H2 Topic 14 – Current of Electricity

One of William Gilbert's (1544 - 1603) contribution to the field of science was his study on static electricity. He established the "amber effect" (now called the triboelectric effect) – that certain materials such as amber can become electrically charged by friction with other materials. As Amber is called elektron in Greek, and electrum in Latin, Gilbert decided to refer to the phenomenon by the adjective "electricus", giving birth to the word "electricity".

Content

- Electric current
- Potential difference
- Resistance and resistivity
- Electromotive force

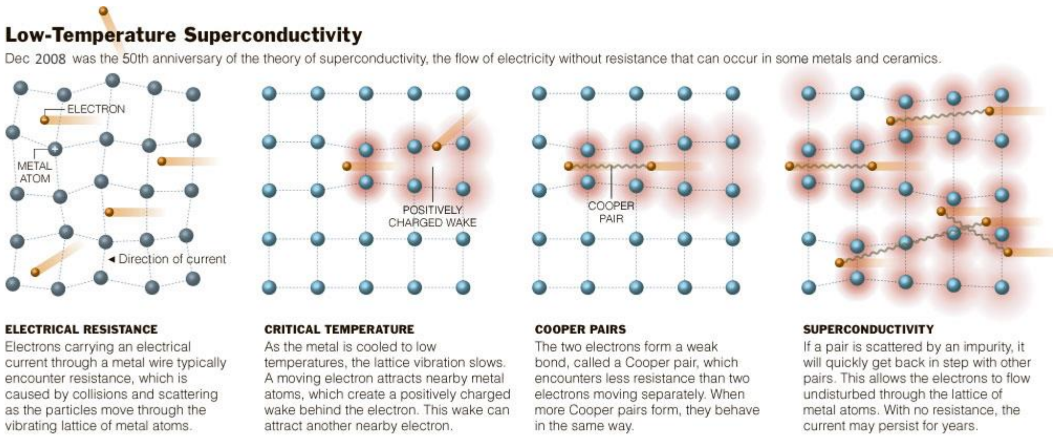
Learning Objectives:

Candidates should be able to:

(a) show an understanding that electric current is the rate of flow of charge
(b) derive and use the equation $I = nAvq$ for a current-carrying conductor, where n is the number density of charge carriers and v is the drift velocity
(c) recall and solve problems using the equation $Q = It$
(d) recall and solve problems using the equation $V = W/Q$
(e) recall and solve problems using the equations $P = VI$, $P = I^2R$ and $P = V^2/R$
(f) define the resistance of a circuit component as the ratio of the potential difference across the component to the current passing through it and solve problems using the equation $V = IR$
(g) sketch and explain the I–V characteristics of various electrical components such as an ohmic resistor, a semiconductor diode, a filament lamp and a negative temperature coefficient (NTC) thermistor
(h) sketch the resistance-temperature characteristic of an NTC thermistor
(i) recall and solve problems using the equation $R = \rho l/A$
(j) distinguish between electromotive force (e.m.f.) and potential difference (p.d.) using energy considerations
(k) show an understanding of the effects of the internal resistance of a source of e.m.f. on the terminal potential difference and output power.

14.0 Introduction

Previously from oscillations through waves, we used a simplified *microscopic* model of individual particles oscillating and linked them to the *macroscopic* behaviour of wave properties. The study of the large-scale behaviour saves us the effort of having to model all of the individual behaviour of particles, and allows us to investigate interactions and changes, such as multiple waves meeting and overlapping with each other.



Electrical resistance is one example of a macroscopic behaviour manifesting from the microscope model of individual electrons colliding with the vibrating lattice of metallic ions.

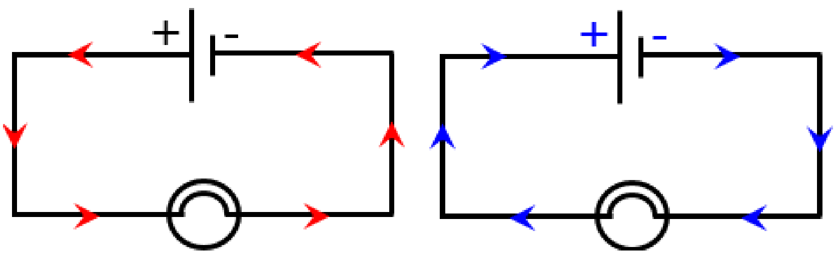
Here we study the macroscopic behaviour of electrons flowing through components and wires.

14.1 Electric Current

Electric current is the rate of flow of charge

When an electrical conductor is conducting electricity, an electric current is said to flow through it. This current is made up of a *net flow* of charged carriers (or charged particles) such as electrons (negatively charged), protons (positively charged) or ions (either polarities).

Unless specified, *conventional flow* is assumed when working with current flow.



conventional current flow   electron flow

Current is a scalar quantity; it does not obey vector addition laws.

Electric current I is one of the 7 base S.I. quantities and has the S.I. unit of Ampere (A). It is *chosen* as a base quantity because it is easier to define the unit of electric current (from the *macroscopic* model)), than to derive the coulomb from the flow of charged particles (*microscopic mode*).

Since current is the rate of flow of charge $I = \frac{dQ}{dt}$. For constant current: $I = \frac{Q}{t}$ and $Q = It$.

14.1.1 Drift Velocity

For a current-carrying conductor,

$$I = nAvq$$

I : current (A)
n : number density of charge carriers (m^{-3})
A : cross sectional area (m^{-2})
V : drift velocity ($m\ s^{-1}$)
q: charge (C)

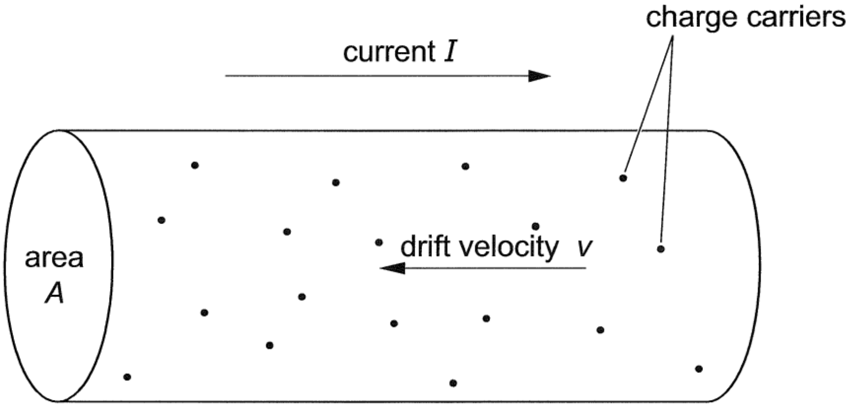
Drift velocity v is the *net* velocity of charge carriers in a certain direction under an externally-applied electric field.

We distinguish this from *thermal velocity*, which is the random, haphazard motion in which there is *no net direction* (similar to Brownian motion). Thermal velocity does not contribute to current because the statistical average is zero in terms of net flow.

Electrons in metals exhibit such random, haphazard motion because they are delocalised from the metal atoms (which form a giant lattice of metal cations), and make up a sea of free, delocalised electrons. The electrons collide with the lattice or amongst themselves, giving rise to *resistance*.

Derivation

For a current I in a wire that causes the charge carriers to have a drift velocity v, derive an equation relating current to the number density of charge carriers in the wire n, the cross sectional area of the wire A and the charge on each charge carrier q.



Solution

For a wire segment of length L, total charge in this volume is

$$\begin{aligned} Q &= (\text{number of charge carriers in volume})q \\ &= n(\text{volume})q \\ &= nALq \end{aligned}$$

Time taken t for charge carriers to have net displacement of L

$$L = vt$$

Current is the rate of flow of charge

$$I = \frac{Q}{t} = \frac{nALq}{L / v} = nAvq$$

drift velocity	thermal agitation
zero when there is no potential difference across conductor	zero in terms of statistical average individually zero only at temperature of 0 K
associated with net flow of charge carriers	results in zero net flow of charge carriers does not contribute to current
speed ~ 10^{-4} m s^{-1}	speed ~ 10^{5} m s^{-1}

Example 1

A copper wire of diameter of 0.500 mm carries a current of 1.0 A. Estimate the number density of free, delocalised electrons in the wire.

Solution

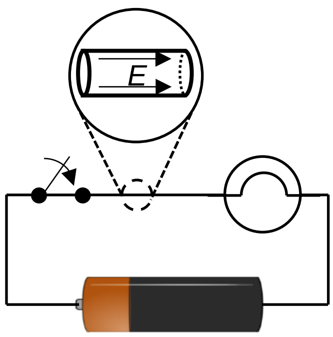
Assuming all charge carriers are free delocalised electrons:

$$I = nAvq$$

$$n = \frac{I}{Avq} = \frac{I}{(\pi r^2)vq} = \frac{I}{\left(\pi\left(\frac{d}{2}\right)^2\right)vq}$$

$$\approx \frac{1}{\left(\pi\left(\frac{0.5\times10^{-3}}{2}\right)^2\right)(10^{-4})(1.6\times10^{-19})}$$

$$= 3.2\times10^{29}\ \text{m}^{-3}$$



Example 1 illustrates 2 important concepts:

- An electron that contributes to current moves at about 10^{-4} m s^{-1}; it may take forever for an electron to travel from a battery to a light bulb, *but it doesn't need to*.
- An electron carries a small finite amount of charge $q = 1.6\times10^{-19}$ C but there are *many* moving (albeit slowly) at the same time.

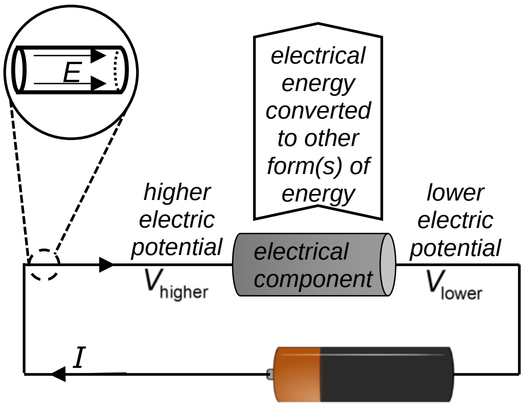
Why does the lightbulb light up almost immediately when the switch is closed?

When a circuit is closed,
the electric field inside the circuit is
set up almost instantaneously so that
all electrons in wire and bulb filament
start to move at the same time.

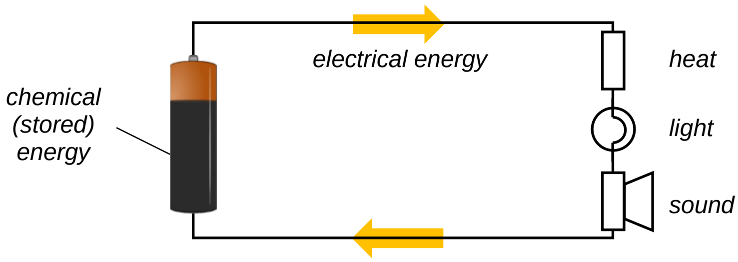
14.2 Electric Potential

When the terminals of a battery are connected across an electrical component (such as a light bulb or resistor), current flows through the component from the point of higher electric potential to the point of lower electric potential.

Recall that gravitational potential refers to the work done *per unit mass* in bringing a small test mass from infinity to that point. Instead of a unit mass (as per a *gravitational* field), for circuits we reference a *unit charge*.



Electromotive force (e.m.f.) is	**Potential difference (p.d.)** is
the energy transformed from chemical to electrical per unit charge when charge is driven round a complete circuit.	the energy transformed from electrical to other forms per unit charge when charge passes through an electrical component.
**e.m.f. is not a force. It is scalar as it does not obey vector addition, and exists with or without current flow*	$V = \frac{W}{Q}$ V: potential difference (V) W : work done (J) Q : charge (C)



We can regard "electrical energy" as the medium of transfer from stored energy in a source of e.m.f. to energy that is dissipated in devices.

Example 2

An electric runs off a 230 V supply for 15 minutes, during which it produced 720 kJ of heat.

(a) Find the amount of charge that flow through the heating element.

(b) State an assumption that you have made in your calculations for part (a).

Solution

Assuming that all electrical energy is converted to heat at 100% efficiency,

$$V = \frac{W}{Q}$$

$$Q = \frac{W}{V} = \frac{720 \times 10^3}{230} = 3130 \text{ C}$$

14.3 Electric Power

From H205 Work, Energy and Power, we learnt that power can be regarded in several ways:

- work done per unit time / rate of work done
- rate of transfer of energy
- **rate of conversion of energy**

The definitions for both e.m.f. and p.d. emphasize the *conversion* of energy from one form to another and so it is clear that the third interpretation is of relevance to this topic:

power supplied by source of e.m.f.	power output of electrical component
$P = I_{\text{total}}\varepsilon = I_{\text{total}}^2 R_{\text{total}} = \dfrac{\varepsilon^2}{R_{\text{total}}}$	$P = I_{\text{device}}V = I^2 R_{\text{device}} = \dfrac{V^2}{R_{\text{device}}}$

Example 3

An electric kettle operates at 230 V and draws a current of 9.0 A from the mains. It brings a full kettle of water from room temperature to boiling point in 5 min, a process is which known to require 5.5×10^5 J of energy. Find the

(a) useful power output of the kettle,
(b) power supplied to the kettle from the power source, and
(c) the efficiency of the kettle.

Solution

(a)
$$P_{\text{useful}} = \frac{W}{t} = \frac{\Delta E}{t} = \frac{5.5\times10^5}{5\times60} = 1830 \text{ W}$$

(b)
$$P_{\text{source}} = IV = I\varepsilon = (9)(230) = 2070 \text{ W}$$

(c)
$$\text{efficiency} = \frac{1830}{2070}\times100\% = 88.6\%$$

OHM NEVER FORGOT HIS DYING UNCLE'S ADVICE.

14.3 Resistance

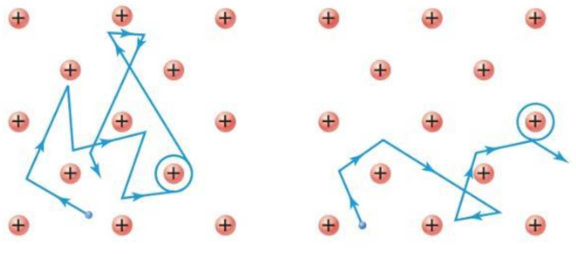
Resistance is the

ratio of
potential difference across a component
to
the current passing through it

$$R = \frac{V}{I}$$

R : resistance (Ω)
V : potential difference (V)
I : current (A)

Resistance is the property of a conductor that limits current flow. The unit for resistance is the ohm (Ω); 1 Ω is the resistance of a conductor when the potential difference across it is 1 V and the current flowing through it is 1 A.

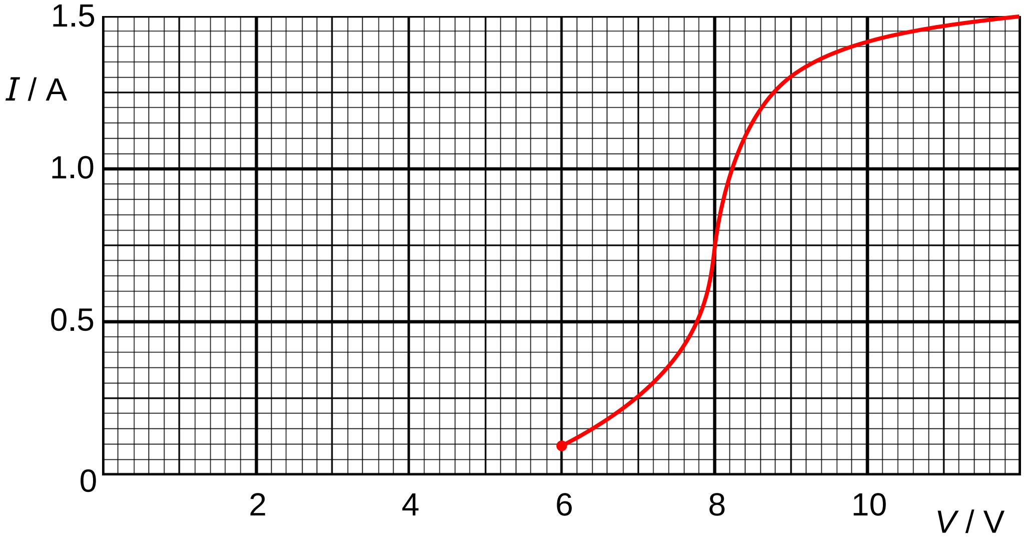


thermal effect; no net flow

net flow (with p.d.) affected by collisions with lattice

Example 4

The variation of current flow through an electric component with the potential difference applied across it is shown below. Find



(a) the resistance when the p.d. is 7.0 V,
(b) the resistance when the p.d. is 9.0 V, and
(c) the p.d. at maximum resistance.

Solution

(a)
$$R = \frac{V}{I} = \frac{7.0}{0.3} = 23.3\ \Omega$$

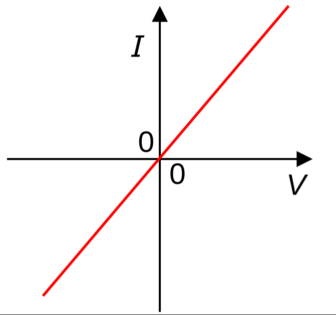
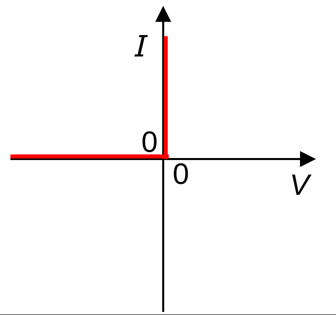
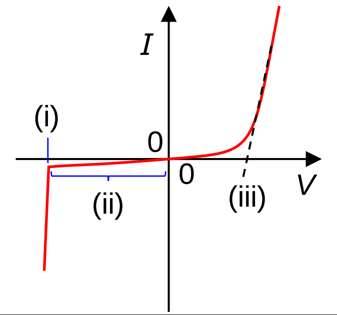
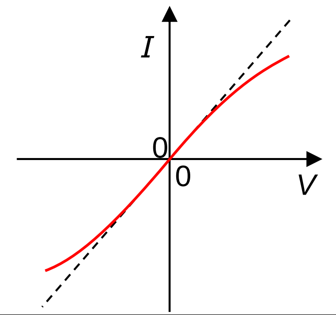
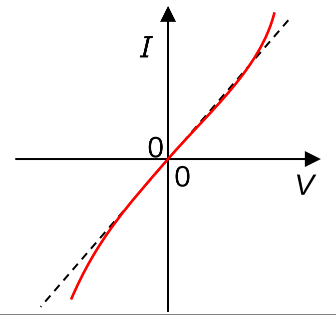
(b)
$$R = \frac{V}{I} = \frac{9.0}{1.3} = 6.92\ \Omega$$

(c) $V = 6.0$ V
$$R_{max} = \frac{6}{0.1} = 60.0\ \Omega$$

Note: the gradient of an I-V or V-I has no useful meaning – only the value of V and I at each point on the curve can allow us to find the resistance.

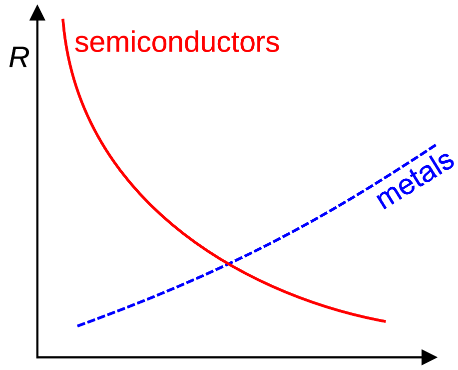
14.3.1 *I*-*V* Curves

E.g. 4 shows an *I*-*V* curve, which seems counter-intuitive as resistance is defined as *V*/*I*. When industries characterise components, engineers vary the p.d. applied across (independent variable) and measure the current passing through it (dependent variable), hence the convention *I* against *V*.

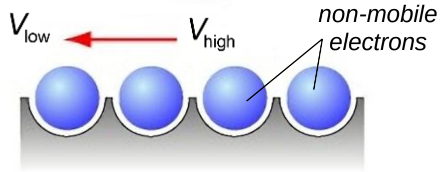
component	**ohmic resistor**	**semiconductor diode**		**filament lamp**	**NTC thermistor**
description	a resistor in which the current flow is directly proportional to potential difference across it, if temperature and other physical conditions are kept constant	a component which allows current to flow in one direction from higher potential to lower potential **more treatment in H218 Alternating Currents*		a light bulb containing a thin coil of wire, which heats up when current passes through it and glows as a result	a **N**egative **T**emperature **C**oefficient thermistor is a resistor that has decreasing resistance with increasing temperature
		ideal diode	non-ideal diode		
I-*V* curve					
traits	• straight line passing through origin • constant resistance • (components to the right → are **not** ohmic)	• pure conductor (zero resistance) once there is p.d. in forward bias • infinite resistance when there is p.d. in reverse bias	i. breakdown voltage, beyond which diode cannot prevent flow of current in reverse bias ii. leakage current is insignificant in reverse bias so very high resistance iii. threshold voltage, beyond which current increases exponentially (low resistance)	• ‘s’-shape curve passing through origin • current increases at decreasing rate with p.d. • resistance increases with higher p.d.	• reverse-‘s’-shape curve passing through origin • current increases at increasing rate with p.d. • resistance decreases with higher p.d.

14.3.2 Temperature Affects Resistance

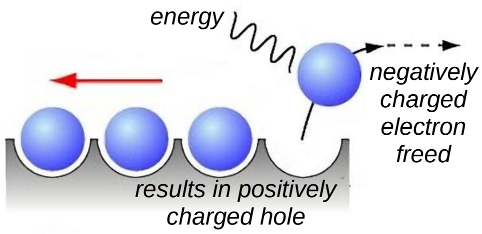
The resistance of a component depends on two factors:
(a) the number density of charge carriers,
(b) the extent of vibration of the atomic lattice.

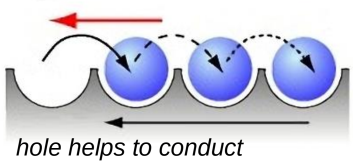


Resistors are typically made of metals – generally the resistance of metals increase with increasing temperature.
(a) valence electrons of metal atoms make up the sea of delocalised electrons responsible for conduction – the number density is therefore fixed.
(b) higher temperatures result in greater vibration of the giant metallic lattice, more frequent collisions with delocalised electrons and disrupts charge flow.



Thermistors are typically semiconductors - the resistance generally decrease with temperature.
(a) With higher temperatures, more electrons are set free to carry charge. The freed electrons also creates holes that can act as mobile *positive* charge carriers.
(b) The increased number density of charge carriers is greater in effect than the increased vibration of lattice causing more frequent collisions with charge carriers.





14.3.3 Geometry Affects Resistance

Resistance is related to geometry via

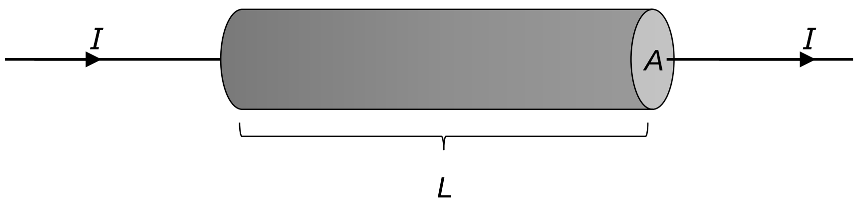
$$R = \frac{\rho L}{A}$$

R : resistance (Ω)
ρ : resistivity (Ω m)
L : length (m)
A : cross sectional area (m^{-2})

With all other physical conditions being constant (including temperature), the resistance of a component will also be affected by its geometry.

We characterise the resistance of a component using a property of the material called resistivity ρ (units is Ω m). Do not confuse it with density since they are represented by the same symbol.

L is the length of the conductor along the direction of current flow, and A is the cross-sectional area "normal" to the current flow (quotation marks because current is a scalar quantity and does not have a direction to define a normal with).



Example 5

Two wires X and Y are of the same material but wire X is twice the length and half the diameter of wire Y. Find the ratio of $\frac{R_X}{R_Y}$.

Solution

$$R = \frac{\rho L}{A} = \frac{\rho L}{\pi r^2} = \frac{\rho L}{\pi\left(\frac{d}{2}\right)^2} = \frac{4\rho L}{\pi d^2}$$

$$\frac{R_X}{R_Y} = \left(\frac{L_X}{L_Y}\right)\left(\frac{d_Y}{d_X}\right)^2 = (2)(2)^2 = 8$$

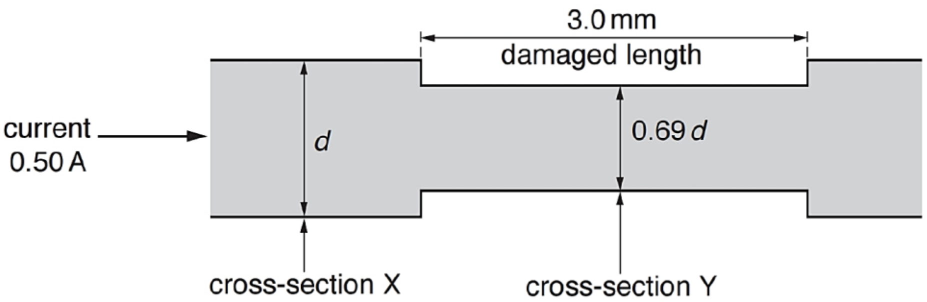
Example 6

A metal wire in a circuit is damaged. The resistivity of the metal is unchanged at but the cross-sectional area is reduced over a length of 3.0 mm as shown. The undamaged part of the wire has a resistance per unit length of $1.7\times10^{-2}\ \Omega\ \text{m}^{-1}$.

(a) Find $\dfrac{\text{average drift speed of free electrons at cross-section Y}}{\text{average drift speed of free electrons at cross-section X}}$.

(b) Find the resistance per unit length of the damaged length of wire.

(c) Find the power dissipated in the damaged length of wire



Solution

(a)
$$I = nAvq = \text{same in X \& Y}$$
$$Av = \text{constant}$$
$$\frac{v_Y}{v_X} = \frac{A_X}{A_Y} = \frac{\pi r_X^2}{\pi r_Y^2} = \left(\frac{d_X}{d_Y}\right)^2 = \left(\frac{1}{0.69}\right)^2 = 2.10$$

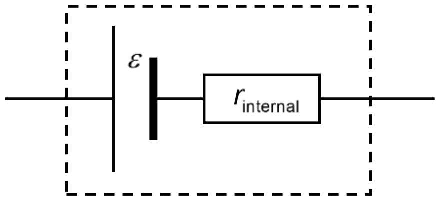
(b)
$$R = \frac{\rho L}{A}$$
$$\left(\frac{R}{L}\right) = \frac{4\rho}{\pi d^2}$$
$$\frac{\left(\frac{R}{L}\right)_{\text{damaged}}}{\left(\frac{R}{L}\right)_{\text{normal}}} = \left(\frac{d_{\text{normal}}}{d_{\text{damaged}}}\right)^2$$
$$\left(\frac{R}{L}\right)_{\text{damaged}} = \left(\frac{d_{\text{normal}}}{d_{\text{damaged}}}\right)^2\left(\frac{R}{L}\right)_{\text{normal}} = \left(\frac{1}{0.69}\right)^2(1.7\times10^{-2}) = 0.0357\ \Omega\ \text{m}^{-1}$$

(c)
$$P = I^2R = I^2\left[\left(\frac{R}{L}\right)_{\text{damaged}} L_{\text{damaged}}\right] = (0.5)^2(0.035707)(3\times10^{-3}) = 2.68\times10^{-5}\ \text{W}$$

14.3.4 Internal Resistance

Most real-world e.m.f. sources are not ideal in that they inherently has some resistance. We represent a *practical* cell on circuits as a combination of an ideal source of e.m.f. ε housed together with a resistor representing the *internal resistance*.

Terminal p.d. V_T is the p.d. measured across the practical cell:



Terminal p.d.:

$$V_T = \varepsilon - Ir$$

ε : e.m.f (V)
I : current (A)
r : internal resistance (Ω)

Example 7

Explain why the terminal p.d. of a cell with internal resistance may be less than the e.m.f..

Solution

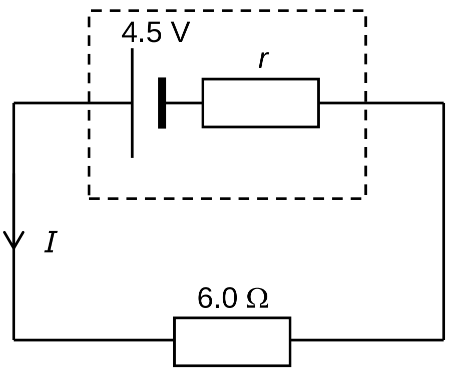
e.m.f. represents total amount of energy per unit charge around complete circuit.
when cell supplies a current, some of the available energy is transformed into other forms of energy in the internal resistance,
causing a drop in potential difference across the internal resistance

Note: some texts describe the process as power being wasted in the cell

Example 8

The current in the circuit shown is 0.65 A. Find

(i) the terminal p.d. of the battery,
(ii) the internal resistance r of the battery,
(iii) the power dissipated in the (external) resistor
(iv) the efficiency of the battery.



Solution

(i) $V = IR = (0.65)(6.0) = 3.9$ V

(ii) $V_T = \varepsilon - Ir$

$r = \dfrac{\varepsilon - V_T}{I} = \dfrac{4.5 - 3.9}{0.65} = 0.923\ \Omega$

(iii) $P = I^2R = (0.65^2)(6) = 2.54$ W

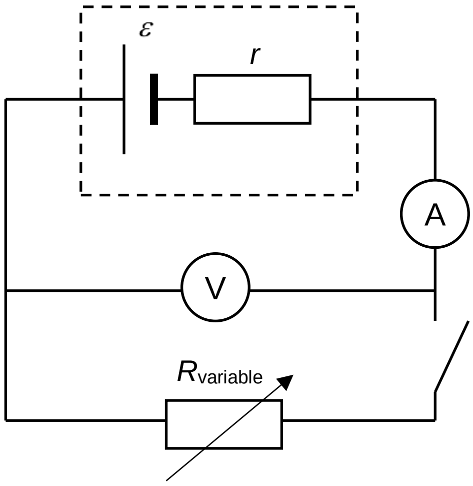
(iv) efficiency $= \dfrac{P_{output}}{P_{source}} = \dfrac{I^2R}{I\varepsilon} = \dfrac{(0.65^2)(6)}{(0.65)(4.5)} = 0.867$

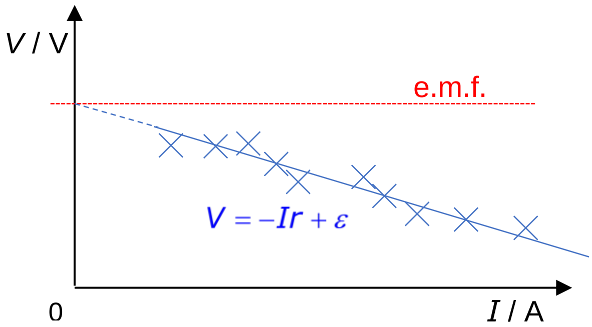
Note: if a battery is shorted, *all* available energy is transformed into heat within internal resistance, the battery heats up rapidly and risks exploding.

14.3.4 Experimental Determination of Internal Resistance

There is no way of physically separating the internal resistance within a practical cell. We can use the following set-up to find the value of the internal resistance of a practical cell.

1) Measure e.m.f. ε using voltmeter when switch is open.
2) Measure the terminal potential difference V using a voltmeter when switch is closed.
3) Measure the current flowing through the circuit I using an ammeter when switch is closed.
4) Repeat Steps 2 – 3 for 10 times, each time varying the resistance of the variable resistor.
5) $V = \varepsilon - Ir$ so a plot of V against I should yield a straight line graph with gradient the value of the internal resistance r and y-intercept e.m.f. ε which can be verified against value obtained from Step 1.



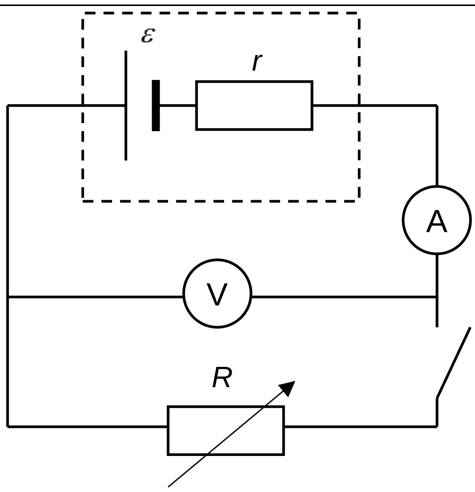


If the e.mf. source is ideal, r = 0, the terminal p.d. will always be equals to e.m.f.

For practical e.m.f. sources, as long as a current flows, the terminal p.d. all always be lower than e.m.f.

Example 9

In the set up show, the voltmeter reads 4.0 V when the variable resistor is at 6.0 Ω, and 4.4 V when the variable resistor is 10 Ω. Find the value of the internal resistance and the e.m.f. of the battery.



Solution

$V_T = IR$

$I_1 = \frac{4}{6}; \quad I_2 = \frac{4.4}{10}$

$V_T = \varepsilon - Ir$

$4 = \varepsilon - \left(\frac{4}{6}\right)r \quad$ ___(1)

$4.4 = \varepsilon - \left(\frac{4.4}{10}\right)r \quad$ ___(2)

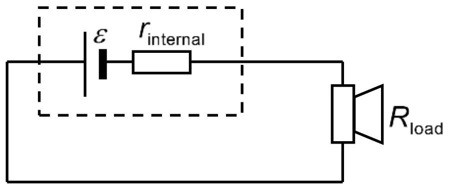
solve simultaneously

$\varepsilon = 5.18$ V

$r = 1.76\ \Omega$

14.4 Efficiency vs Power Transfer

When we design simple electronics and electrical appliances, efficiency and power output are opposing trade-offs that needs to be balanced according to needs.



For the following section we will consider a simplified circuit shown here. The speaker is a component which converts available p.d. into sound waves. The volume ("loudness") of the speaker is measured by the power output across its resistance $P_{\text{load}} = IV_{\text{load}} = I^2 R_{\text{load}} = \dfrac{V_{\text{load}}^2}{R_{\text{load}}}$.

14.4.1 Maximising Efficiency

$$\text{efficiency} = \frac{P_{\text{load}}}{P_{\text{source}}} = \frac{I^2 R_{\text{load}}}{I^2 R_{\text{total}}} = \frac{R_{\text{load}}}{R_{\text{load}} + r}$$

To increase efficiency,
minimise internal resistance of source of e.m.f. or
maximise the resistance of component

For efficiency, we study the ratio of the total power supplied and the useful power output.

Since current is the same throughout the circuit, efficiency increases with increasing load resistance because more of the proportion of power is shifted over to the component.

The maximum limit of 100% efficiency then occurs when the source has no internal resistance.

When working with practical sources of e.m.f. where internal resistance is present, the condition to increase efficiency necessitates increasing the load resistance.

However, because overall circuit resistance would have increased, the actual amount of power (not ratio) being output at the component will be decreased as a result.

14.4.2 Maximum Power Transfer Theorem

Maximum power transfer happens when

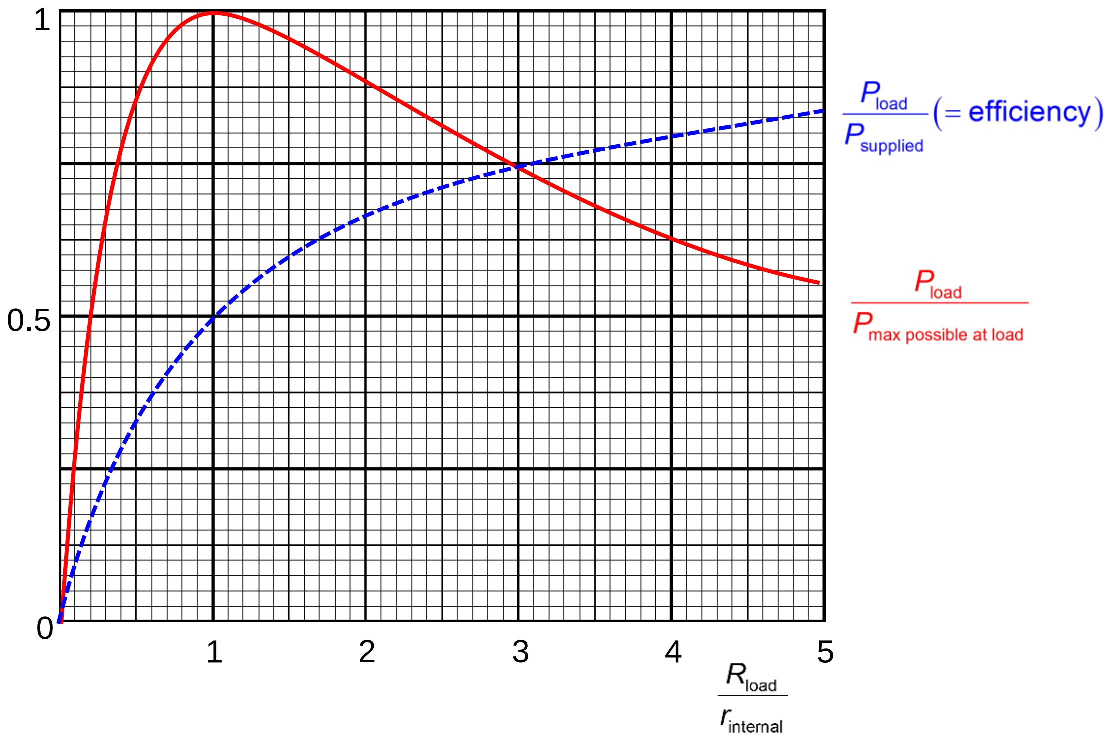
resistance of external load
is the same resistance
as the internal resistance of source of e.m.f.

The power transfer refers to the *actual* amount of power that the component outputs.

At the condition for maximum power transfer, the efficiency is 50%:

$$\text{efficiency} = \frac{R_{\text{load}}}{R_{\text{load}} + r} = \frac{R_{\text{load}}}{R_{\text{load}} + R_{\text{load}}} = 50\%$$

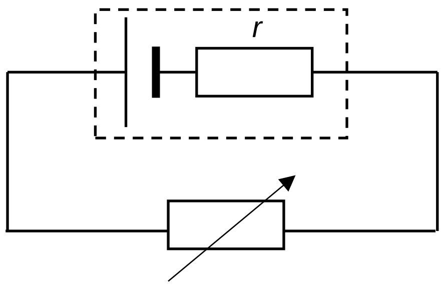
Using the example of the speaker, we trade-off by either choosing (i) a very efficient circuit where most of the power is being used by the speaker but the speaker output is very soft or (ii) the speaker operates at the loudest possible given the fixed internal resistance of the e.m.f. source but operates at 50% efficiency.



Example 10

A practical cell has a constant e.m.f. and is connected to a variable resistor where the resistance is increased from zero to 4*r*.

Which option best describes the effect s of this change on terminal p.d. and power dissipated in the variable resistor?



	terminal p.d.	power dissipation
A	constant	increases then decreases
B	constant	increases
C	increases	increases then decreases
D	increases	increases

Solution:

C

Max power is dissipated in variable resistor when external resistance matches internal resistance so power dissipation increases then decreases; eliminate options B and D

$V_{terminal} = \varepsilon - Ir_{internal}$ so terminal p.d. cannot remain constant with a changing current in circuit.

14.5 Ending Notes

This topic emphasizes the fundamental understanding of components and their individual behaviour when subject to potential differences.

In the next topic H215 DC circuits we will consider how do different components interact with each and delve deeper into analysis of circuits.