1 A ball is rolling in a straight line such that its distance away from the starting point, s cm, can be modelled using the equation

$$s = at + \frac{b}{\sqrt{t+4}} + c\,,$$

where t is the time taken in seconds, and a, b and c are real constants.

The ball is at the starting point when $t = 0$, and moved 10 cm in the first 5 seconds. It moved another 9 cm in the next 16 seconds. Find the ball's distance away from the starting point when $t = 50$. [4]

2 On a single diagram, sketch the graphs of $y = |2x - p|$ and $y = qx$ where the following conditions are satisfied, indicating the axial intercepts.

- p and q are constants, $p > 1$ and $q > 0$, and
- the graphs have only one point of intersection. [2]

(a) State the least value of q. [1]

(b) Solve the inequality $|2x - p| > qx$, leaving your answer in terms of p and q. [2]

3 Find

(a) $\int \tan^2(x-1)\,\mathrm{d}x$, [2]

(b) $\int \sin^{-1} 2x\ \mathrm{d}x$. [3]

4 **Do not use a calculator in answering this question.**

(a) It is given $w = -\sqrt{3} + \mathrm{i}$.

(i) Find arg w. [1]

(ii) Express $\mathrm{i}w^8$ in the form $r\mathrm{e}^{\mathrm{i}\theta}$ where $r > 0$ and $-\pi < \theta \le \pi$. [3]

(b) **(i)** It is given that $(1 + a\mathrm{i})^2 = -3 - 4\mathrm{i}$. Find the value of the real constant a. [2]

(ii) Hence solve the equation $2z^2 + (-3 + 2\mathrm{i})z + (1 - \mathrm{i}) = 0$. [3]

5 The points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively. C is the point on line OB such that AC is perpendicular to OB.

(a) By using a suitable scalar product, or otherwise, show that $\overrightarrow{OC} = \dfrac{(\mathbf{a}\cdot\mathbf{b})}{|\mathbf{b}|^2}\mathbf{b}$. [3]

(b) Give a geometrical interpretation of $\dfrac{|\mathbf{a}\cdot\mathbf{b}|}{|\mathbf{b}|}$. [1]

(c) It is given that $\mathbf{a} = \begin{pmatrix} -3 \\ 3 \\ -1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 1 \\ h \\ 0 \end{pmatrix}$. Given also that the length of the line segment AB is 5 units and angle AOB is an obtuse angle, find the exact value of h. [4]

6 The curve C is defined by the parametric equations

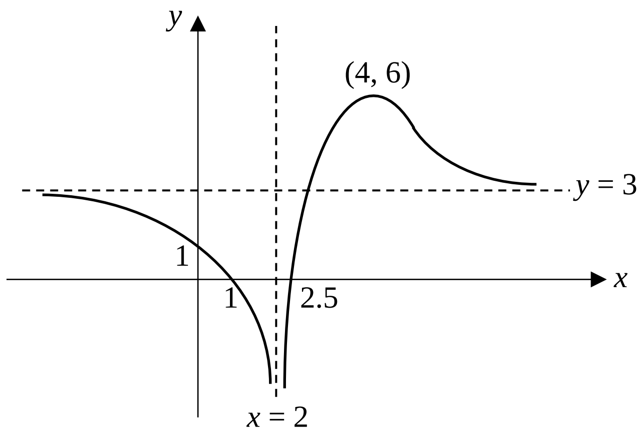
$$x = 1 - \cos t, \quad y = \sin 2t, \quad \text{where } 0 \le t \le \frac{\pi}{2}.$$

(a) Sketch C, giving the exact coordinates of the points where C meets the x-axis. [1]

(b) The normal to C at the point where $t = \dfrac{\pi}{2}$ cuts the y-axis at D. Show that the y-coordinate of D is $-\dfrac{1}{2}$. [4]

(c) Find the exact area of the region bounded by C, the normal in part **(b)** and the y-axis. [5]

7 **(a)** The diagram shows the curve with equation $y = \mathrm{f}(x)$. The curve crosses the x-axis at $x = 1$ and $x = 2.5$, crosses the y-axis at $y = 1$ and has a maximum point at $(4,\ 6)$. The equations of the asymptotes are $x = 2$ and $y = 3$. Sketch the graph of $y = \mathrm{f}\,'(x)$, giving the equations of asymptotes, coordinates of turning points and axial intercepts, where possible. [2]



(b) The curve C has equation $y = \dfrac{x^2 + kx - 1}{x+1}$, where k is a non-zero constant.

(i) Find the range of values of k for which C has no stationary points. [4]

(ii) Given that $y = x + 3$ is an asymptote of C, show that $k = 4$. [2]

(iii) State a sequence of transformations which transform the graph of $y = \dfrac{x}{4} - \dfrac{1}{x}$ onto the graph of $y = \dfrac{x^2 + 4x - 1}{x+1}$. [3]

8 The function f and g are defined by

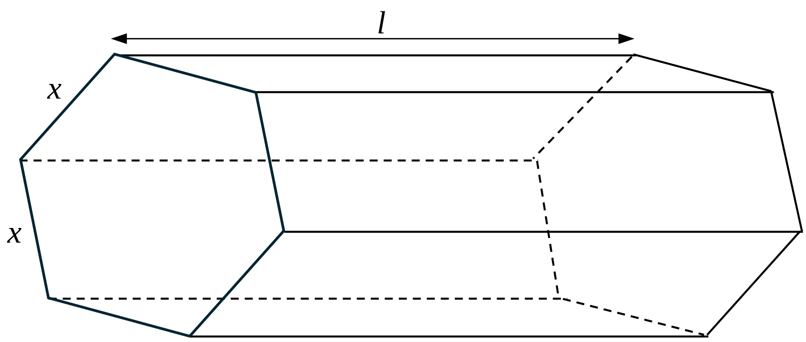
$$\mathrm{f} : x \mapsto -(x-4)^2 + 5, \quad x \in \mathbb{R},\ x \le 3,$$
$$\mathrm{g} : x \mapsto \mathrm{e}^{|x-3|}, \quad x \in \mathbb{R},\ x \le 10.$$

(a) Show that the composite function gf exists. [2]

(b) Find an expression for gf(x) and state the domain of gf. Hence find the value of x such that $x = (\mathrm{gf})^{-1}(1)$. [5]

(c) The function g has an inverse if its domain is restricted to $\alpha \le x \le 10$. State the smallest possible value of α and find $\mathrm{g}^{-1}(x)$, stating its domain. [4]

9



The figure above shows a metal rod with length l cm. The cross-section of the rod is a regular hexagon with sides of length x cm.

(a) A regular hexagon is made up of six identical triangles. Show that the area of the cross-section of the rod is $\dfrac{3\sqrt{3}x^2}{2}$ cm^2. [2]

(b) Suppose the rod has a fixed volume of C cm^3, show that the total surface area, S cm^2, of the rod may be expressed as $S = 3\sqrt{3}x^2 + \dfrac{4C}{\sqrt{3}x}$. [3]

(c) By using differentiation, find the value of x, in terms of C, which minimises S. [4]

Lucas heats up one of these metal rods. When heated, the metal rod expands uniformly such that it always retains its shape. At time t seconds, the length of each side of the hexagon is x cm, the length of the rod is l cm and the volume of the rod is V cm^3.

(d) Given that x and l are both increasing at a constant rate of 0.0025 cms^{-1}, find the rate of increase of V at the instant when $x = 2$ and $l = 5$. [2]

10 Anand writes a computer programme to simulate a population of organisms in a controlled environment. It is assumed that none of the organisms die or leave the environment within the duration of a simulation.

(a) In Simulation A, 200 organisms are introduced to the environment on Day 1. At the start of each subsequent day, 48 more organisms are introduced to the environment. Find the first day when the number of organisms in the environment exceeds 2025 at the end of that day. [2]

(b) In Simulation B, 15 organisms are introduced to the environment on Day 1. At the start of each subsequent day, each organism in the environment spawns two more organisms of the same type, i.e there are 45 organisms at the end of Day 2. Find the number of organisms in the environment at the end of Day 20. [2]

(c) In Simulation C, 5 organisms are introduced to the environment on Day 1. At the start of each subsequent day, the organisms in the environment will spawn in either one of the following ways.

I: Each organism will spawn three more organisms of the same type.

II: Each organism will spawn five more organisms of the same type.

On Day 2 to Day 9, the organisms undergo process **I** on m days and process **II** on the other days. Given that there are 1,105,920 organisms at the end of Day 9, find the value of m. [2]

Anand then adjusts the programme such that the simulation would allow for organisms to die at certain junctures.

(d) In Simulation D, 100 organisms are introduced to the environment at the start of Day 1. At the end of each day, 10% of the total population in the environment would die. At the start of Day 2 and each subsequent day, 20 organisms are introduced to the environment.

(i) Find an expression for the population size, P, in the environment at the start of Day n, after the organisms have been introduced. Leave your answer in the form $s - t\left(r^{n-1}\right)$, where s and t are positive integers and r is a real number. [4]

(ii) Describe what happens to the population size in the environment in the long term. [1]

(iii) Explain why the conclusion in **(ii)** does not depend on the population size in the environment on Day 1. [1]

11 A metal ball is released from the surface of the liquid in a tall cylinder. The ball falls vertically through the liquid and the distance, x cm, that the ball has fallen in time t seconds is measured. The speed of the ball at time t seconds is v cms^{-1}. The ball is released in a manner such that $x = 0$ and $v = 0$ when $t = 0$.

(a) The motion of the ball is modelled by the differential equation

$$\frac{d^2x}{dt^2} + \frac{1}{2}\frac{dx}{dt} - 10 = 0.$$

It is given that $v = \frac{dx}{dt}$.

(i) Show that the differential equation can be written as

$$\frac{dv}{dt} = 10 - \frac{1}{2}v.$$ [1]

(ii) Using the differential equation in **(a)(i)**, find v in terms of t. **Hence** find x in terms of t. [6]

(b) The metal ball is now released in another tall cylinder filled with a different liquid. However, for this liquid, the motion of the ball is modelled by the differential equation

$$\frac{dv}{dt} = 10 - k^2v^2, \text{ where } k \text{ is a positive constant.}$$

It is given that $x = 0$ and $v = 0$ when $t = 0$.

(i) Find v in terms of t and k. [5]

(ii) When the ball falls through this liquid, its speed will approach its "terminal speed" which is the speed it will attain after a long time. Find the ball's terminal speed in terms of k. You must show sufficient working to justify your answer. [2]