Title	Junior College 'A' Levels H1/H2 Mathematics – Normal Distribution
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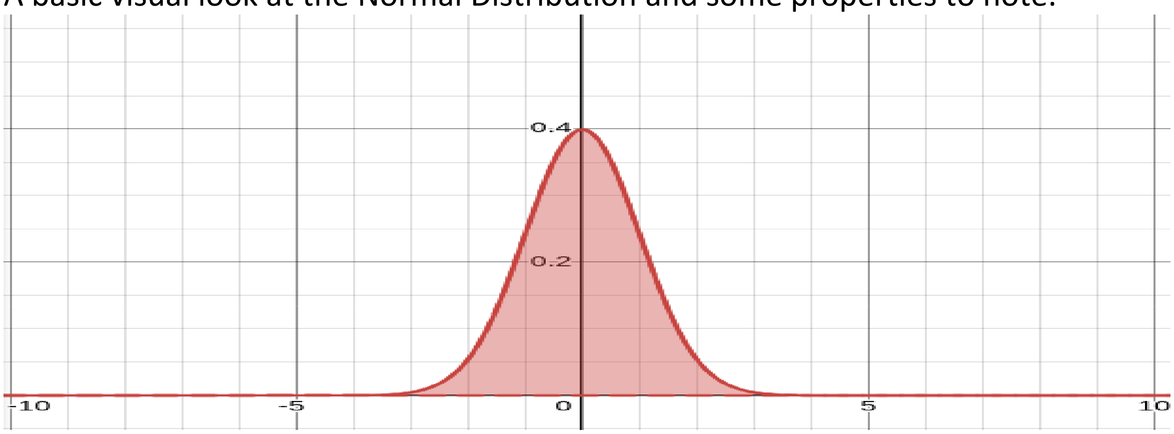
In many situations out there, many types of continuous random variables, such as test scores, height and weight of students of a certain age would show the following characteristics once data got collected and plotted as it would be on a probability distribution.

- You would realize that majority of the data are centred in the middle, with extreme small and big values tailing off symmetrically on the left and right of the average measurement respectively.

When this happens, the random variable X is said to follow a Normal Distribution with parameters μ and σ^2, where $X \sim N(\mu, \sigma^2)$, with μ referring to the mean and σ^2 referring to the variance of the Normal Distribution.

(*Be careful when dealing with notations, certain books, software and calculators deal with Normal Distribution using the Standard Deviation σ parameter rather than variance which is σ^2. In such cases, the obvious first step you should take is to square-root the variance to get the value of standard deviation σ.)

A basic visual look at the Normal Distribution and some properties to note.



- The probability value is the area between the curve and the x-axis. (Which is also the definite integral of the Normal Distribution in question.)
- The probability of getting a very specific value in a Normal Distribution is basically zero since area under curve cannot be created on a continuous random variable just with specific values. Instead of defining specific value on a Normal Distribution, we usually define a range of values to calculate probability in a Normal Distribution. [Therefore $P(X = x) = 0$]

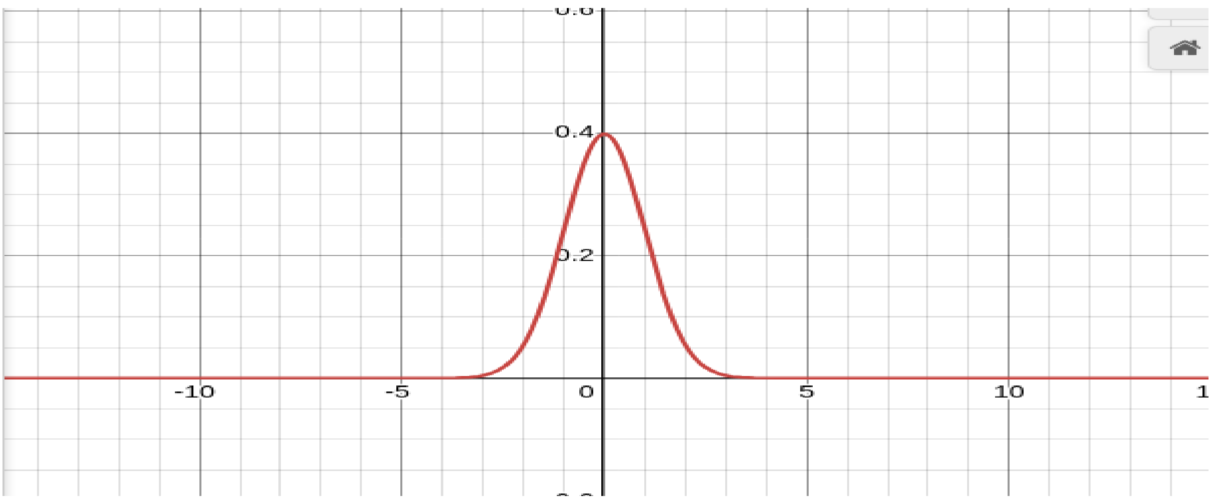
- Any Normal Distribution is symmetrical at the mean μ. (This property is important as certain questions you will encounter requires you to understand this symmetrical property of any Normal Distribution.)

Understanding the concept of a Standard Normal Distribution

- Any Normal Distribution can technically be transformed to a Standard Normal Distribution.
- A Standard Normal Distribution has the property of which the area under curve from negative infinity to positive infinity is exactly 1.
- A Standard Normal Distribution also has property of mean $\mu = 0$ and $\sigma^2 = 1$, for this reason, a Standard Normal Distribution will also have standard deviation $\sigma = 1$ as well.

(Note: In order to input "negative infinity" in Graphing Calculator, press -E99. In order to input a value of "positive infinity" in Graphing Calculator, press E99.)

A visual look at the standard normal distribution.



Understanding the concept of Z-score in Standard Normal Distribution

- Z-score refers to the number of standard deviations from the mean in a standardized normal distribution where the Z -score can take on any finite values. $(-\infty < Z < \infty)$
- Z-score of any normal distribution can be computed using the below formula.

$$Z = \frac{X - \mu}{\sigma}$$

Z refers to the Z-score
X refers to the position of the normal random variable on the X-axis as it is in original unstandardized form.
μ refers to the mean of the normal distribution as it is in original unstandardized form.
σ refers to the standard deviation as it is in the original unstandardized form.

Process of Finding Probability in a Standard Normal Distribution:

In order to find probability in a Standard Normal Distribution, we use the TI-84 graphing calculator in the following manner.

Example 1. Find the probability of the following

a) $P(Z < 1.96)$
b) $P(Z > -0.586)$
c) $P(0.43 < Z < 1.23)$

Procedures (Example 1a)	Output on Graphing Calculator Screen
Press [2ND] followed by [VARS]	DISTR DRAW 1:normalpdf (2:normalcdf (3:invNorm (4:invT (5:tpdf (6:tcdf (7:χ^2pdf (
Look for "normalcdf" option and press [ENTER]	normalcdf Lower: Upper: μ: 0 σ: 1 Paste
For probability values less than Z We key in -E99 in the field "lower" and we key in 1.96 in the upper field. Since Standard Normal Distribution has a value of $\mu = 0$ and $\sigma = 1$, we input $\mu = 0$ and $\sigma = 1$	normalcdf Lower: -E99 Upper: 1.96 μ: 0 σ: 1 Paste
Press the down arrow after checking the inputs and press down arrow until the cursor is on "Paste" and press [ENTER] twice. The probability is 0.975 (3sf)	normalcdf(-E99, 1.96, 0, 1) .9750021748

Procedures (Example 1b)	Graphing Calculator Output
Press [2ND] followed by [VARS]	DISTR DRAW 1:normalpdf (2:normalcdf (3:invNorm (4:invT (5:tpdf (6:tcdf (7:χ^2pdf (
Look for "normalcdf" option and press [ENTER]	normalcdf Lower: Upper: μ: 0 σ: 1 Paste
For probability values more than Z. We key in -0.586 in the "Lower" field and E99 into the "Upper" field. Since Standard Normal Distribution has a value of $\mu = 0$ and $\sigma = 1$, we input $\mu = 0$ and $\sigma = 1$.	normalcdf Lower: -0.586 Upper: E99 μ: 0 σ: 1 Paste
Press the down arrow after checking the inputs and press down arrow until the cursor is on "Paste" and press [ENTER] twice. The probability is 0.721 (3sf)	normalcdf(-0.586, E99, 0, 1) .7210622905

Procedures (Example 1C)	Graphing Calculator Output
Press [2ND] followed by [VARS]	DISTR DRAW 1:normalpdf (2:normalcdf (3:invNorm (4:invT (5:tpdf (6:tcdf (7:χ^2pdf (
Look for "normalcdf" option and press [ENTER]	normalcdf Lower: Upper: μ: 0 σ: 1 Paste
Since the question mentioned we have to find the probability for which the Z-score is in between 0.43 and 1.23. We key in 0.43 in the "Lower" field and 1.23 in the "Upper" field. We set $\mu = 0$ and $\sigma = 1$.	normalcdf Lower: 0.43 Upper: 1.23 μ: 0 σ: 1 Paste
After checking that the values are correct, we can press down arrow key on calculator until the cursor is on "Paste", press enter twice. The probability of obtaining Z-score between 0.43 and 1.23 is 0.224 (3sf)	normalcdf (0.43, 1.23, 0, 1) .2242492346

Example 2

Find the probability of the following

(a) $P(|Z| < 1.234)$

(b) $P(|Z| \geq 2.17)$

(c) $P(|Z - 1| > 1.1389)$

2(a) $|Z| < 1.234$ is to be rewritten as $-1.234 < Z < 1.234$

Steps taken	Graphing Calculator Output
Enter “normalcdf” functionality of the graphing calculator	normalcdf lower: upper: μ: σ: Paste
Set “lower” as -1.234 Set “upper” as 1.234 Set μ as 0 Set σ as 1 Press down arrow key until the cursor is on “Paste”	normalcdf lower: -1.234 upper: 1.234 μ:0 σ:1 Paste
Press [ENTER] twice and the following should appear. The probability is 0.783 (3sf)	normalcdf(-1.234, 1.234, 0, 1) .7827969667

2(b) $|Z| \geq 2.17$ is to be written as $Z < -2.17$ OR $Z > 2.17$

Steps Taken	Graphing Calculator Output
Enter “normalcdf” functionality of calculator	normalcdf lower: upper: μ: σ: Paste

Input lower as - E99 Input upper as -2.17 Set $\mu = 0$ and $\sigma = 1$ Press arrow down key until the cursor is at paste.	normalcdf lower: -E99 upper: -2.17 μ:0 σ:1 Paste
Press [ENTER] twice.	normalcdf(-E99,2.17,0, 1) .150033693
Enter "normalcdf" functionality of calculator again. Input lower as 2.17 Input upper as E99 Set $\mu = 0$ and $\sigma = 1$ Press arrow down key until the cursor is at paste.	normalcdf lower: 2.17 upper: E99 μ:0 σ:1 Paste
Press [ENTER] twice.	normalcdf(2.17, E99, 0, 1) .150033693
$P(\|Z\| \geq 2.17)$ = 0.150033693+0.150033693 = 0.300 (3sf)	

2(c)

Rewrite $P(|Z - 1| > 1.1389)$ as $Z - 1 < -1.1389$ as well as $Z - 1 > 1.1389$ which can be transformed as follows:

$Z < -1.1389 + 1$ OR $Z > 1.1389 + 1$

$Z < -0.1389$ OR $Z > 2.1389$

Steps Taken	Graphing Calculator Output
Enter "normalcdf" functionality of calculator	normalcdf lower: upper: μ: σ: Paste
Key in the values as follows Lower: -E99 Upper: -0.1389 μ:0 σ: 1	normalcdf lower:-E99 upper:-0.1389 μ:0 σ:1 Paste
Press down arrow until the cursor is at "paste" and press [ENTER] key twice	normalcdf(-E99, -0.1389, 0, 1) 0.447645561
Enter "normalcdf" functionality of calculator again	normalcdf lower: upper: μ: σ: Paste
Key in the values as follows Lower: 2.1389 Upper: E99 μ:0 σ: 1	normalcdf lower: 2.1389 upper: E99 μ:0 σ:1 Paste

Press down arrow until the cursor is on "Paste" and press [Enter] key twice	normalcdf(2.1389, E99, 0, 1) 0.0162218257

Therefore $P(|Z-1|>1.1389)=$ 0.447645561+0.0162218257=0.461 (3sf)

Process of Finding Probability Values of Normal Distribution without consideration for Standardization.

Example 3.
Given that $X \sim N(23{,}9)$, find the following probabilities

(a) $P(20<X<25)$
(b) $P(X<19)$
(c) $P(X \geq 14)$

(a)

Enter "normalcdf" functionality of calculator	normalcdf lower: upper: μ: σ: Paste
Key in the following values Lower: 20 Upper: 25 μ:23 σ: 3	normalcdf lower:20 upper: 25 μ:23 σ:3 Paste
Press down arrow until the cursor is on "Paste" and press [ENTER] twice to get probability value	normalcdf(20,25,23,3) .5888522734

$P(20<X<25)=0.589$ (3sf)

(b)

Enter "normalcdf" functionality of calculator	normalcdf lower: upper: μ: σ: Paste
Key in the following values Lower: -E99 Upper: 19 μ:23 σ: 3	normalcdf lower: -E99 upper:19 μ:23 σ:3 Paste
Press down arrow until the cursor is on "Paste" and press [ENTER] twice to get probability value	normalcdf(-E99,19,23,3) .0912112819

$P(X < 19) = 0.0912$ (3sf)

(c)

Enter "normalcdf" functionality of calculator	normalcdf lower: upper: μ: σ: Paste
Key in the following values Lower: 14 Upper: E99 μ:23 σ: 3	

	normalcdf lower: 14 upper: E99 μ:23 σ:3 Paste
Press down arrow until the cursor is on "Paste" and press [ENTER] twice to get probability value	normalcdf(14,E99,23,3) .9986500328

$P(X \geq 14) = 0.999$(3sf)

Example 4.
Given that $X \sim N(15{,}3)$, find the following probabilities

(a) $P(|X - 15| < 4)$
(b) $P(|X - 15| > 2)$

4(a)
$P(|X - 15| < 4)$ can be rewritten as,
$P(-4 < X - 15 < 4)$
$P(11 < X < 19)$

Enter normalcdf functionality of calculator	normalcdf lower: upper: μ: σ: Paste

Key in the following values as follows Lower: 11 Upper: 19 μ:15 σ:$\sqrt{3}$	normalcdf lower:11 upper:19 μ:15 σ: $\sqrt{3}$ Paste
Press arrow down key until the cursor is on top of “paste” and press enter twice.	normalcdf(11, 19, 15, $\sqrt{3}$) .9790787186

$P(|X-15|<4)=0.979$ (3sf)

4(b) $P(|X-15|>2)$
Rewritten, it will look like the following
$P(X-15<-2)$ OR $P(X-15>2)$
$P(X<13)$ OR $P(X>17)$

Enter normalcdf functionality of graphing calculator	normalcdf lower: upper: μ: σ: Paste
Key in the following into the fields Lower: -E99 Upper: 13 μ:15 σ:$\sqrt{3}$	normalcdf lower:-E99 upper:13 μ:15 σ: $\sqrt{3}$ Paste
Press arrow down key until the cursor is on top of “paste” and press enter twice.	normalcdf(-E99, 13, 15, $\sqrt{3}$) .1241065934

Enter normalcdf functionality of graphing calculator again	normalcdf lower: upper: μ: σ: Paste
Key in the following into the fields as follows: Lower: 17 Upper: E99 μ:15 σ:$\sqrt{3}$	normalcdf lower:17 upper:E99 μ:15 σ: $\sqrt{3}$ Paste
Press arrow key until the cursor is on top of "Paste" and press [Enter] twice	normalcdf(17, E99, 15, $\sqrt{3}$) .1241065934

Adding both probability values, we get the following
0.1241065934+0.1241065934=0.248 (3sf)

Using inverse Normal Distribution functionality to find Z-Score or number of standard deviations away from mean by input of p-values from negative infinity of a normal distribution.

After studying how to find probability upon knowing the values of Z-Score or number of standard deviations away from mean, along with parameters μ and σ. It will be a logical next step to wonder if the reverse is also possible. The TI-84 family of calculator has a functionality that allows students to deduce the Z-score after knowing the probability value from negative infinity of the normal distribution up to the $Z - score$, along with parameters μ and σ.

The instructions below explain how to get into the "InvNorm" functionality of Ti-84 graphing calculators.

Steps Taken	Calculator Output

Press [2ND] followed by [VARS]	DISTR DRAW 1:normalpdf (2:normalcdf (3:invNorm (4:invT (5:tpdf (6:tcdf (7:χ^2pdf (
Press arrow down key until cursor is on top of "3:InvNorm" and press [ENTER]	InvNorm Area: μ: σ: Tail: LEFT CENTER RIGHT Paste:

Example 5

Given that $W \sim N(6,6)$, find the value of a or the range of values of a for each of the following:

(a) $P(W < a) = 0.00144$

(b) $P(W \geq a) > 0.25$

(c) $P(6 < W < a) > 0.4999$

(d) $P(|W| < a) = 0.01$

(e) $P(|W| \geq a) = 0.975$

(a)

Go to InvNorm Functionality of Graphing Calculator	Area: μ: σ: Tail: LEFT CENTER RIGHT Paste
Key in the fields as follows Area:0.00144 μ:6 σ: $\sqrt{6}$ Tail: Select "Left"	Area: 0.00144 μ:6 σ: $\sqrt{6}$ Tail: LEFT CENTER RIGHT Paste:
Press down arrow until the cursor is at "Paste" and press [ENTER] twice and the following should appear on the screen	invNorm (0.00144, 6, $\sqrt{6}$, LEFT) -1.300126258

$a = -1.30$ (3sf)

(b)

Go to InvNorm Functionality of Graphing Calculator	Area: μ: σ: Tail: LEFT CENTER RIGHT Paste
Key in the fields as follows Area: 0.25 μ: 6 σ: $\sqrt{6}$ Tail: RIGHT	Area: 0.25 μ:6 σ:$\sqrt{6}$ Tail: LEFT CENTER RIGHT Paste
Press down arrow until the cursor is at "Paste" and press [ENTER] twice and the following should appear on the screen	invNorm(0.25, 6, $\sqrt{6}$, RIGHT) 7.652155723

$a < 7.65$ (3sf)

(c)

Since $\mu = 6$ as well, we can agree that $P(W \leq 6) = 0.5$ which implies

$P(W < a) > 0.5 + 0.4999$

$P(W < a) > 0.9999$

Go to "invNorm" functionality of graphing calculator	InvNorm Area: μ: σ: Tail: LEFT CENTER RIGHT Paste:
Key in the fields as follows Area: 0.9999 μ: 6 σ: $\sqrt{6}$ Tail: Left	InvNorm Area: 0.9999 μ: 6 σ: $\sqrt{6}$ Tail: LEFT CENTER RIGHT Paste:
Press arrow down key until the cursor is on top of "Paste" and press [ENTER] key twice.	invNorm(0.9999,6,$\sqrt{6}$, LEFT) 15.10969287

$a > 15.1$(3sf)

(d)
$P(|W| < a) = 0.01$
Rewritten we get the following:
$P(-a < W < a) = 0.01$

Press [Y=] button and press [2ND] followed by [VARS] button. Select “normalcdf” option and fill in as follows: Lower: -X Upper: X μ:6 σ: $\sqrt{6}$	normalcdf lower: -X upper: X μ: 6 σ: $\sqrt{6}$ Paste
Scroll down until cursor is on top of “Paste” and press [Enter]	Y1= normalcdf(-X, X, 6, $\sqrt{6}$) Y2= Y3= Y4= Y5= Y6= Y7= Y8=
Press [MATH] and scroll down to look for numeric solver and press [ENTER]	Equation Solver E1: E2:
Press [ALPHA] followed by trace and a pop up should appear, select Y1 and press [ENTER]	Equation Solver E1: Y1 E2:

Scroll down to the box named "E2" and key in 0.01	Equation Solver E1: Y1 E2: 0.01 OK
Press [GRAPH] and the following should appear	Y1 = 0.01 X=0 Bound= {-1E99, 1E99} Solve
Set X = 0.5	Y1 = 0.01 X=0.5 Bound= {-1E99, 1E99} Solve
Press [GRAPH] and the following should appear on the calculator	Y1 = 0.01 X=0.5883012575898 Bound= {-1E99, 1E99} Solve

$a = 0.588$(3sf)

(e)
$P(|W| \geq a) = 0.975$ can be rewritten as the following
$P(|W| < a) = 1 - 0.975 = 0.025$ and hence,
$P(-a < W < a) = 0.025$

Press [Y=] button and press [2ND] followed by [VARS] button. Select "normalcdf" option and fill in as follows: Lower: -X Upper: X μ:6 σ: $\sqrt{6}$	normalcdf lower: -X upper: X μ: 6 σ: $\sqrt{6}$ Paste

Scroll down until cursor is on top of "Paste" and press [Enter]	Y1= normalcdf(-X, X, 6, $\sqrt{6}$) Y2= Y3= Y4= Y5= Y6= Y7= Y8=
Press [MATH] and scroll down to look for numeric solver and press [ENTER]	Equation Solver E1: E2:
Press [ALPHA] followed by trace and a pop up should appear, select Y1 and press [ENTER]	Equation Solver E1: Y1 E2:
Scroll down to the box named "E2" and key in 0.025	Equation Solver E1: Y1 E2: 0.025 OK
Press [Graph] and the following should appear	Y1 = 0.025 X=0 Bound= {-1E99, 1E99} Solve

Set X=0.5	Y1 = 0.025 X=0.5 Bound= {-1E99, 1E99} Solve
Press [Graph]	Y1 = 0.025 X=1.2611029611347 Bound= {-1E99, 1E99} Solve

$a = 1.26$