Name:		Index Number:		Class:	

DUNMAN HIGH SCHOOL
Preliminary Examination
Year 6

MATHEMATICS (Higher 2) — 9740/02

22 September 2014

Paper 2 — **3 hours**

Additional Materials: Answer Paper
List of Formulae (MF15)

READ THESE INSTRUCTIONS FIRST

Write your Name, Index Number and Class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.
Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

At the end of the examination, **attach the question paper to the front of your answer script.**

The total number of marks for this paper is **100**.

For teachers' use:

Qn	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Total
Score												
Max Score	**9**	**10**	**10**	**11**	**4**	**6**	**8**	**9**	**10**	**11**	**12**	**100**

Section A: Pure Mathematics [40 marks]

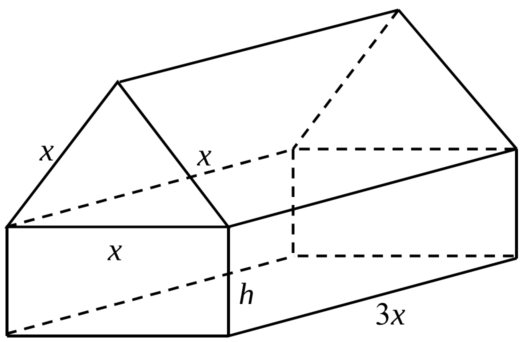
1 **(i)** Prove by method of mathematical induction that

$$\frac{3}{(1)(2)}+\frac{13}{(2)(3)}+\frac{37}{(3)(4)}+...+\frac{n^2(n+1)+1}{n(n+1)}=\frac{n(n+1)^2-2}{2(n+1)}+1$$

for all positive integral values of n. [4]

(ii) Hence, by considering $\frac{r^2(r+1)+1}{r(r+1)}=r+\frac{A}{r}+\frac{B}{r+1}$, where A and B are constants to be determined, find an expression for $\sum_{r=1}^{n} r$ in terms of n. [5]

2



An event company builds a tent (as shown in the diagram) which has a uniform cross-sectional area consisting of an equilateral triangle of sides x metres and a rectangle of width x metres and height h metres. The length of the tent is $3x$ metres.

(i) It is given that the tent has a fixed volume of k cubic metres and is fully covered (except the base) with canvas assumed to be of negligible thickness. Show that the area of the canvas used, A, in square metres, is given by $A = 6x^2 - \frac{3\sqrt{3}}{2}x^2 + \frac{8k}{3x}$.

Use differentiation to find, in terms of k, the value of x which gives a stationary value of A. Determine if A is minimum or maximum for this value of x. [7]

(ii) It is given instead that the tent has a volume of 360 cubic metres and the area of canvas used is 300 square metres. Find the value of x and the value of h. [3]

3 **(i)** Given $w^4 - 2w^2 + 2 = 0$, find the roots of the equation, giving your answers in the exact form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \le \pi$. [4]

(ii) Show the roots on an Argand diagram. [2]

(iii) The roots in **(i)** represented by w_1, w_2, w_3 and w_4 are such that $-\pi < \arg(w_1) < \arg(w_2) < \arg(w_3) < \arg(w_4) < \pi$. Explain why the locus of all points z such that $\arg(z - w_1) = \frac{\pi}{8}$ passes through the point that represents w_3. Draw this locus on your Argand diagram and find its exact cartesian equation. [4]

4 The line l has equation $\frac{x-2}{-1} = \frac{z-a}{1}$, $y = -1$, where a is a real constant and the plane p_1 has equation $3x + y + 2z = 5$. The point A has the position vector $2\mathbf{i} + 2\mathbf{j}$ with respect to the origin O.

(i) Find the acute angle between l and p_1. [2]

(ii) Find the perpendicular distance from the point A to p_1. [3]

(iii) Given that l is the line of intersection of the planes p_2 and p_3 with equations $x - 4y + z = 6$ and $x - y + bz = c$, where b and c are real constants, find b and c. [3]

(iv) The point B varies such that the midpoint of AB is always in p_1. Find a cartesian equation for the locus of B. [3]

Section B: Statistics [60 marks]

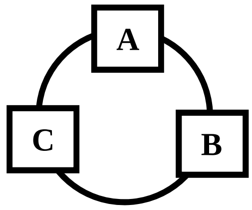
5 The organiser of an overseas education fair would like to survey 1% of the people attending the one-day event on their opinions about the fair.

(a) The organiser has identified that the people at the event comprise students and working adults and has decided to target these two groups for his survey. Give one advantage of using a stratified sample in comparison to a sample obtained by quota sampling. Suggest why it would be difficult for the organiser to obtain a stratified sample. [2]

(b) Explain how the event organiser can conduct his survey using a systematic sample. [2]

6 For a particular activity, three boxes A, B and C are placed in a circular arrangement.

Box A contains 5 red and 5 white balls.
Box B contains 4 red and 6 white balls.
Box C contains 3 red and 7 white balls.



For each round, a participant of this activity will draw a ball randomly from a box at any one time without replacement. The participant begins his first draw from box A, and the subsequent draws are based on the following rules:

- If a red ball is drawn, the next draw is from the box in the clockwise direction.
- If a white ball is drawn, the next draw is from the box in the anti-clockwise direction.

(i) Show that the probability that a participant has not drawn from box C from the first to the sixth draw is $\frac{1}{36}$. [2]

(ii) Given that a participant drew from box B on the fourth draw, find the probability that he has not drawn from box C from the first to his sixth draw. [4]

7 A group of eight girls and four boys take part in a Mathematics competition. Among this group, there are six Chinese, four Malay, and two Indian students.

For the individual segment, all students are arranged randomly in a row. Find the probability that

(i) the boys are all separated from one another. [2]

(ii) a particular boy is between two girls. [2]

For the team segment, all students are arranged in a circle to facilitate discussion.

(iii) Find the probability that the boys are all next to one another. [1]

The group won the first prize and only seven students are chosen to represent the group to receive the prize.

(iv) Find the probability that at least one student from each race are chosen to represent the group. [3]

8 **(i)** Sketch a scatter diagram that might be expected for the case when x and y are related approximately by $y = a + b\ln x$, where a is positive and b is negative. Your diagram should include 6 points, approximately equally spaced with respect to x, and with all x- and y-values positive. [1]

About a year ago, Amy decided to go on a healthy eating lifestyle and an exercise regime in order to lose weight. She monitored her weight, y kg, x months after she started and the data is provided below.

No. of months, x	1	3	5	7	9	11
Weight, y	61.4	60.1	59.5	58.9	58.5	58.2

(ii) Draw the scatter diagram for these values, labelling the axes. [1]

(iii) Explain which model, $y = a + b\ln x$ or $y = c + dx$, is better for modelling these values. [1]

For the better model that you have identified in part **(iii)**,

(iv) give a contextual interpretation of the value of a or c and calculate the product moment correlation coefficient. [2]

(v) use a suitable regression line to estimate the month in which she will reach a weight of 55 kg. You may assume that the model is suitable for short term predictions. [3]

(vi) explain, in context, why the model is not suitable for long term predictions. [1]

9 A baby drinks either BabyGrow or InfanGrow milk powder. The sleeping hours of babies on a particular night follow independent normal distributions with means and standard deviations as given in the table, according to the brand of milk powder they drink.

	Mean	Standard deviation
BabyGrow	8.0	σ
InfanGrow	6.5	0.795

(i) Given that 85% of babies who drink BabyGrow slept less than 9 hours, find the value of σ. [2]

(ii) Twelve babies who drink BabyGrow are chosen at random. Find the probability that at least ten babies slept more than 7 hours. [3]

(iii) Find the probability that the total sleeping hours of three randomly chosen babies who drink BabyGrow exceeds four times the sleeping hours of a randomly chosen baby who drinks InfanGrow, by more than one hour. [3]

(iv) A study shows that 20% of babies who slept less than 5.5 hours developed obesity when they grew up. A random sample of 300 babies who drink InfanGrow is selected. Find the expected number of these babies who will develop obesity when they grow up. [2]

10 To improve their learning, students book consultation appointments with their subject tutors. For the Math subject, it is found that there is an average of 1.8 appointments in a day.

Assume that the number of appointments follows a Poisson distribution.

(i) Find the probability that there are at most 6 Math appointments in a 5-day period. What is the most likely number of appointments in the same period? [2]

(ii) The probability that there are exactly 3 Math appointments in a day is 0.161. Out of a period of 30 days, find the least value of n such that the probability that there are at most n days with exactly 3 Math appointments in a day is at least 0.95. [3]

State a reason, in context, why the Poisson distribution may not be a good model for the number of appointments in a year. [2]

Students also book consultation appointments in the Science subject, where the number of appointments in a day is an independent random variable with the distribution $\text{Po}(2.2)$.

(iii) By using a suitable approximation, find the probability that in a 30-day period, the number of Science appointments exceeds that of Math by more than 12. [4]

11 In Factory A, it is claimed that the mean mass of each bag of beans produced is 22 kg. To investigate this claim, the mass, x kg, of a random sample of 50 bags of beans are obtained and summarised by:

$$\sum(x-18)=165.3 \text{ and } \sum(x-18)^2=876.5.$$

(i) What do you understand by the term "unbiased estimate"? [1]

(ii) Find the unbiased estimates of the population mean and variance. [2]

(iii) Test at the 5% level of significance whether this claim is valid. [4]

(iv) State the meaning of the p-value obtained in part **(iii)**. [1]

In Factory B, masses of bags of beans produced follow a normal distribution. It is claimed that the mean mass of each bag of beans produced is μ_0 kg. A random sample of 15 bags of beans has a mean mass of 8 kg and an unbiased estimate of the population standard deviation is 0.2 kg. Given that there is sufficient evidence at the 5% level of significance to conclude that this claim has been overstated, find the set of possible values of μ_0. [4]

END OF PAPER