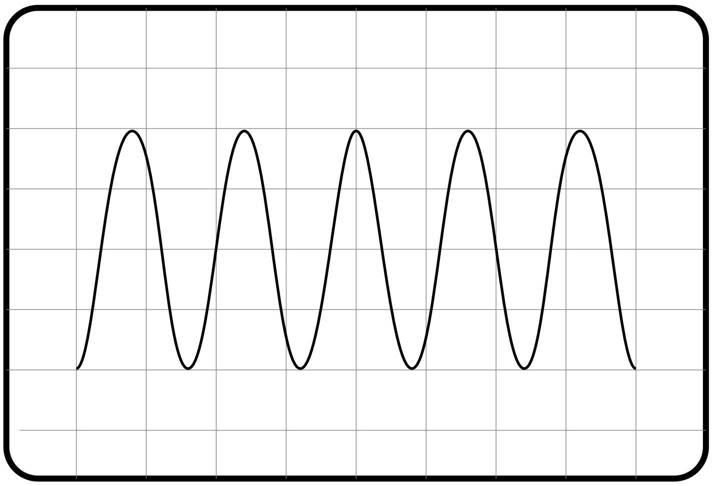


Solution

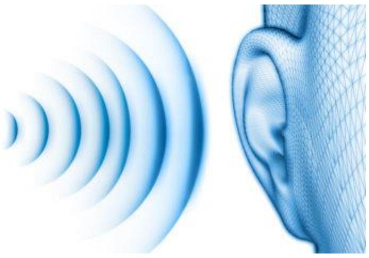
(a) duration of 2.5 wavelengths: $2.5T = (8)(2\times10^{-3})$

$$\text{frequency } f = \frac{1}{T} = \frac{2.5}{(8)(2\times10^{-3})} = 156\text{ Hz}$$

(b)



11.5 Waves Transfer Energy from a Source to a Receiver



Recall that a progressive wave is one in which energy is transferred from the *source of disturbance*, through surrounding regions.

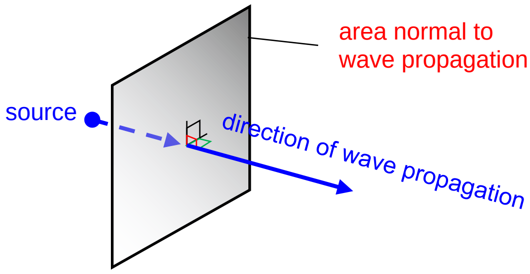
At the "micro" perspective, the oscillating particles in a wave transfers its energy from one particle to the next. To quantify the energy at a "macro" level, we typically discuss the

(i) source power,
(ii) geometry (shape) of how a wave spreads in space,
(iii) intensity of the wave at different distances from the source, and
(iv) received power.

11.5.1 Intensity of a Wave

The **intensity** of a wave is the rate of energy flow per unit area that is perpendicular to the direction of wave propagation.

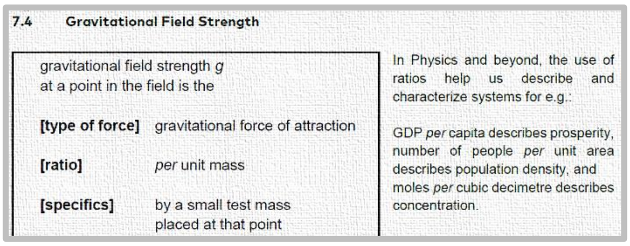
$$I = \frac{P}{A}$$



Intensity I is given by $\frac{\text{power}}{\text{area}}$, therefore the units for intensity I is W m^{-2}.

This *per unit* area here is imaginary – as a way of comparing the energy passing through that region.

We previously referred to this concept of using ratios (*per* capita, *per* dm^3, *per* unit mass) for comparing quantities in Gravitational Field:



7.4 Gravitational Field Strength

gravitational field strength *g* at a point in the field is the

[type of force]	gravitational force of attraction
[ratio]	*per* unit mass
[specifics]	by a small test mass placed at that point

In Physics and beyond, the use of ratios help us describe and characterize systems for e.g.:

GDP *per* capita describes prosperity, number of people *per* unit area describes population density, and moles *per* cubic decimetre describes concentration.

11.5.2 Intensity and Amplitude

Recall from H210 Oscillations that the total energy (kinetic energy and potential energy) of an oscillating system is directly proportional to the square of the amplitude: $E_{\text{total}} = \left[\frac{1}{2}m\omega^2\right]x_0^2$.

Since a wave comprises of many oscillating particles, for waves of the same type and frequency,

$$I = \frac{P}{A} = \frac{1}{A}\left(\frac{E_{\text{total}}}{t}\right)$$

$$I \propto x_0^2$$

intensity $\propto$ (amplitude)^2

$$\left(I \propto x_0^2\right)$$

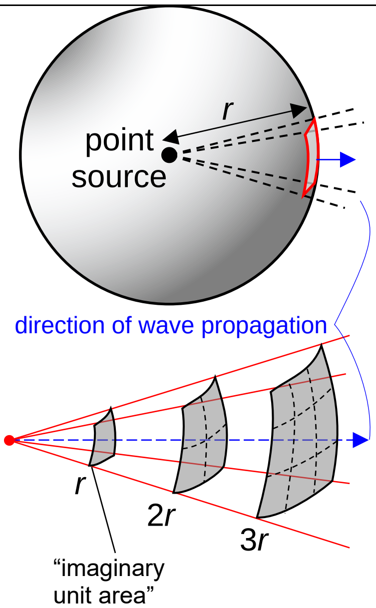
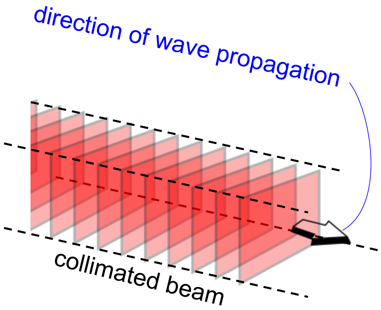
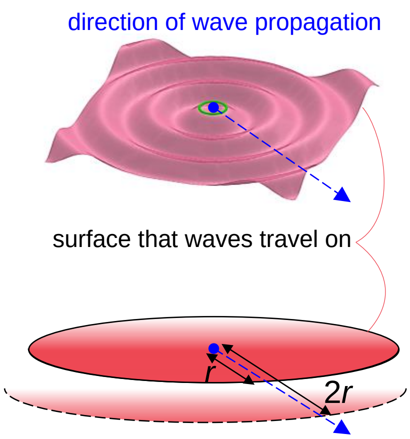
11.5.3 The Geometry of Spreading

As waves from a source spread out through space, the distribution energy may drop. The rate at which the energy density drops depends on the shape that the wavefronts are spreading at:

<table>
<tr><th>spherical wavefronts</th><th>planar wavefronts</th><th>circular wavefronts</th></tr>
<tr><td>a wave from a point source and travelling without loss of energy

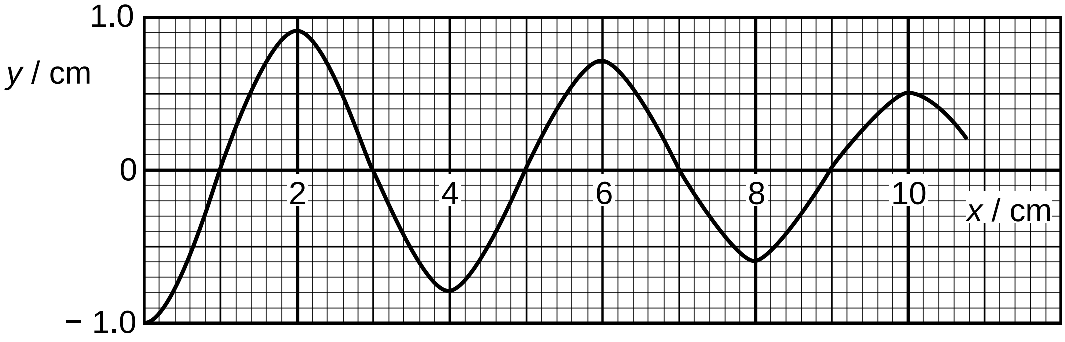
radiates outwards uniformly in all directions</td><td>waves from a source travels without loss of energy

along a collimated beam</td><td>waves from a point source, and travelling without loss of energy

confined to a 2D surface</td></tr>
<tr><td>
</td><td>
</td><td>
</td></tr>
<tr><td>$$I = \frac{P}{A} = \frac{P}{4\pi r^2}$$
$$I \propto \frac{1}{r^2}$$</td><td>$$I = \frac{P}{A}$$</td><td>energy distribution
$$= \frac{P}{\text{circumference}} = \frac{P}{(2\pi r)} \propto \frac{1}{r}$$</td></tr>
<tr><td>the intensity I obeys an inverse square law</td><td></td><td>think of it as energy distributed along circumference (a “linear density” instead of “aerial density”)</td></tr>
</table>

Example 10

A wave is produced on the surface of a liquid. At one particular time, the variation of the vertical displacement *y* with distance *x* along the surface of the liquid is shown.



The wave has energy density I_1 at distance *x* = 2.0 cm and energy density I_2 at distance *x* = 10.0 cm. Find the ratio $\frac{I_2}{I_1}$. Hence, suggest if the wave is a circular wave.

Solution

$$\text{intensity} \propto (\text{amplitude})^2$$

$$\frac{I_2}{I_1} = \left[\frac{(\text{amplitude})_2}{(\text{amplitude})_1}\right]^2$$

$$= \left(\frac{0.5}{0.9}\right)^2 = 0.309$$

Assuming that the source of the wave is a point source, the waves are confined to the liquid surface and travel uniformly via circular wave fronts, the energy density should be inversely proportional to the distance travelled:

$$\text{if} \quad I \propto \frac{1}{r}$$

$$\frac{I_2}{I_1} = \frac{r_1}{r_2}$$

$$= \frac{2}{10}$$

$$= 0.200$$

As the value obtained is 0.200 which is different from the value of 0.309 obtained previously, we can tell that it is likely not a circular wave. This means that the wave may be directed towards a sector of the circle and is not a circular wave.

Example 11

A point source emits sound waves uniformly in all directions. The loudness of the sound waves is initially measured at a distance of 7.0 m away from it. The power of the source is then halved. Find the new distance from the point source where the loudness is perceived to be the same as before.

Solution

loudness is a measure of intensity:

$$I = \frac{P}{A} = \frac{P}{4\pi r^2}$$

$$\frac{\cancel{P}}{\cancel{4\pi}(7)^2} = \frac{\frac{1}{2}\cancel{P}}{\cancel{4\pi}(r_{new})^2}$$

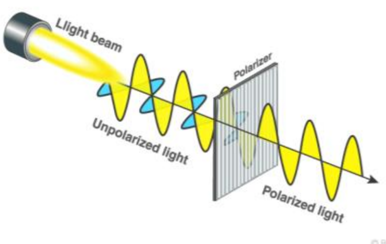
$$r_{new} = \frac{1}{\sqrt{2}}(7)$$
$$= 4.95 \text{ m}$$

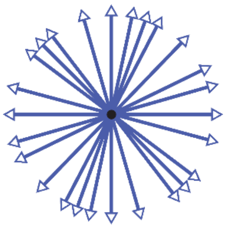
Note: Logic check: the power is reduced so to hear the same loudness we need to be nearer the source.

11.6 Polarisation of a transverse wave

In a polarised wave, the

oscillations are along one direction only,
in a single plane that is
normal to the direction of energy transfer of the wave



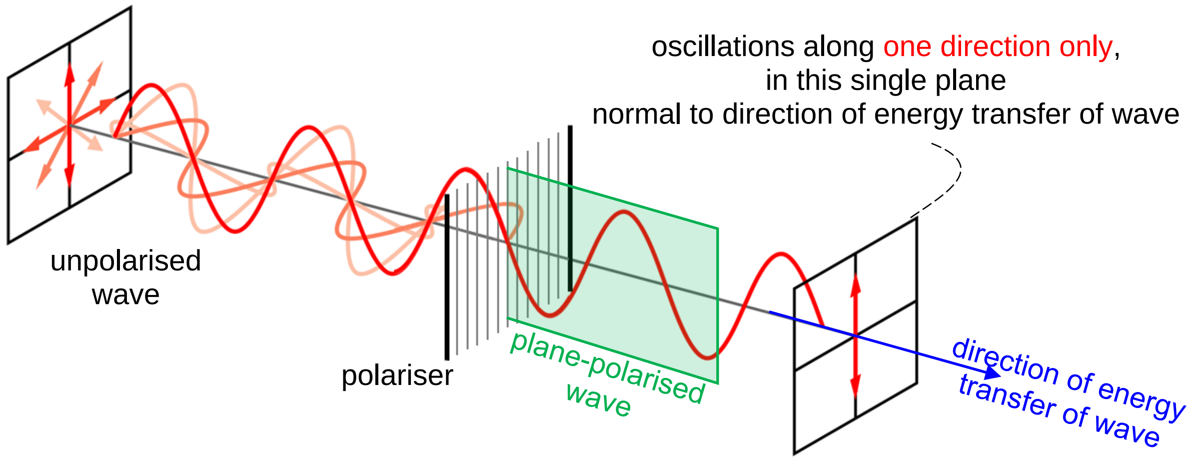


Conversely, an unpolarized wave heading towards you will look like the *planes of polarization* are all random and changing:

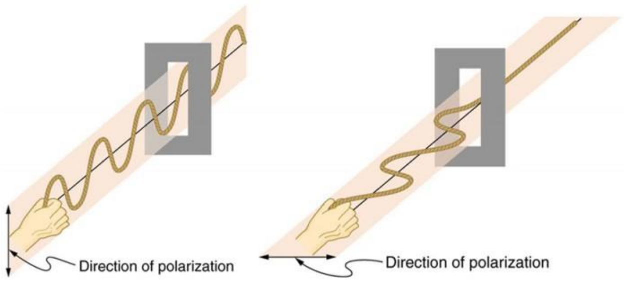
As polarisation deals with oscillations along directions that are *normal* to the direction of propagation, longitudinal waves cannot exhibit polarisation.

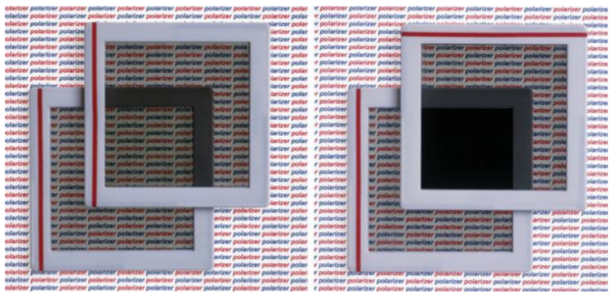
Only transverse waves can be polarised

We can pass an unpolarised wave through a (linear) polarizer to get a *linearly polarised* wave or a *plane polarised* wave. A polariser only allows polarization along 1 specific direction to pass through.

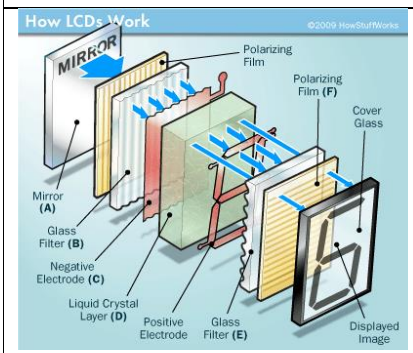
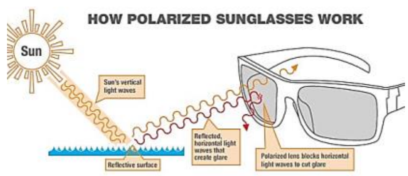
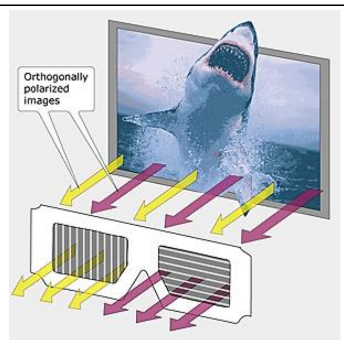


We can imagine a stretched rope and a slit as a mechanical analogy. This means that if we stack 2 ideal linear polarisers at right angles to each other, we can cut out the waves.





Selectively cutting out light has many useful applications. Some examples follow:

LCD screens	sunglasses	3D glasses
		
"Old school" LCDs (e.g. watches and calculators) reflects ambient light through 2 polarizers to your eyes. The dark parts have polarizations set at 90° relative to each other. Colour LCDs work similarly and give polarised light. OLED screens do not work this way and produce unpolarised light instead.	Reflections off road puddles and snow tend to be largely polarized along one direction, so sunglasses manufacturers align their lenses (if it is the polarised variants) 90° to the reflections to cut glare. Photographers can use linear polarizers to reduce or enhance reflections off glass surfaces as well.	An earlier generation of 3D glasses use linear polarizers set at 90° to each other so that each eye sees a slightly different image. However the effect is reduced if the head tilts. Later generations will use *circular polarisers* instead of linear polarizers for getting the eyes to see different images.

11.6.1 Malus' Law

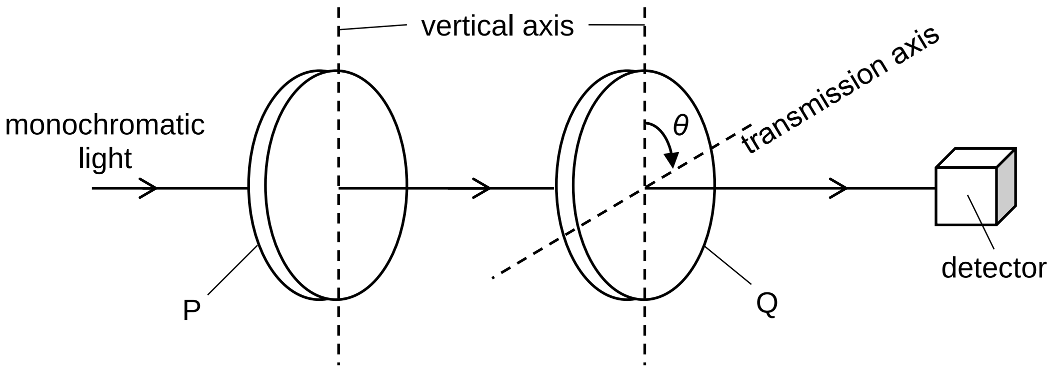
Malus' Law applies to an (already) plane-polarised electromagnetic wave (e.g. beam of light) going through a polariser (polarising filter).

We often use this form: $I = I_0 \cos^2\theta$ where I is the intensity of light **after** passing through the polariser, I_0 is the intensity of incident polarised light before the polariser, and θ is the **relative** angle between the polarization of the incident light and that of the polarizer.

Malus' Law:
intensity $\propto \cos^2\theta$
$\left(I = I_0 \cos^2\theta\right)$

Example 12

Two polarisers P and Q are initially arranged such that their polarisation axes are parallel and vertical. The intensity of the emergent light after Q is I_0. Q is then rotated such that the emergent intensity is reduced by 90%. Find (i) the angle by which Q is rotated, and (ii) the ratio of the amplitudes of the electric fields of light waves: $\dfrac{\text{amplitude between P and Q}}{\text{amplitude after Q}}$.



Solution

(i) before rotation, intensity of light before Q and after Q is the same I_0

after rotation, intensity of light before Q is I_0 and after Q is 0.1 I_0

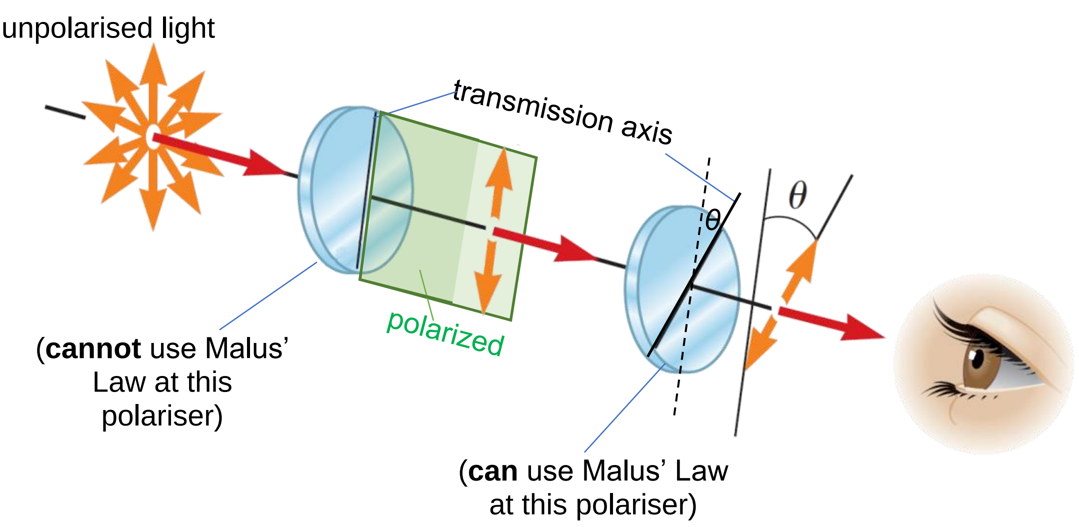
$$I = I_0 \cos^2\theta$$
$$\frac{I}{I_0} = \frac{0.1\ \cancel{I_0}}{\cancel{I_0}} = 0.1 = \cos^2\theta$$
$$\theta = 71.6^\circ$$

(ii) intensity $\propto$ (amplitude)^2

$$I \propto x_0^2$$
$$\frac{x_{0,\,PQ}}{x_{0,\,\text{past Q}}} = \sqrt{\frac{I_0}{I}} = \sqrt{\frac{\cancel{I_0}}{0.1\cancel{I_0}}} = 3.16$$

Note: Logic checks for (i): yes intensity is almost diminished completely so angle should be near 90° and (ii) yes amplitude, which indicates energy, should be larger before polariser.

11.6.2 Malus' Law in terms of Amplitude of Waves



The oscillations of a wave exhibit *displacement*, which is a vector quantity and so can be resolved into components.

When **polarised** light reaches a polariser, the component of the displacement that is

- parallel to the transmission axis is allowed to pass,
- perpendicular to the transmission axis is absorbed by the polariser

So Malus' Law, when expressed as *amplitudes* (maximum displacement) reads as

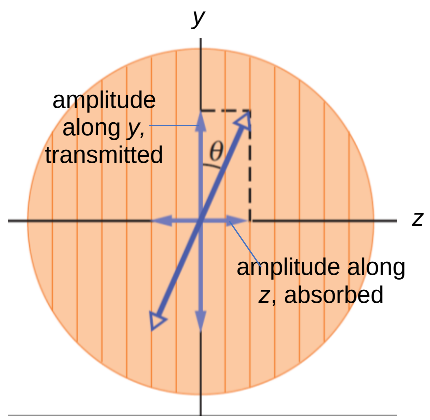
$$x_{\text{transmitted}} = x_{\text{before polariser}} \cos\theta$$

$$\text{since} \quad \text{intensity} \propto (\text{amplitude})^2$$

$$I_{\text{transmitted}} \propto x_{\text{transmitted}}^2$$

$$\propto (x_{\text{before polariser}} \cos\theta)^2$$

$$\propto \cos^2\theta$$



This polariser allows only vertical components of a wave's oscillations to pass through.

Example 13

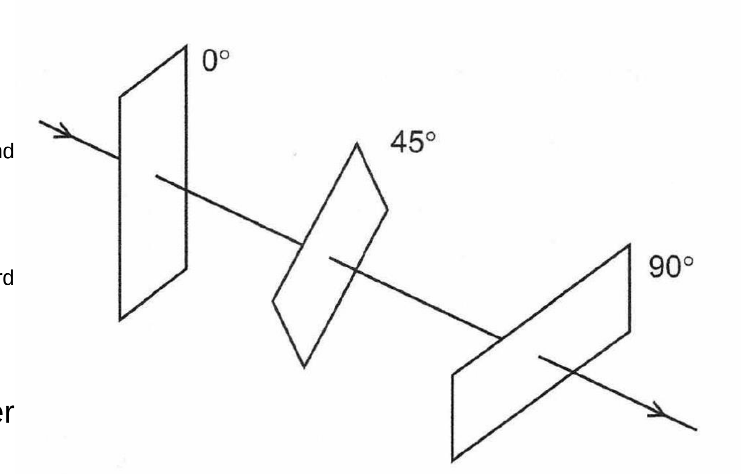
A narrow, parallel beam of unpolarised light is directed towards three ideal polarising filters. The beam exits the first filter with its plane of polarisation vertical. The plane of polarisation of the second filter is at angle of 45° to the first filter. The beam emerges from the second filter with amplitude $\frac{A}{\sqrt{2}}$. The third filter has its plane of polarisation at 90° to the first filter as shown. Find the emergent intensity in terms of intensity after the first filter I.

Solution

Let the region between 1st and 2nd polariser be region P

and the region between 2nd and 3rd polariser be region Q.

Consider amplitudes before and after 2nd polariser:



$$A_Q = A_P \cos\theta$$
$$\frac{A}{\sqrt{2}} = A_P \cos(45°)$$
$$A_P = A$$

Consider amplitudes before and after 3rd polariser:

$$A_{emergent} = A_Q \cos\theta$$
$$= \frac{A}{\sqrt{2}} \cos(45°)$$
$$= \frac{A}{2}$$

so $$\frac{I_{emergent}}{I} = \left(\frac{A_{emergent}}{A_P}\right)^2 = \left(\frac{A/2}{A}\right)^2$$
$$= \frac{1}{4}$$

$$I_{emergent} = \frac{1}{4}I$$

Note: In this H211 Waves, be sure to distinguish between A for amplitude of waves and A for the area when considering wave intensity. Also, we could have applied Malus' Law twice to solve this question: $I_{emergent} = I_Q \cos^2(90° - 45°) = I \cos^2(45°)\cos^2(90° - 45°) = I \cos^4(45°)$

11.7 Ending Notes

This topic forms a large and important basis for the next topic, H212 Superposition.

Superposition has been a topic that JC students consistently found challenging to master – securing the foundation in this topic is therefore crucial to a smoother advancement in your Physics journey.

You may use the space below for your own summary and mind-map(s):