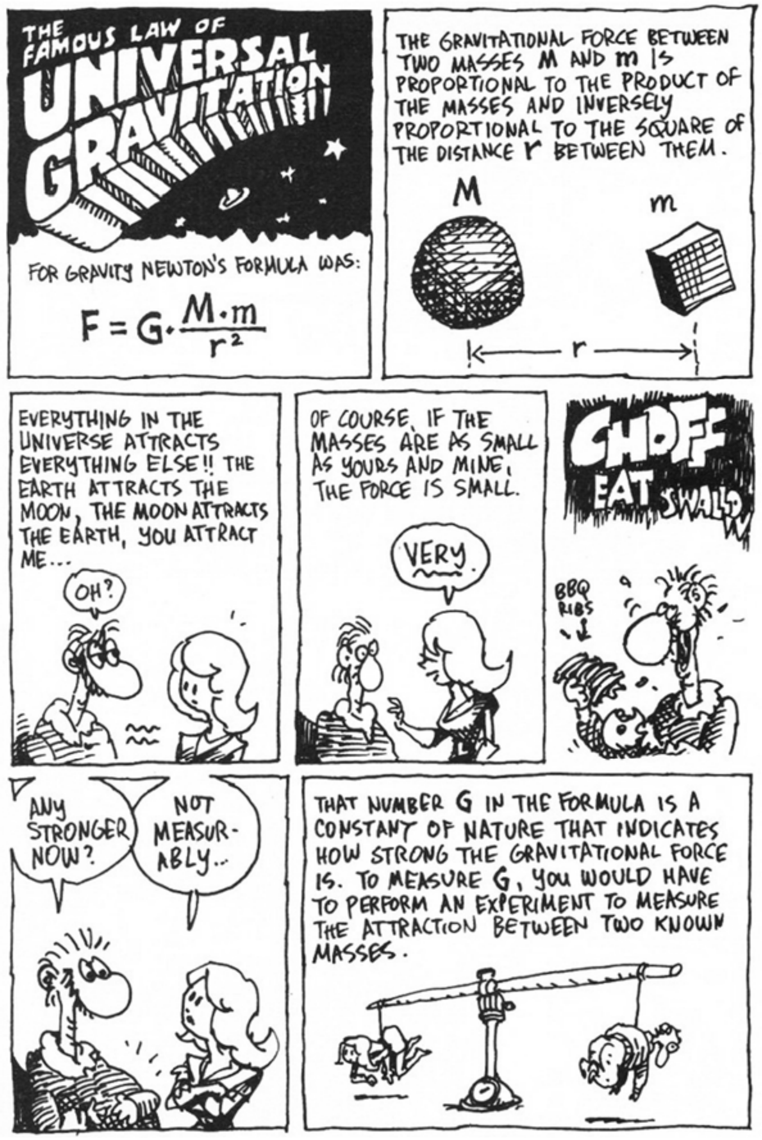
Chapter 7

Gravitational Field



Playlist of Lecture Examples and Concept Videos can be found at
https://youtube.com/playlist?list=PL_b5cjrUKDlbcIl2_xwdbabhxq0fs40Xw.

Alternative resource (with videos) at xmphysics:
https://xmphysics.com/xmGravitation/

Chapter 7: Gravitational Field

H2 Physics Syllabus 9749

Content

- Gravitational field
- Gravitational force between point masses
- Gravitational field of a point mass
- Gravitational field near to the surface of the Earth
- Gravitational potential
- Circular orbits

Learning Outcomes

Candidates should be able to:

(a) show an understanding of the concept of a gravitational field as an example of field of force and define the gravitational field strength at a point as the gravitational force exerted per unit mass placed at that point.

(b) recognize the analogy between certain qualitative and quantitative aspects of gravitational and electric fields (will be done in the chapter on electric fields).

(c) recall and use Newton's law of gravitation in the form $F = \frac{Gm_1m_2}{r^2}$.

(d) derive from Newton's law of gravitation and the definition of gravitational field strength, the equation $g = \frac{GM}{r^2}$ for the gravitational field strength of a point mass.

(e) recall and apply the equation $g = \frac{GM}{r^2}$ for the gravitational field strength of a point mass to new situations or to solve related problems.

(f) show an appreciation that near the surface of the Earth, gravitational field strength is approximately constant and equal to the acceleration of free fall.

(g) define the gravitational potential at a point as the work done per unit mass in bringing a small test mass from infinity to the point.

(h) solve problems by using the equation $\phi = -\frac{GM}{r}$ for the gravitational potential in the field of a point mass.

(i) analyse circular orbits in inverse square law fields by relating the gravitational force to the centripetal acceleration it causes.

(j) show an understanding of geostationary orbits and their application.

7.1 Newton's Law of Universal Gravitation

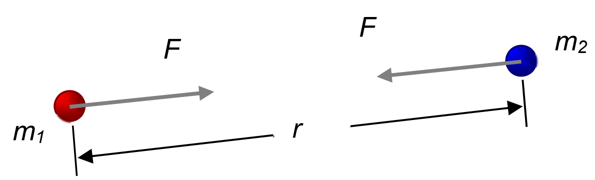
Newton's Law of Universal Gravitation states that:

Every **point mass attracts** every other **point mass** with a **force** that is directly **proportional to the product of their masses** and **inversely proportional to the square of the distance between them**.

Consider two point masses of m_1 and m_2 separated by a distance r. Each will exert a force F on the other, and its magnitude is given by

$$\boxed{F = G\frac{m_1 m_2}{r^2}}$$

where
- F : gravitational force between the two point masses.
- m_1, m_2 : masses of the two point masses.
- r : distance between the two point masses.
- G : gravitational constant



The two forces form an **action-reaction pair** and have the following characteristics,

- are equal in magnitude,
- are opposite in direction,
- act on different bodies
- are of the same type (gravitational force).

G is called the *gravitational constant* (or constant of universal gravitation), which has been measured experimentally to be: $\mathbf{G = 6.67 \times 10^{-11}\ N\ m^2\ kg^{-2}}$. It is given in the list of constants provided on the second page of all tests and exams.

Important points to note about Newton's Law of Gravitation

1. Newton's Law of Gravitation is a **universal** law. It applies everywhere in the universe.

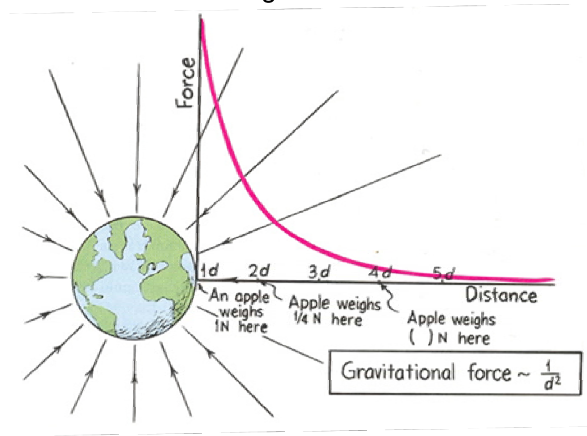
2. **Attractive nature of gravitational force:** Note that the masses in this case are always attracted to each other.

3. The gravitational force is a **field force** that always exists between two masses regardless of the medium that separates them and the two masses need not be in contact with other. In fact, it would still exist if there were no medium between them.

4. **Inverse-square law:**
   Newton's Law of Gravitation is an example of an **inverse-square law**, i.e., the force is inversely proportional to the square of the separation of the masses.

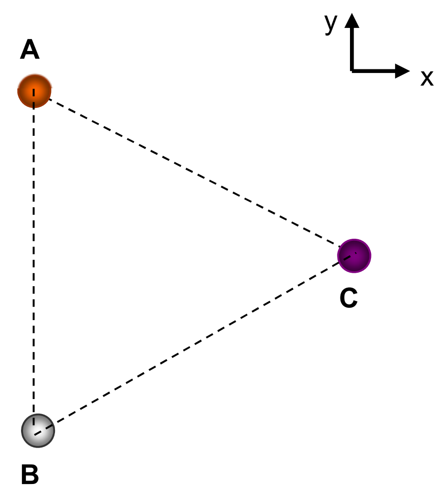
$$F \propto \frac{1}{r^2}$$

5. Newton's Law of Gravitation can be applied to spherical bodies as though they were point masses with all their mass concentrated at their geometrical centres.



Example 1: Finding Resultant Gravitational Force

Three identical masses, each of mass *m,* are located on a table at the corners of an equilateral triangle of side *d*. Determine the resultant gravitational force on mass C due to masses A and B.



Solution:

> **Concept:** If several masses are present, each pair will experience a mutual gravitational attraction. The resultant gravitational force on a given mass is **the vector sum of the separate attractive forces acting on it due to the masses interacting with it**.

7.2 Gravitational Field Strength

The gravitational field is a **region of space** where a **mass** will experience a **gravitational force**.

Consider a particle of mass *m* placed at a point in a gravitational field (due to another mass *M*).

The particle would experience a **gravitational force**, $\boldsymbol{F_g}$, in the gravitational field, given by:

$$\boldsymbol{F_g} = m\boldsymbol{g}$$

<u>Definition: Gravitational Field Strength, *g*</u>

The gravitational field strength *g* at a point is the **gravitational force per unit mass** acting on a small test mass placed at the point.

$$\boldsymbol{g} = \frac{F_g}{m}$$

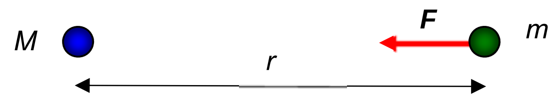
where
- $\boldsymbol{g}$: gravitational field strength at that point
- $\boldsymbol{F_g}$: gravitational force acting on mass *m* placed at that point in the gravitational field.

<u>Notes:</u>

1. S.I. unit of ***g*:** N kg^{-1} or m s^{-2}.
2. ***g*** is a <u>vector</u> quantity. Its **direction is determined by the gravitational force acting on a mass *m* placed at the point** in the gravitational field.
3. Mass *m* is a particle that is placed in the gravitational field. It is not the mass that is creating the gravitational field. Hence, mass *m* is often known as the *test* mass. The mass *M* that is creating the gravitational field is known as the *source* mass. The presence of the test mass is not necessary for the field to be present.
4. ***g*** is also commonly known as the *acceleration due to gravity* or *gravitational acceleration*.

7.2.1 Gravitational Field of a Point Mass

If a point mass M is placed at some point in space, it will set up a gravitational field around itself. Suppose we wish to determine the gravitational field strength $\boldsymbol{g}$ at a distance r away from this point mass M. We need to place a test mass m at the distance r away from this source mass M.



The test mass would experience a gravitational force $\boldsymbol{F}$ as it is in the gravitational field created by M.

By Newton's Law of Gravitation,

⇒ The magnitude of the gravitational force acting on m due to the gravitational field set up by M,

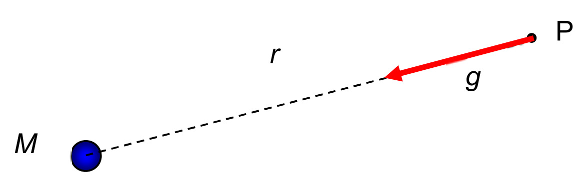
$$F = G\frac{Mm}{r^2}$$

From the definition of gravitational field strength,

⇒ The magnitude of gravitational field strength due to M at a distance r away from M,

$$\begin{aligned} g &= \frac{\text{Magnitude of gravitational force acting on test mass } m}{\text{Mass of test mass } m} \\ &= \frac{F}{m} \\ &= \frac{\left(G\dfrac{Mm}{r^2}\right)}{m} \\ &= G\frac{M}{r^2} \end{aligned}$$

<u>Gravitational Field Strength due to a Point Mass</u>



The magnitude of the gravitational field strength at a point due to a point mass M at a distance r away is given by:

$$\boxed{g = G\frac{M}{r^2}}$$

Note:
The equation is also valid for gravitational field strength outside a spherical body of uniform density or spherical shells of uniform density, where its whole mass is concentrated at its geometric centre.[1] r would then represent the distance between the test mass and the geometric centre of the source mass.

[1] Read about the gravitational field outside and within a sphere in Appendix A.

7.2.2 Representation of Gravitational Field

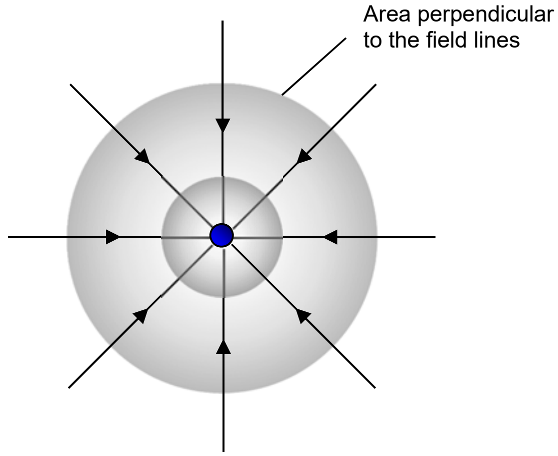
We use field lines to represent a gravitational field. The gravitational field lines are related to the gravitational field in the following manner:

1. **Direction**
   The direction of the gravitational field strength *g* is **tangent** to the gravitational field line at each point in space.

2. **Magnitude**
   The magnitude of the gravitational field strength is **proportional to the number of lines per unit area** through a surface perpendicular to the lines.
   Therefore, *g* is large when the lines are close together and small when they are far apart.

Gravitational Field Line Diagram

<u>For a point mass</u>



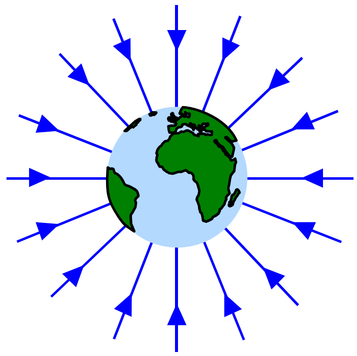
- The gravitational field in the vicinity of a point mass is always directed *<u>radially towards</u>* the point mass.

- The magnitude of the gravitational field is inversely proportional to the square of the distance from the point mass. Therefore, the number of field lines per unit area decreases at further distances from the point mass, indicating the diminishing strength of the field.

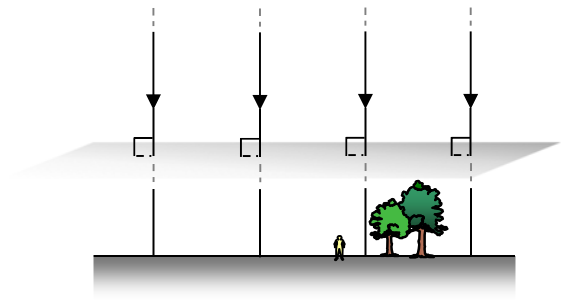
- The field is really 3-dimensional, but on paper, we can only take a 2-dimensional slice of it. This is a radial field.

- Note that
  - Fields lines end on a mass and extend back all the way to infinity.
  - Field lines never cross one another.

For Earth (spherical mass)



Far away



Close to the surface of the Earth

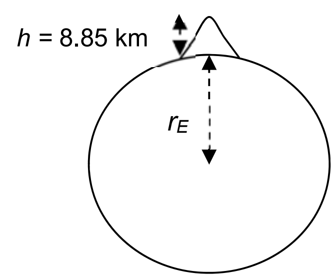
- At a large distance from the Earth (sphere), the field lines look similar to that of a point mass.

- Close to the surface of Earth,
    - The field lines are directed downwards (the direction in which a body near the Earth's surface would feel a gravitational force).
    - The field lines are *parallel and equidistant indicating that the field is constant*, or uniform.
    - The number of field lines per unit area is approximately constant for heights up to about 10 km above the Earth surface. The field strength is approximately 9.81 m s^{-2}.

Example 2: Gravitational Field Strength near the Surface of the Earth

Assume that the Earth is not rotating and is a perfect sphere of uniform density.

(i) Calculate the gravitational field strength of the Earth at its surface.
(ii) Given that Mount Everest is 8.85 km high, compute the gravitational field strength of Earth at the peak of Mount Everest.
(iii) Comment on your values computed above.



You are given the following information:

- Mass of the Earth, $m_E = 5.98 \times 10^{24}$ kg
- Average radius of Earth, $r_E = 6.37 \times 10^{6}$ m

Solution:

7.2.3 Gravitational Field Strength (Acceleration due to Gravity)

If the only force acting on a mass m is the gravitational force, which is given by mg, where g is the gravitational field strength at the point, then applying Newton's second law,

$$m\mathbf{a} = m\mathbf{g}$$
$$\mathbf{a} = \mathbf{g}$$

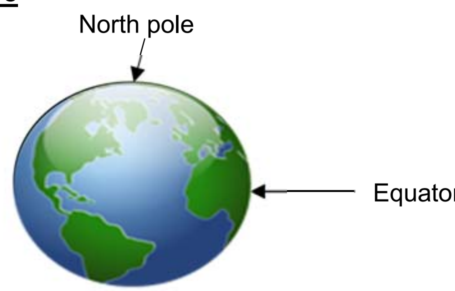
Hence the mass will have an acceleration equal to the gravitational field strength g. Thus the gravitational field strength g is also known as *acceleration due to gravity* or *gravitational acceleration*.

7.2.4 Variation of *g*

However, when we try to measure the value of $\mathbf{g}$ at various regions on Earth, the value of $\mathbf{g}$ varies. At the poles, $\mathbf{g}$ is 9.83 m s^{-2}, while at the equator, it is 9.78 m s^{-2}. In Singapore, it is about 9.79 m s^{-2}.

What accounts for this variation? There are mainly three factors affecting the value of g measured.

(1) <u>The Earth is not a perfect sphere</u>



Earth is approximately an ellipsoid, flattened at the poles and bulging at the equator. Its equatorial radius is greater than its polar radius by 21 km. Thus a point at the poles is closer to the dense core of the Earth than a point on the equator. This is one reason the gravitational field strength increases as one proceeds, at sea level, from the equator towards the poles.

(2) <u>The density of the Earth is not uniform</u>

The density of the Earth varies from region to region over the Earth's surface. The northern hemisphere, for instance, has far more land area than the southern hemisphere. Accurate measurements of the gravitational pull are used to identify different densities of materials beneath the Earth's surface. These measurements are used in prospecting for oil, coal and other minerals.

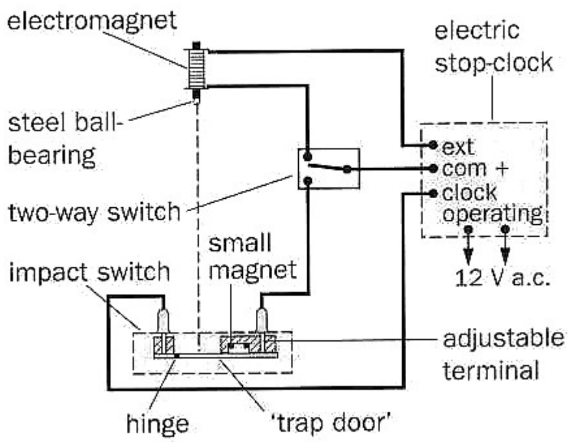
(3) <u>The Earth is rotating</u>

The Earth rotates about an axis passing through the poles. Because of the Earth's rotation, the gravitational force on a body that is not at the poles has to also provide the body with the centripetal acceleration required for rotation.

The farther the body is from the poles and the nearer it is to the equator the larger the centripetal acceleration required. Therefore, the *measured* gravitational acceleration gets smaller as one approaches the equator.

The measured acceleration is also called the acceleration of free fall (or the apparent gravitational field strength).

7.2.5 Measurement of acceleration of free fall



In this experiment, a steel ball bearing is made to fall a known distance, *s*, from rest and the time taken to fall this distance is measured using an electric stop-clock.

When the two-way switch is changed to the 'down' position, the electromagnet releases the ball and simultaneously the clock starts. At the end of its fall, the ball opens the 'trap door' of the impact switch and the clock stops.

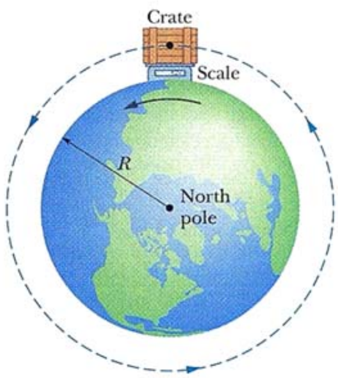
Assuming air resistance is negligible and applying the equations of motion,

$$s = ut + \frac{1}{2}at^2 \quad \Rightarrow \quad a = \frac{2s}{t^2}$$ where *t* is the time taken to fall the distance *s*.

Accurate measurements of *s* and *t* will allow the acceleration of free fall to be obtained.

Example 3: How Earth's rotation affects the weight measured

Consider a crate of mass *m* resting on a weighing scale at the equator.



Take the Earth to be a uniform sphere of mass *M* and radius *R*, spinning on its axis.

(a) On the diagram, draw all the forces acting on the crate.

(b) Write down an expression for the reading of the weighing scale supporting this mass.

(c) Write down an expression for the 'apparent' gravitational acceleration indicated by the weighing scale and determine its value.

Solution:

7.3 Satellites

A satellite is a moon, planet or machine that orbits a planet or star. For example, Earth is a satellite because it orbits the sun. Likewise, the moon is a satellite because it orbits Earth. Usually, the word "satellite" refers to a machine that is launched into space and moves around Earth or another body in space.

Earth and the moon are examples of natural satellites. Thousands of artificial, or man-made, satellites orbit Earth. Some take pictures of the planet that help meteorologists predict weather and track hurricanes. Some take pictures of other planets, the sun, black holes, dark matter or faraway galaxies. These pictures help scientists better understand the solar system and universe.

Still other satellites are used mainly for communications, such as beaming TV signals and phone calls around the world. A group of more than 20 satellites make up the Global Positioning System, or GPS. If you have a GPS receiver, these satellites can help figure out your exact location.

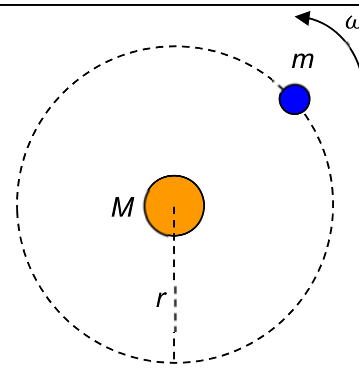
Example 4: Investigating satellite motion

Consider a satellite of mass *m* revolving around a larger mass *M* in a circle of radius *r*.

Determine the relationship between the radius of the orbit and

(a) the velocity of the satellite;

(b) the period of rotation.



<u>Solution:</u>

As the satellite revolves, its velocity keeps changing as the direction of motion keeps changing. Hence, the satellite is accelerating. From the topic of Circular Motion, we know that a resultant force must be providing this centripetal acceleration.

The gravitational attraction of the large mass *M* on the satellite keeps the satellite in orbit, i.e., **the gravitational force acting on the satellite (by the mass *M*) provides for the centripetal force required for the satellite to maintain circular orbit about the mass *M*.**

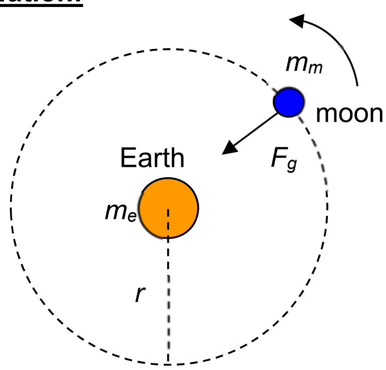
Example 5: How do you 'weigh' the Earth?

When Newton first discovered his Law of Gravitation, he had neither the value of *G* nor the mass of the Sun.

In 1798, a hundred and twenty-one years after Newton proposed the Universal Law of Gravitation, Henry Cavendish[2] conducted the first experiment to measure the force of gravity between masses in the laboratory, and obtained accurate values for the gravitational constant, *G*, and the mass density of the Earth. With the value of *G*, it was possible to obtain a value for the mass of the Earth.

By considering the motion of the moon about the Earth, show that the mass of the Earth m_e is approximately 6 x 10^{24} kg. You may assume that the distance between Earth and the moon is $r = 4.0 \times 10^8$ m.

Solution:



The **gravitational force** acting on the moon by the Earth's gravitational field **provides** for **the centripetal force** required by the moon to move in orbit about the Earth.
(NOTE: the statement above is required to explain the following steps below. Please write this in exams!)

By Newton's 2nd Law,

$$F_g = m_m a_c$$

$$\Rightarrow G\frac{m_e m_m}{r^2} = m_m r \omega_m^2 = m_m r\left(\frac{2\pi}{T_m}\right)^2$$

$$\Rightarrow m_e = \frac{4\pi^2}{G}\frac{r^3}{T_m^{\ 2}}$$

Substituting for *G*, *r* and since we know that the moon takes approximately about 1 month to make one complete revolution around the Earth,

$$\Rightarrow \quad m_e = \frac{4\pi^2}{G}\frac{r^3}{T_m^{\ 2}} = \frac{4\pi^2}{G}\frac{(4\times 10^8)^3}{(30\times 24\times 60\times 60)^2} = 5.64 \times 10^{24} \text{ kg} \sim 6 \times 10^{24} \text{ kg}$$

Hence, we have found the mass of the Earth!

The Cavendish Experiment - Sixty Symbols

[2] https://en.wikipedia.org/wiki/Cavendish_experiment

7.3.1 Geostationary Satellites

Geostationary satellites are satellites in orbits where the satellites are always positioned over the same geographical spot on Earth.[3] They appear stationary when seen from Earth.

Key Features of Geostationary Satellites:

1. **Period**
   - 24 hours (same period as the Earth's rotation about its own axis)

2. **Direction of rotation**
   - same direction as the rotation of the Earth, i.e. from west to east.

3. **Plane of rotation**
   - the plane that contains **the equator**.

The gravitational force on the satellite is always directed towards the centre of the Earth and so any circular orbit must have its centre at the centre of the Earth. If the orbit is not in the equatorial plane, the satellite will sometimes be over the northern hemisphere and sometimes over the southern hemisphere and so will not be geostationary.

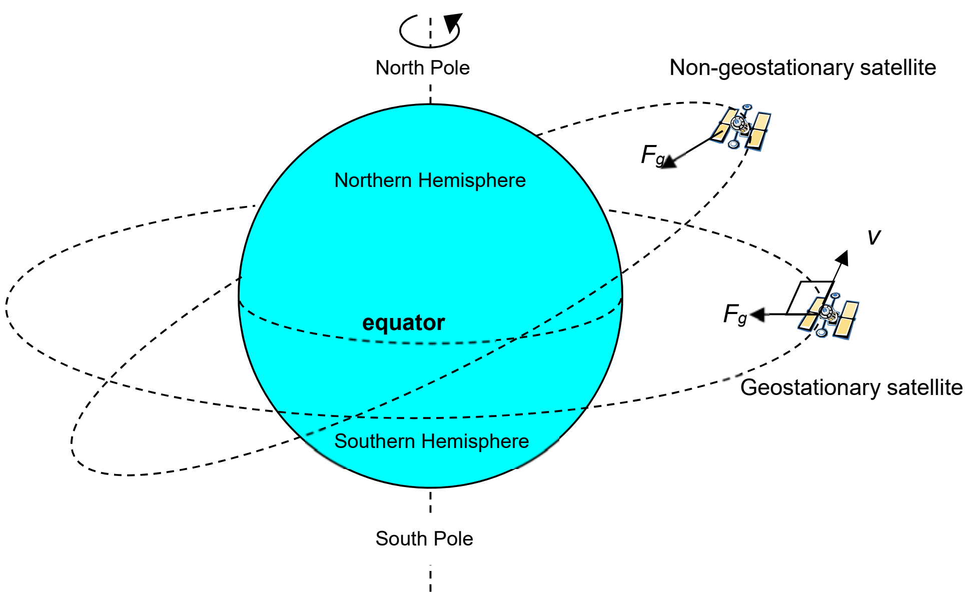


Figure not to scale

Advantages of Geostationary Satellites

1. As they remain stationary above the same point on Earth, they are ideal for use as communication satellites as they require no tracking to receive signals from Earth. This is why there is no need to keep adjusting the satellite dish to receive TV signals from a particular geostationary satellite.

[3] Besides the geostationary orbits, satellites can have other orbits as well. Check out the following sites for different types of satellites, orbits and functions by scanning the QR codes below.

NASA: Earth Observatory

Types and Uses of Satellites

2. As geostationary satellites are positioned at such a high altitude (see Example 6 for its height above the Earth's surface), they can capture almost the full-disk Earth image, rather than a small subsection. Hence, they are ideal for meteorological applications and remote imaging.

Disadvantages of Geostationary Satellites

1. As geostationary satellites are positioned at such a high altitude, the spatial resolution (i.e. the amount of detail shown) of their images tends to be not as good as satellites which are much closer to Earth.

2. Another problem with geostationary satellites is that since they are positioned above the equator, they cannot see the north or south poles and are of limited use for latitudes greater than 60-70 degrees north or south.

Example 6: Geostationary Satellite

A communications satellite of mass *m* is placed in a circular geostationary orbit.
Find its height above the Earth's surface.
(Take radius of the Earth to be 6.38×10^6 m and the mass of the Earth to be 5.98×10^{24} kg).

Solution:

The **gravitational force** on the satellite **provides** the required **centripetal force.**
(NOTE: the statement above is required to explain the following steps below. Please write this in exams!)

By Newton's second law:

7.3.2 Weightlessness

There are two types of weightlessness – true weightlessness and apparent weightlessness.

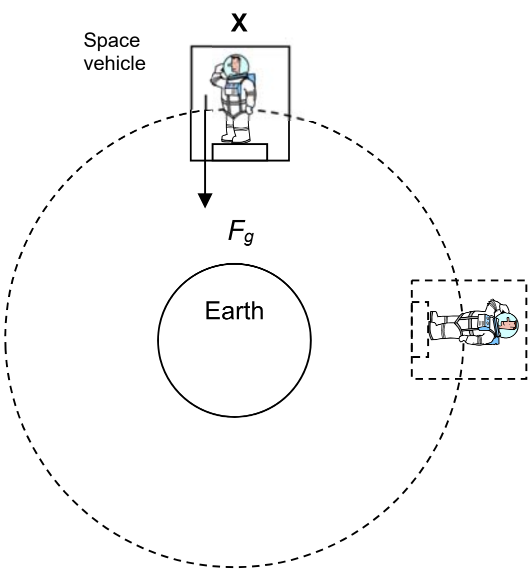
True Weightlessness

An object is truly weightless when there is no net gravitational force acting on it, such as when it is at an infinite distance away from any other body, or when it experiences opposing gravitational forces that exactly cancel one another out.

Apparent Weightlessness

An astronaut orbiting the Earth in a space vehicle with its thrusters off is often said to be 'weightless' as well. To understand why, let us try to 'weigh' this astronaut in the space vehicle.

Suppose an astronaut is standing on a weighing balance in a space vehicle orbiting the Earth. Let us first consider the space vehicle, astronaut and weighing balance as a single system. Let the mass of this system be M_{sys}.



The only force acting on this system is the gravitational force due to the Earth's gravitational field, F_g.

Since the space vehicle is in a circular orbit around the Earth,
$\Rightarrow$ F_g provides for the centripetal acceleration required for the space vehicle to move in a circular orbit.

By Newton's 2nd Law,

$$F_g = M_{sys}\,a$$

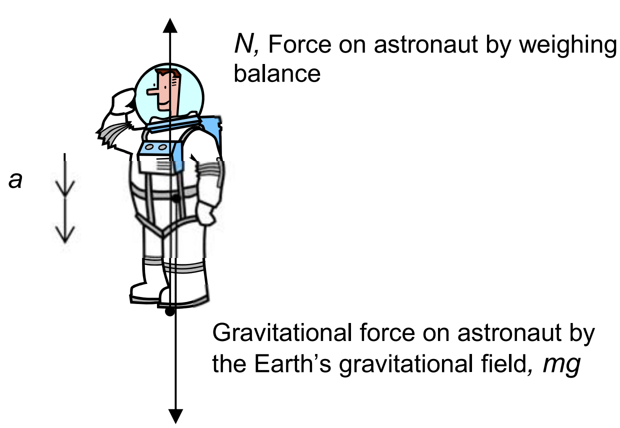
But $F_g = M_{sys}\,g$

$$\Rightarrow M_{sys}g = M_{sys}a$$

Therefore, $a = g$

The space vehicle, the man and the weighing balance all have the same acceleration of g.

Now consider forces acting only on the astronaut at position X in the orbit.
.



By Newton's 2nd Law,

$$mg - N = ma$$

where m is the mass of the astronaut and a is the acceleration of the astronaut.

However, from above, $a = g$.

Therefore, $mg - N = mg$

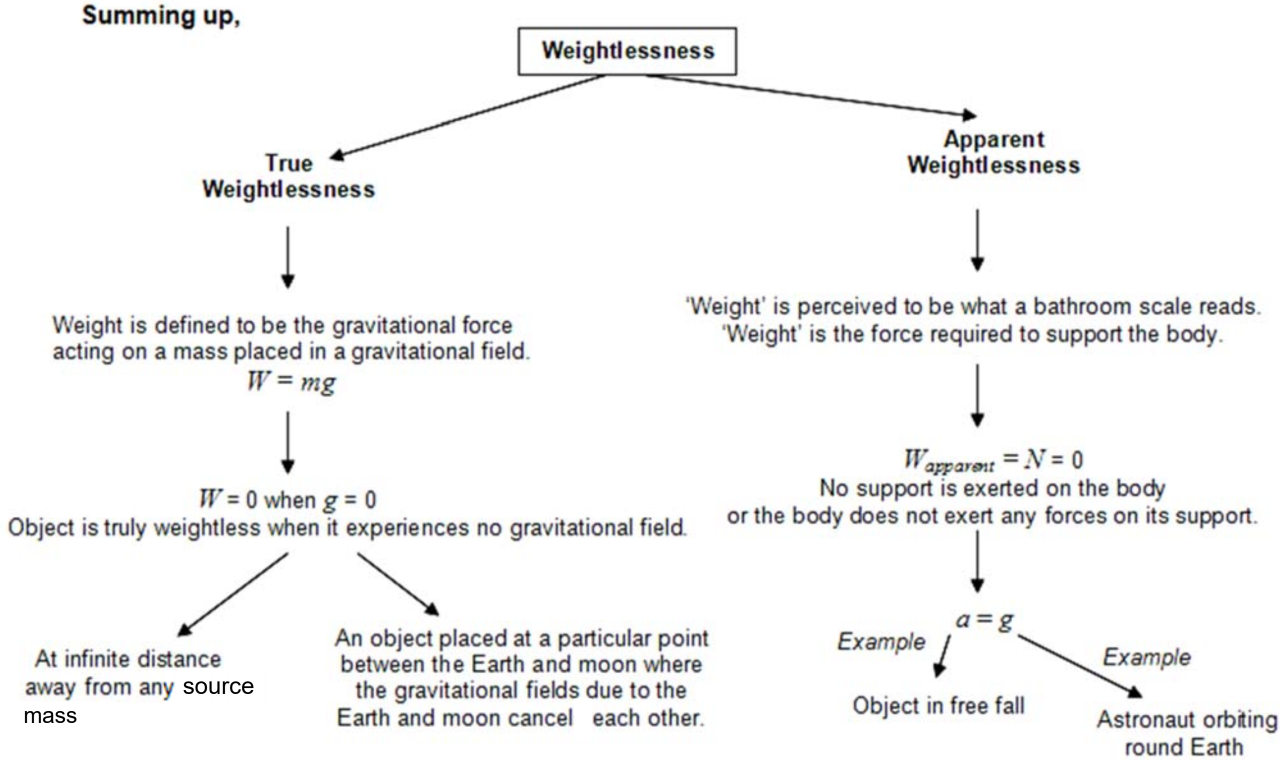
$$\Rightarrow \quad N = 0$$

Therefore, we see that the weighing balance does not exert a support force on the astronaut.

By Newton's 3rd Law, the astronaut does not exert a force on its support, the weighing balance. The reading indicated on the weighing balance is **zero**. The astronaut thus experiences the sensation of 'weightlessness'.

Hence, **an object is considered to be (apparently) weightless when it exerts no contact force on its support.** This happens when the object is accelerating with an acceleration of $\boldsymbol{g}$, e.g. an object in free fall or an object orbiting around the Earth.

Summing up,



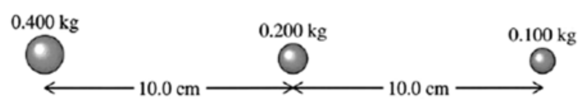
Tutorial 7A: Gravitational Force and Field

Self-Review Questions

S1 The gravitational constant *G* has the SI base unit

A m s^{-2} **B** m^{-1} kg^{-1} s^{-2} **C** m^3 kg^{-1} s^{-2} **D** m^2 kg^{-2} **E** m^3 s^{-2}

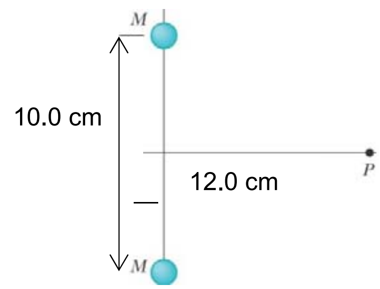
S2 **Coletta, Physics Fundamentals, Page 158, Problem 6.10.**
Find the resultant force on (a) the 0.100 kg mass and (b) the 0.200 kg mass in the diagram below (the masses are isolated from the Earth).



S3 An experimental satellite is found to have a weight *W* when assembled before launching from a rocket site. It is placed in a circular orbit at a height $h = 6R$ above the surface of the Earth (of radius *R*). What is the gravitational force acting on the satellite whilst in orbit?

A *W*/6 **B** *W*/7 **C** *W*/36 **D** *W*/49

S4 **Modified from Serway and Faugh, 6th Edition, P13.25.**
Compute the magnitude and direction of the gravitational field at a point *P* on the perpendicular bisector of the line joining two objects of equal mass, 20.0 kg separated by a distance 10.0 cm as shown in the figure to the right.



S5 The values of the acceleration of free fall, *g*, on the surfaces of two planets will be the same provided that the planets have the same

A mass **B** radius **C** mass/radius **D** mass/(radius)^2 **E** mass/(radius)^3

S6 **Coletta, Physics Fundamentals, Page 161, Problem 6.39.**
Find the radius of a planet made of solid lead, such that gravitational acceleration at the surface of the planet is 1.00 m s^{-2}. The density of lead is 1.13×10^4 kg m^{-3}.

S7 The gravitational field strength outside a uniform sphere of mass *M* is the same as that due to a point mass *M* at the centre of the sphere. The Earth may be taken to be a uniform sphere of radius *r*. The gravitational field strength at its surface is *g*.

What is the gravitational field strength at a height *h* above the surface?

A $\dfrac{gr^2}{(r+h)^2}$ **B** $\dfrac{gr}{(r+h)}$ **C** $\dfrac{g(r-h)}{r}$ **D** $\dfrac{g(r-h)^2}{r^2}$

S8 A small part of the Earth's gravitational field close to the surface of the Earth is uniform. Which of the following statements is **not** correct?

A The field lines are parallel to each other.
B The gravitational force on an object is proportional to its height above the Earth's surface.
C The units of gravitational field strength are equivalent to m s^{-2}.
D The direction of the field lines is towards the Earth.

S9 **J83/II/6 mod** Assuming Earth to be a uniform sphere rotating about an axis through the poles, the weight of a body measured on a weighing scale at the Equator compared with its weight measured on the same weighing scale at a pole would be

A greater, because the angular velocity of the Earth is greater at the equator than at the pole.
B greater, because the weight at the Equator is given by the sum of the gravitational attraction of the Earth and the centripetal force due to the circular motion of the body.
C the same, because the weight is the gravitational attraction of the Earth and for a uniform sphere, even when rotating, this is independent of the body's position on the Earth.
D smaller, because the gravitational attraction of the Earth must provide both the weight and the centripetal force due to the circular motion of the body.
E smaller, because the gravitational attraction at the pole is greater than that at the Equator.

S10 The moon remains in its orbit around the Earth rather than fall to the Earth because

A it is also attracted by the gravitational forces from the sun and other planets
B the net force on the Moon is zero
C the gravitational force exerted by the Earth on the moon provides a net force that provides the Moon's centripetal acceleration.
D the magnitude of the gravitational force from the Earth is too small to cause any appreciable acceleration of the Moon

S11 Which statement about a geostationary satellite is true?

A It can remain vertically above any chosen fixed point on the Earth.
B Its linear speed is equal to the speed of a point on the Earth's equator.
C It has the same angular velocity as the Earth's rotation on its axis.
D It is always travelling from east to west.

S12 **2009/1/6**
An object in a space capsule orbiting the Earth seems to be floating. Which statement describes the forces acting on the object?
A There are no forces on the object.
B The centrifugal force on the object is equal and opposite to its weight.
C The centripetal force on the object is equal and opposite to its weight.
D The weight of the object is the only force acting on it.

S13 **Coletta, Physics Fundamentals, Pg 159, Problem 6.23.**
Find the speed of a satellite moving in a circular orbit just above the surface of the moon. The moon has radius of 1.74×10^{6} m and a mass of 7.36×10^{22} kg. Also find the satellite's orbital period.

Discussion Questions

D1 **Coletta, Physics Fundamentals, page 158. Problem 6.13.**
A particle of mass *m* is between a 1.00×10^2 kg mass and a 4.00×10^2 kg mass, which are 10.0 m apart. Find the distance of the particle from the 1.00×10^2 kg mass such that the resultant force on the particle is zero. (This is also known as the neutral point or null point.)

D2 **2010/1/15**
The neutral point in the gravitational field between the Sun, the Earth and the Moon is the point at which the resultant gravitational field due to the three bodies is zero. The mass of the Earth is about 80 times the mass of the Moon.

At what position is it possible for the neutral point to be? (The diagram is not drawn to scale.)

D3 **Lowe & Rounce, 3rd Edition, page 85, Ex12.2, Q2.**
The acceleration due to gravity at the Earth's surface is 9.81 m s^{-2}. Calculate the acceleration due to gravity on a planet which has
(a) the same mass and twice the radius,
(b) the same radius and twice the density,
(c) half the radius and twice the density.

D4 The acceleration of free fall on the surface of the Earth is 6 times its value on the surface of the Moon. The mean density of the Earth is 5/3 times the mean density of the moon.
What is the ratio of the radius of the Earth over the radius of the Moon?

A 1.9 **B** 3.6 **C** 6.0 **D** 10

D5 **Coletta, Physics Fundamentals, Pg 161, Problem 6.42**
Suppose the Earth rotates on its axis so quickly that an object at the equator has an apparent weight of zero. What would then be the length of an Earth day, given the radius of Earth is 6400 km?

D6 **2009/3/5**

(a)(i) Define gravitational field strength.

(ii) State Newton's law of gravitation and hence, using your definition in **(i)**, show that the gravitational field strength *g* at a distance *R* from a point mass *M* is given by $g = \frac{GM}{R^2}$

(b) A neutron star has mass 5.2×10^{30} kg and radius 1.7×10^4 m.

(i) Calculate the mean density of the star.
(ii) Suggest, with a reason, whether the density is likely to vary with the distance from the centre of the star.

(c) The mass of the star in (b) may be considered to be a point mass at its centre.

(i) Calculate the gravitational field strength at the surface of the star.
(ii) Determine the centripetal acceleration of a particle moving in a circular path of radius 1.7×10^4 m and with a period of rotation of 0.21 s.
(iii) The star rotates about its axis with a period of 0.21 s. Use your answers in (i) and (ii) to suggest whether particles on the surface of the star leave the surface owing to the high speed of rotation of the star.

D7 **CIE J96/III/2(part)**

(c) The Earth may be considered to be uniform sphere of radius 6370 km, spinning on its axis with a period of 24.0 hours. The gravitational field at the Earth's surface is identical with that of a point mass of 5.98×10^{24} kg at the Earth's centre. For a 1.00 kg mass situated at the Equator,

(i) calculate, using Newton's law of Gravitation, the gravitational force on the mass.
(ii) determine the force required to maintain the circular path of the mass,
(iii) deduce the reading on an accurate newton-meter (spring balance) supporting the mass.

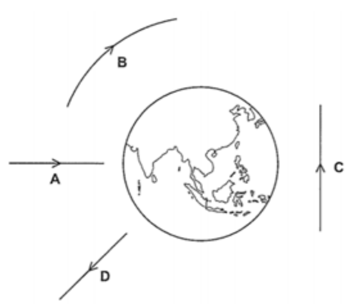
(d) Using your answers to (c), state what would be the acceleration of the mass at the Earth's surface due to
(i) the gravitational force alone,
(ii) the force measured on the newton-meter.

(e) A student situated at the Equator releases a ball from rest in a vacuum and measures its acceleration towards the Earth's surface. He then states that this acceleration is 'the acceleration due to gravity'. Comment on his statement.

D8 **2018/1/11**

The lines on the diagram show parts of four paths of a spacecraft moving near the Earth above any influence of the atmosphere.

Which path is not possible unless the aircraft fires its rockets as it follows the path?



D9 **2021/2/4(b)**

A satellite is in a circular orbit about the Earth with a period of 110 minutes.
For this satellite, the Earth may be considered to be a point mass of 6.0×10^{24} kg situated at its centre.

Determine the gravitational field strength *g* at the location of the satellite.

D10 **2017/2/2 (part)**

In July 2015, the New Horizons space probe made its closest approach of Pluto.

Charon is one of the moons of Pluto. Data, including the radius *r* and period of rotation about the axis *T*, are given for Pluto and Charon the table below.

	r / km	mass / kg	*T* / days
Pluto	1.20×10^3	1.31×10^{22}	6.36
Charon	0.600×10^3	1.52×10^{21}	6.36

When viewed from above, Pluto and Charon rotate in the same direction about their axes.

(a) **(i)** The orbital speed of Charon is 0.200 km s^{-1} and its orbital radius is 1.75×10^4 km. Show that the period of the orbit of Charon around Pluto is 6.36 days.

(ii) A space probe on the surface of Pluto is able to observe Charon over a time of several days.

Suggest what the space probe observes as a result of

1. the period of rotation of Pluto about its axis equalling the orbital period of Charon,

2. equal periods of rotation about their axes for both Pluto and Charon.

(b) Pluto may be considered to be an isolated sphere with the mass concentrated at its centre. Use data from the table to determine the gravitational field strength on the surface of Pluto.

D11 2016/3/9 (part)

A binary star consists of two stars A and B. The two stars may be considered to be isolated in space. The centres of the two stars are separated by a constant distance d, as illustrated in Fig. 9.3.

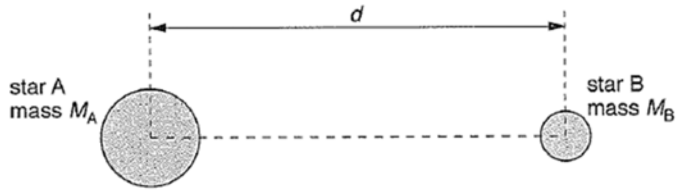


Fig. 9.3

Star A, of mass M_A, has a larger mass than star B of mass M_B, such that $\frac{M_A}{M_B} = 3.0$.

The stars are in circular orbits about each other such that the centre of their orbits is at a fixed point. Viewed from Earth over a period of time equal to the period T of the orbits, the appearance of the stars is shown in Fig. 9.4.

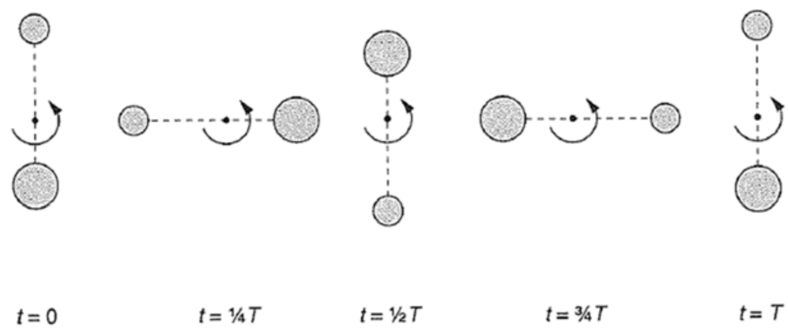


Fig. 9.4

The period T of each orbit is 4.0 years. The separation d of the centres of the stars is 3.0×10^{11} m.

(i) Explain why the centripetal forces acting on the two stars are equal in magnitude.
(ii) Calculate the angular speed ω of star A.
(iii) Determine the radius of the orbit of star A. Explain your working.
(iv) Use data and your answers from the above parts to determine the mass of each star.
(v) The plane of the orbits of the binary star in above parts is normal to the line of sight from Earth to the binary star. A second binary star has the plane of its orbits parallel to the line of sight from Earth. This binary star is so far from Earth that the individual stars cannot be distinguished. Suggest and explain what observation can be made to determine the period of the orbits of the stars.

Numerical solutions

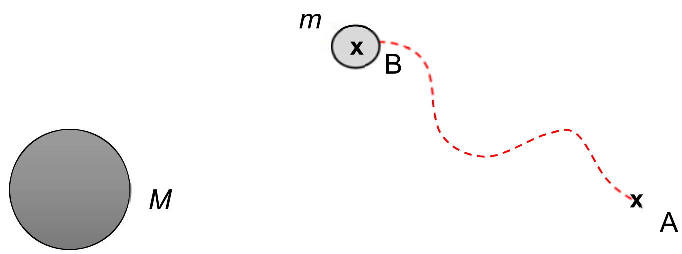
S1) C
S2) (a) 2.00×10^{-10} N to the left, (b) 4.01×10^{-10} N towards the left
S3) D
S4) 1.46×10^{-7} N kg^{-1},
S5) D
S6) 3.17×10^{5} m
S7) A
S8) B
S9) D
S10) C
S11) C
S12) D
S13) 1.68×10^{3} m s^{-1} and 1.81 h

D1) 3.33 m
D3) (a) 2.45 m s^{-2}, (b) 19.6 m s^{-2}, (c) 9.81 m s^{-2}
D5) 5.06×10^{3} s or 1.4 hrs
D6) (b)(i) 2.53×10^{17} kg m^{-3}, (c)(i) 1.20×10^{12} N kg^{-1}, (c)(ii) 1.52×10^{7} m s^{-2}
D7) (c)(i) 9.83 N, (ii) 0.0337 N, (iii) 9.80 N, (d)(i) 9.83 m s^{-2}, (ii) 9.80 m s^{-2}
D9) 6.9 N kg^{-1}
D10) (b) 0.607 N kg^{-1}
D11) (ii) 4.98×10^{-8} rad s^{-1}, (iii) 7.5×10^{10} m, (iv) 2.51×10^{29} kg, 7.53×10^{29} kg

7.4 Gravitational Potential and Gravitational Potential Energy

7.4.1 Gravitational Potential Energy

Consider a region of space in which a gravitational field, created by source mass *M*, is present.



Consider a mass *m* placed in this gravitational field. If no external force is applied on mass *m*, it will accelerate towards mass *M* due to the attractive gravitational force on *m* by *M*. Let's say we apply an external force on the mass to move it between two points, from A to B, without changing its kinetic energy. By conservation of energy, whatever work is done by the external force would be equal to the change in gravitational potential energy of the system of 2 masses, *M* and *m*, i.e.

work done by an external force to move *m* from A to B in the gravitational field,

$$W = \Delta U_{AB} = U_B - U_A$$

where ΔU_{AB} : Change in gravitational potential energy of the system as mass *m* moves from A to B

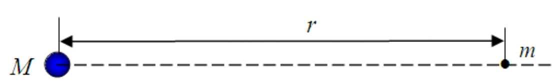
U_A : Gravitational potential energy of system when *m* is at A

U_B : Gravitational potential energy of system when *m* is at B

If point A is at an infinite distance from *M*, then the gravitational field strength due to *M* would be zero there. The gravitational potential energy is zero at that point ($U_A = U_\infty = 0$), then

$$W = \Delta U_{AB} = U_B - U_A = U_B$$

i.e. the work done by an external force to move a mass *m* from A (at infinity) to a point B without a change in kinetic energy is the gravitational potential energy of the system when mass *m* is at B.



From the concept of work done by a force, the **gravitational potential energy**[4] ***U* of a system of two point masses *M* and *m*** separated by a distance *r* is given by $\int_\infty^r \frac{GMm}{r^2} dr$. It can be shown that

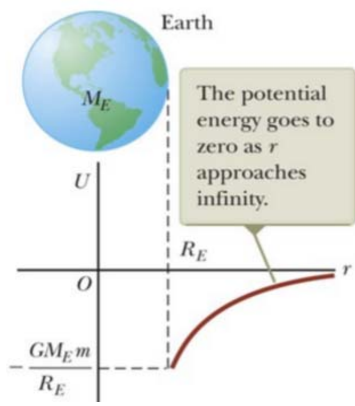
$$\boxed{U = -G\frac{Mm}{r}}$$

[4] A more rigorous derivation of the Gravitational Potential Energy (GPE) between 2 point masses and the gravitational potential of a point mass can be found in Appendix B.

Notes:

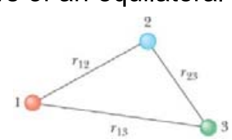
1. S.I. unit for U: joule (J).
2. The **gravitational potential energy is actually a property of the system of the two masses** rather than of either mass alone. There is no way to divide this energy and say that so much belongs to one mass and so much to the other. Therefore, we say that it is the **gravitational potential energy of the system of *M* and *m***.

   However, if $M >> m$, as for the case of the Earth and the apple, we often use the phrase "the potential energy of the apple". This is because when an apple moves in the vicinity of Earth, changes in the potential energy of the Earth-apple system appear almost entirely as changes in kinetic energy of the apple, since the changes in the kinetic energy of the Earth are negligible.
3. U is a scalar quantity. A scalar quantity can be positive or negative in value. However, gravitational potential energy is always negative. The negative sign has nothing to do with direction.
4. **Why is *U* negative and what is the significance of the negative sign?**
   - Gravitational force is attractive in nature.
   - To bring a mass from infinity to the point in the field without changing its kinetic energy, an external force opposite to the gravitational force needs to be applied.
   - Since the external force and displacement of the mass are in opposite directions, the external force does negative work on the mass in the process.
   - As GPE at infinity is taken as zero, the GPE at any point must be negative.



Example 7: Gravitational Potential Energy of a System of 3 Masses

A system consists of three particles, each of mass m located at the corners of an equilateral triangle of side x. Determine the gravitational potential energy of the system.



Solution:

Example 8: Gravitational Potential Energy Changes near the Earth's Surface

Show that the expression $U = -G\dfrac{Mm}{r}$ can be reduced to the expression $mg\Delta h$ for calculations of changes in gravitational potential energy near the Earth's surface, i.e. for Δh << radius of the Earth R_E.

Solution

Example 9: Escape Speed

The minimum speed with which a mass should be projected from the Earth's surface in order to escape Earth's gravitational field is known as the **escape speed** of the body from Earth.

Show that the escape speed of the mass *m* placed at the surface of the Earth is $\sqrt{2gR_E}$, given that the radius of the Earth is R_E and gravitational field strength at the Earth's surface is given by *g*.

Solution:

Example 10: Mechanical Energies of a Satellite in Orbit

A satellite of mass *m* orbits Earth of mass *M* in a circular path. The satellite has speed *v* and the radius of its orbit about Earth is *R.*

(a) Derive an expression for the KE and the total energy of the satellite in terms of G, *M, m* and *R*.

(b) Plot a graph showing how each of the following physical quantities associated with the satellite varies with distance from the centre of Earth:

(i) kinetic energy;

(ii) gravitational potential energy;

(iii) total energy.

Solution:

7.4.2 Gravitational Potential

Whenever a field is present in a region, we can always define a scalar property of the field at each point in the region. This property is called the *potential* of the field. This *potential* of a field is associated with the *potential energy per unit mass* that a test mass will possess when it is placed at the point in the field.

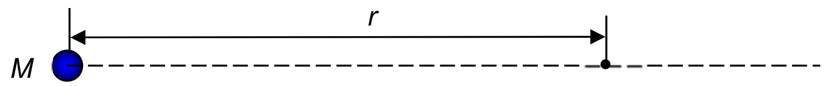
Definition: Gravitational Potential, ϕ

The **gravitational potential** at a point in a gravitational field is the work done per unit mass, by an external force, in bringing a small test mass from infinity to that point, without any change in kinetic energy.

Notes:

1. S.I. unit of gravitational potential ϕ: J kg^{-1}.
2. The gravitational potential is a **scalar quantity**.
3. The gravitational potential at infinity is taken to be zero.

Gravitational Potential due to a Point Mass :

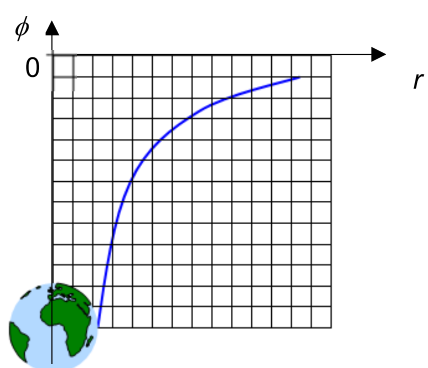


The gravitational potential ϕ at a point a distance r away from a source point mass M is given by:

$$\boxed{\phi = -\frac{GM}{r}}$$

Note:

1. The equation is also valid for any point outside masses of radial symmetry. This means that, for instance, we can assume the whole mass of the Earth to be concentrated at the Earth's centre. The gravitational potential *beyond the Earth's radius* can then be calculated using the formula given above.



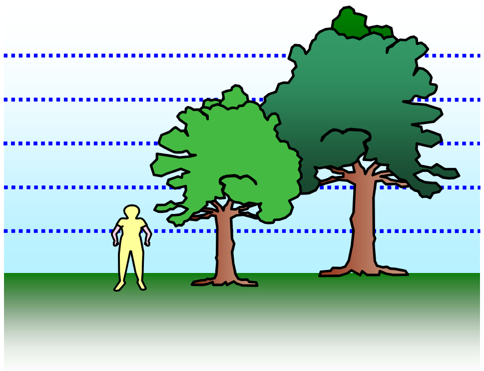
The gravitational potential at a point in the field is the work done per unit mass in bringing a test mass from infinity to that point.

Since the gravitational field is attractive and zero potential is at infinity, the gravitational potential is always negative.

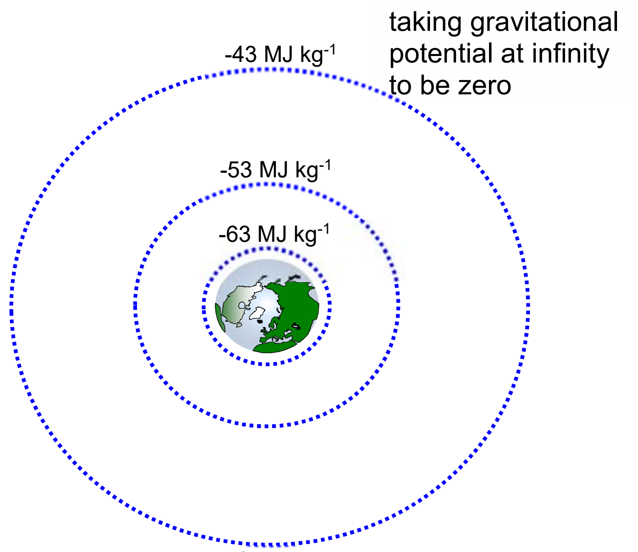
2. All points at the same distance from the centre of the Earth will have the same gravitational potential. Points of the same potential are known as points of **equipotential**.

   Equipotential lines/surfaces join points of equal potential. Below are diagrams showing the cases of uniform fields (near the surface of the Earth) and radial fields (for point and spherical masses):

Equipotential surfaces: ....

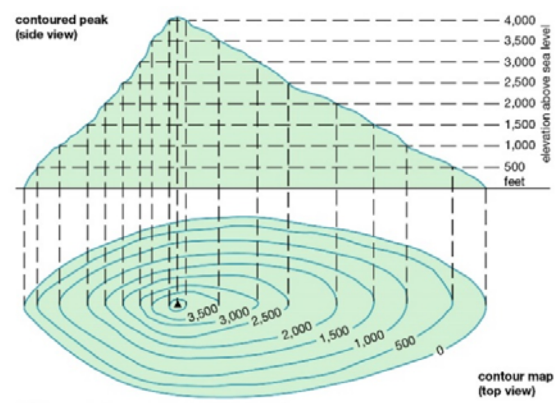


The gravitational field is uniform close to the surface of the Earth. This results in uniformly spaced equipotential lines.



The gravitational field obeys the inverse square law. Hence the distance between adjacent equipotential lines increases as distance from Earth increases.

Equipotential lines are always perpendicular to field lines. Diagrams of equipotential lines give us information about the gravitational field in much the same way as contour maps give us information about geographical heights which show regions of same heights as shown below.



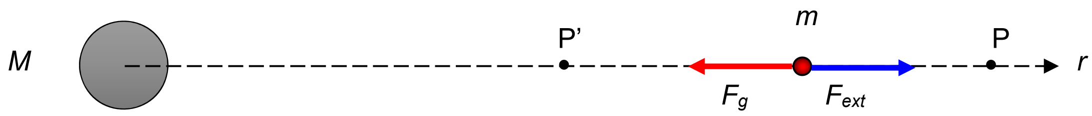
<u>Relationship between Gravitational Potential Energy U and gravitational potential ϕ</u>

$$\boxed{U = m\phi}$$

As we move a mass from one point to another, the change in gravitational potential energy is given by

$$\boxed{\Delta U = m\Delta\phi = m(\phi_f - \phi_i)}$$

7.4.3 Relationship between Potential and Field Strength



Consider a mass *M* fixed in space. Suppose we move a mass *m* from point P to point P' in the gravitational field of *M* by applying an external force F_{ext} such that there is no change in the kinetic energy of *m*. Then

$$\Rightarrow \qquad F_{ext} = -F_g$$

where F_g: gravitational force acting on *m*. (Note that F_{ext} and F_g vary with *r*.)

The change in gravitational potential energy of *m*, ΔU, is equal to the work done W_{ext} by external force F_{ext} in moving a mass *m* from point P to point P' (without an increase in kinetic energy) i.e.,

$$\Rightarrow \qquad W_{ext} = \Delta U$$

But $W_{ext} = \int_{r_P}^{r_{P'}} F_{ext} dr$

$$\Rightarrow \qquad \int F_{ext} dr = \Delta U$$

$$\Rightarrow \qquad F_{ext} = \frac{dU}{dr}$$

$$\Rightarrow \qquad F_g = -\frac{dU}{dr}$$

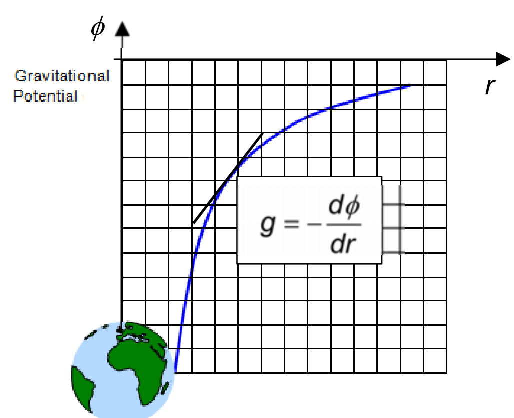
Relationship between Gravitational Force F_g and Gravitational Potential Energy *U*

$$\boxed{F_g = -\frac{dU}{dr}}$$

Dividing both sides of $F_g = -\frac{dU}{dr}$ by *m*, we obtain:

Relationship between Gravitational Field Strength *g* and Gravitational Potential ϕ

$$\boxed{g = -\frac{d\phi}{dr}}$$



Notes:

1. $\frac{d\phi}{dr}$ is known as the **potential gradient**.

2. The magnitude of the potential gradient at a point in the gravitational field gives the magnitude of the gravitational field strength at the point.

3. The **negative sign** indicates that gravitational field strength points in the direction of decreasing ϕ.

Gravitational Field Relationships

$$F = mg$$

$$\boxed{F = \frac{GMm}{r^2}} \qquad \boxed{g = \frac{GM}{r^2}}$$

Vectors:
1. Resultant force or field strength requires **vector** addition

$$F = -\frac{dU}{dr} \qquad g = -\frac{d\phi}{dr}$$

$$\boxed{U = -\frac{GMm}{r}} \qquad \boxed{\phi = -\frac{GM}{r}}$$

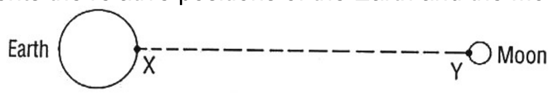
Scalars:
1. Total potential energy or potential requires **scalar** addition
2. The negative sign is necessary

$$U = m\phi$$

Note: The four relationships in the boxes are only valid for point masses (and spherical objects) while the other four relationships are valid for all cases.

Example 11: Gravitational Potential between 2 Masses (Mod TYS Pg 91 Qn 38 N93/I/5; N2000/I/7)

The diagram (not to scale) represents the relative positions of the Earth and the Moon.



The line XY joins the surface of the Earth to the surface of the Moon.

(a) Sketch on the same axes graphs representing

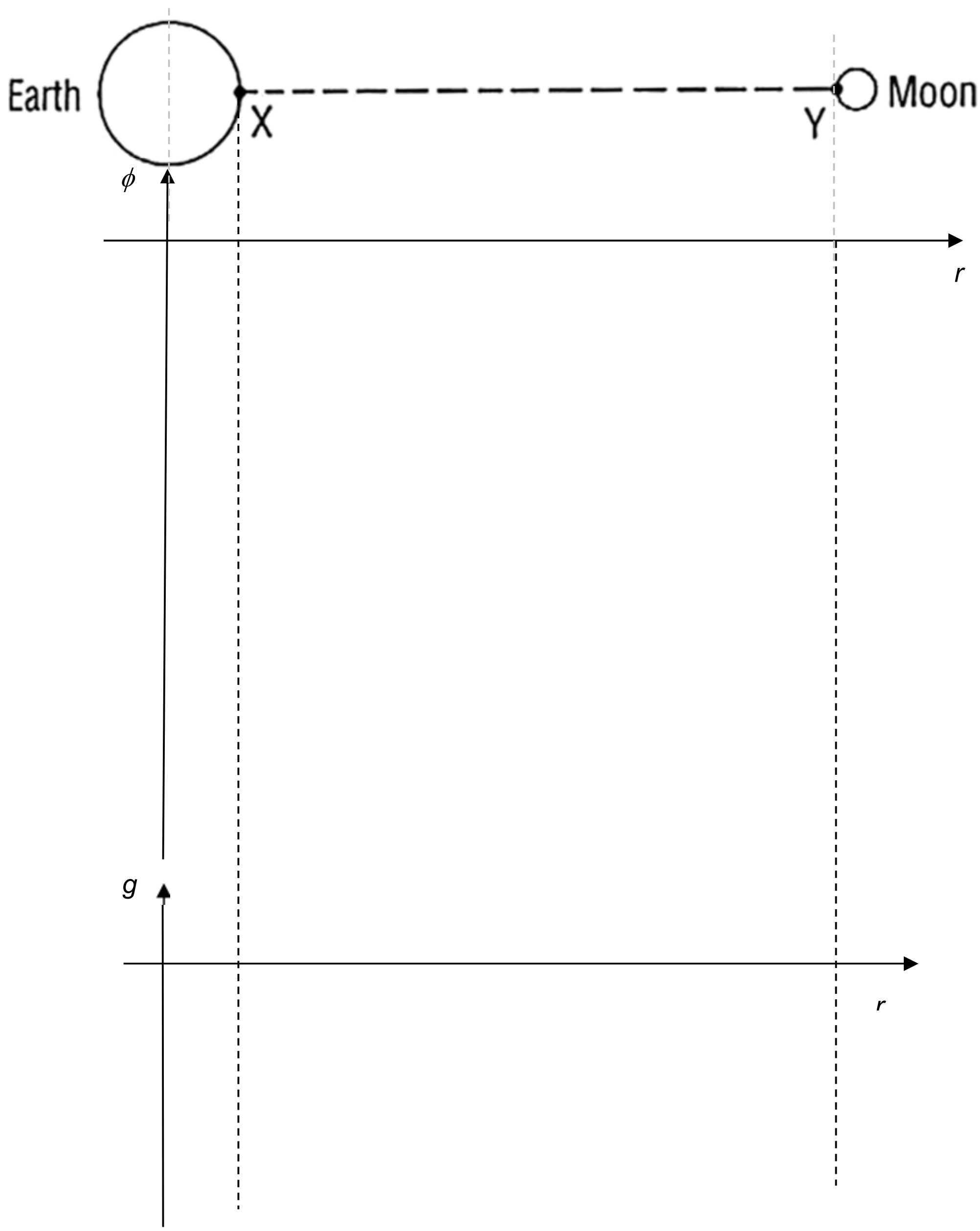
(i) the gravitational potential due to the Earth between X and Y;

(ii) the gravitational potential due to the Moon between X and Y;

(iii) the resultant gravitational potential between X and Y.

(b) Sketch the corresponding variation of gravitational field strength with distance on the axis below.

Solution:



Appendices:

Appendix A: Variation of the Gravitational Field Strength above and below the Earth's Surface

Consider the Earth to be a non-rotating spherical planet of uniform density ρ. Take the radius of the Earth to be r_E, and the mass of the Earth to be m_E.

To determine the variation of g with respect to the distance from the centre of the Earth r, the following results are useful:

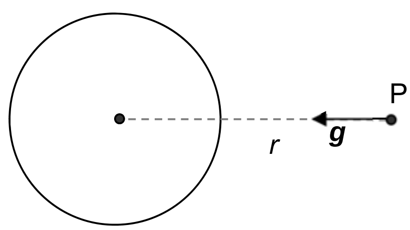
Result 1: The acceleration due to gravity **outside** a spherical body of uniform density is the same as if the entire mass of the body were concentrated at its centre.

Result 2: The acceleration due to gravity at all points **inside** a spherical shell of uniform density is zero.

With these results, we are able to obtain expressions for the acceleration due to gravity both above and below the Earth's surface.

<u>Gravitational Field Strength above the Earth's surface ($r \geq r_E$)</u>

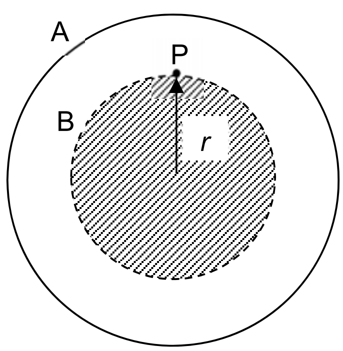
It follows from Result 1 that for $r \geq r_E$, $g = G\dfrac{m_E}{r^2}$; and at $r = r_E$, $g = G\dfrac{m_E}{r_E^{\,2}}$



Since G and m_E are constants, $g \propto \dfrac{1}{r^2}$.

<u>Gravitational Field Strength below the Earth's surface ($r \leq r_E$)</u>[5]

Consider a point P below the Earth's surface and at a distance r away from the centre of the Earth.



The gravitational field at P,

g = g at P due to spherical shell A + g at P due to spherical mass B

By Result 2,
Gravitation field at P due to shell A = 0, and

By Result 1,
Gravitation field at P due to spherical mass B

$= G\dfrac{M_B}{r^2}$ where M_B is the mass of the spherical mass B

[5] The gravitational field strength below the Earth's surface is not explicitly required in the syllabus, it is only introduced here for completeness.

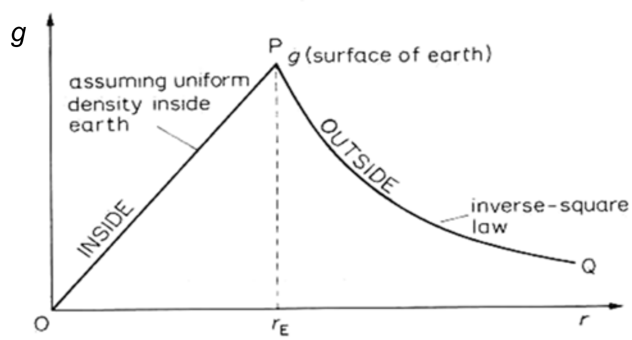
But $M_B = \rho$ (Volume of spherical mass B) = $\rho(\frac{4\pi r^3}{3})$

Hence the gravitational field at P (with r < radius of the Earth) is:

$$g = \frac{G}{r^2}(\frac{4\pi r^3 \rho}{3}) = (\frac{4\pi G\rho}{3})r$$

Since $\frac{4\pi G\rho}{3}$ is a constant, we see that for $r < r_E$, $g \propto r$.

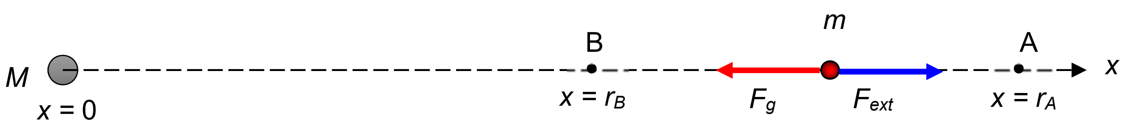
A sketch of g against r is shown below.



Appendix B: Derivation of the Gravitational Potential Energy and Gravitational Potential

Consider a particle of mass *m* moving between two points A and B in the gravitational field exerted by another particle *M.* To find an expression for the gravitational potential energy of this system, we need to first find the work done by an external force in moving the mass *m* between the two points without a change in kinetic energy of *m*.

Suppose we set up the coordinate system as below.



By Newton's Law of Gravitation, the gravitational force[6] F_g acting on the mass *m* at a distance *x* away from *M*,

$$F_g = -G\frac{Mm}{x^2}$$

The negative sign here indicates that the gravitational force is pointing in the negative *x* direction.

For the kinetic energy of *m* to remain constant, we must move *m* at a **constant velocity** (acceleration = 0).

Therefore, the external force F_{ext} acting on *m* is opposite in direction and equal in magnitude to F_g.

$$F_{ext} = G\frac{Mm}{x^2}$$

Since $\mathbf{F_{ext}}$ is not constant but varies with distance *x* away from *M*, to find the work done W by this external force in moving the mass *m* between A and B,

$$W = \int_{r_A}^{r_B} F_{ext}\,dx = \int_{r_A}^{r_B} G\frac{Mm}{x^2}dx = GMm\left[-\frac{1}{x}\right]_{r_A}^{r_B} = -GMm\left[\frac{1}{r_B}-\frac{1}{r_A}\right]$$

By conservation of energy, since there is no increase in the kinetic energy of the system,

W = Change in gravitational potential energy of the system, ΔU

Therefore, $$W = \Delta U = U_B - U_A = -GMm\left[\frac{1}{r_B}-\frac{1}{r_A}\right] = \left[-G\frac{Mm}{r_B}\right]-\left[-G\frac{Mm}{r_A}\right]$$

If the reference point A is chosen to be at an infinite distance away from M, i.e. $r_A = \infty$, the gravitational potential energy for *m* at A, U_A is equal to zero.

---

[6] Note that is gravitational force is an internal force to the system of *M* and *m*. To consider work done on *m*, we must always treat *M* and *m* as a single system, as *m* only experiences a force due to the gravitational field exerted by *M*.

$$U_A = U_\infty = \left[-G\frac{Mm}{r_A}\right] = 0$$

Then, $$U_B - U_A = U_B = -G\frac{Mm}{r_B}$$

Therefore, the gravitational potential energy U of a test mass m placed at a distance of r away from the source mass M, is given by

$$\boxed{U = -G\frac{Mm}{r}}$$

From the definition of the potential at a point, the potential ϕ at a point at a distance r away from the source mass M,

$$\phi = \frac{U}{m}$$

$$\Rightarrow \quad \boxed{\phi = -\frac{GM}{r}}$$

Tutorial 7B: Gravitational Potential and Potential Energy

Self-Review Questions

S1 CIE J80/II/5 (modified) *X* and *Y* are two points at respective distances *R* and 2*R* from the centre of the Earth, where *R* is greater than the radius of the Earth. The gravitational potential at *X* is –800 kJ kg^{-1}. When a 3 kg mass is taken from *X* to *Y*, the work done on the mass is

A –400 kJ **B** –1200 kJ

C +400 kJ **D** +1200 kJ

S2 N12/1/15 (modified) A satellite of mass 810 kg is to be raised from the Earth to a height of 92.0 km above the surface of the Earth. What is the necessary increase in the potential energy of the satellite?

The mass of the Earth is 5.98×10^{24} kg. The radius of the Earth is 6370 km.

S3 Lowe and Rounce, 3rd Edition, Pg. 86, Ex. 12.3, Q2. The moon has mass 7.7×10^{22} kg and radius 1.7×10^{6} m. Calculate

(a) the gravitational potential at its surface and

(b) the work needed to completely remove a 1.5×10^{3} kg space craft from its surface into outer space.

Neglect the effect of earth, other planets, sun etc.

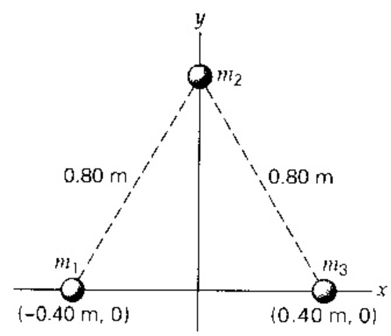
S4 Lowe and Rounce, 3rd Edition, Pg. 86, Ex. 12.3, Q4. A neutron star has radius 10 km and mass 2.5×10^{29} kg. A meteorite is drawn into its gravitational field. Calculate the speed with which it will strike the surface of the star. Neglect the initial speed of the meteorite.

S5 N09/1/17 (modified) A satellite orbits a planet at a distance *r* from its centre. Its gravitational potential energy is -3.2 MJ. Another identical satellite orbits the planet at a distance 2*r* from its centre. What is the sum of the kinetic energy and the gravitational potential energy of this second satellite?

Discussion Questions

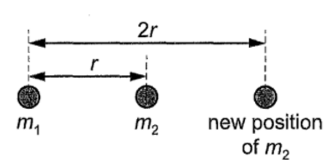
D1 **(a)** What is the total gravitational potential energy of the configuration shown in the figure on the right if all the masses are 1.0 kg?

(b) What is the work done (by an external agent) if we were to add in an additional 1.0 kg mass into the centre of the system of masses? Explain why this answer is negative.



D2 **2020/1/12**

Two objects are separated by a distance *r*. The objects can be considered to be point masses of mass m_1 and m_2. The object of mass m_1 is fixed in position. The object of mass m_2 is moved so that the separation of the two objects is 2*r*.



What is the increase in gravitational potential energy of the object of mass m_2?

A $\dfrac{Gm_1m_2}{2r}$ **B** $\dfrac{-Gm_1m_2}{2r}$ **C** $\dfrac{3Gm_1m_2}{4r^2}$ **D** $\dfrac{-3Gm_1m_2}{4r^2}$

D3 CIE J85/II/9 (part)

The curve in Fig. 5 shows the way in which the gravitational potential energy of a body of mass *m* in the field of the Earth depends on *r*, the distance from the centre of the Earth, for values of *r* greater than the Earth's radius R_E.

(a) What does the gradient of the tangent to the curve at $r = R_E$ represent?

The body referred to above is a rocket which is projected vertically upwards from the Earth. At a certain distance *R* from the centre of the Earth, the total energy of the rocket (i.e. its gravitational potential energy plus its kinetic energy) may be represented by a point on the line PQ. Five points A, B, C, D, E have been marked on this line. Which point (or points) could represent the total energy of the rocket

(b) if it were momentarily at rest at the top of its trajectory,

(c) if it were falling towards the Earth,

(d) if it were moving away from the Earth, with sufficient energy to reach an infinite distance?

In each case, explain briefly how you arrive at your answer.

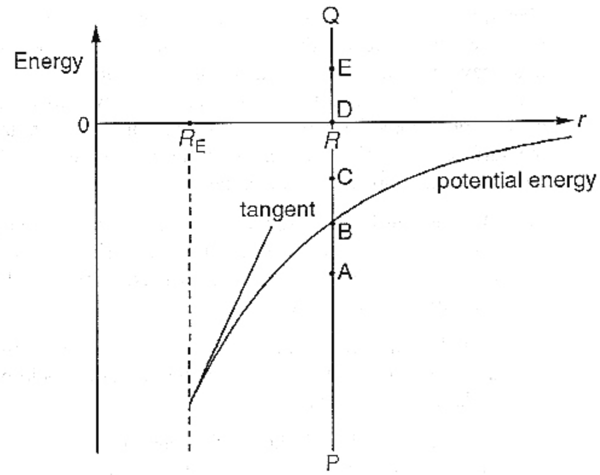


Fig. 5

D4 2021/3/2

(a) The gravitational potential *Φ* at a distance *x* from a point mass *M* is given by the expression

$\phi = -\dfrac{GM}{x}$, where G is the gravitational constant.

Explain why gravitational potential is a negative quantity.

(b) A planet of diameter 6.8×10^3 km has a mass of 6.2×10^{23} kg. The planet has no atmosphere and it may be assumed to be isolated in space. The mass of the planet may be considered to be a point mass at its centre.

A meteorite collides with the planet. This causes a rock of mass 2.8 kg to be thrown up from the surface of the planet with a speed of 3.8×10^3 m s^{-1}.

(i) Calculate the gravitational potential at the surface of the planet.

(ii) Use energy considerations, and your answer in **(b)(i)** to determine whether the rock returns to the surface of the planet or travels out into space.

D5 A space station of mass 450,000 kg is in a low Earth orbit (LEO) at an altitude of 415 km. Calculate the energy required to bring it to a medium Earth orbit (MEO) of altitude 20,200 km. You are given that the mass of the Earth is 6.0 x 10^{24} kg and its radius is 6,378 km. You may assume that, as the space station is in LEO or MEO, it is in uniform circular motion about the centre of the Earth.

D6 Adapted from UCLES N08/II/3 and N04/II/3

A satellite of mass *m* orbits a planet of mass *M* and radius R_p with a speed of *v*. The radius of the orbit is *R*. The satellite and the planet may be considered to be point masses with their masses concentrated at their centres. They may be assumed to be isolated in space.

(a) **(i)** Show that speed *v* is given by the expression

$$v^2 = \frac{GM}{R}.$$

(ii) The mass of the satellite is *m*. Determine an expression for the kinetic energy E_k of the satellite in terms of *G*, *M*, *m* and *R*.

(b) **(i)** State an expression, in terms of *G*, *M*, *m* and *R*, for the gravitational potential energy E_p of the satellite. Explain why it is negative.

(ii) Hence, show that the satellite in orbit, the ratio of its *gravitational potential energy* to the *kinetic energy* is equal to -2.

(iii) Also show that the total energy E_t of the satellite is given by $E_t = -\frac{GMm}{2R}$.

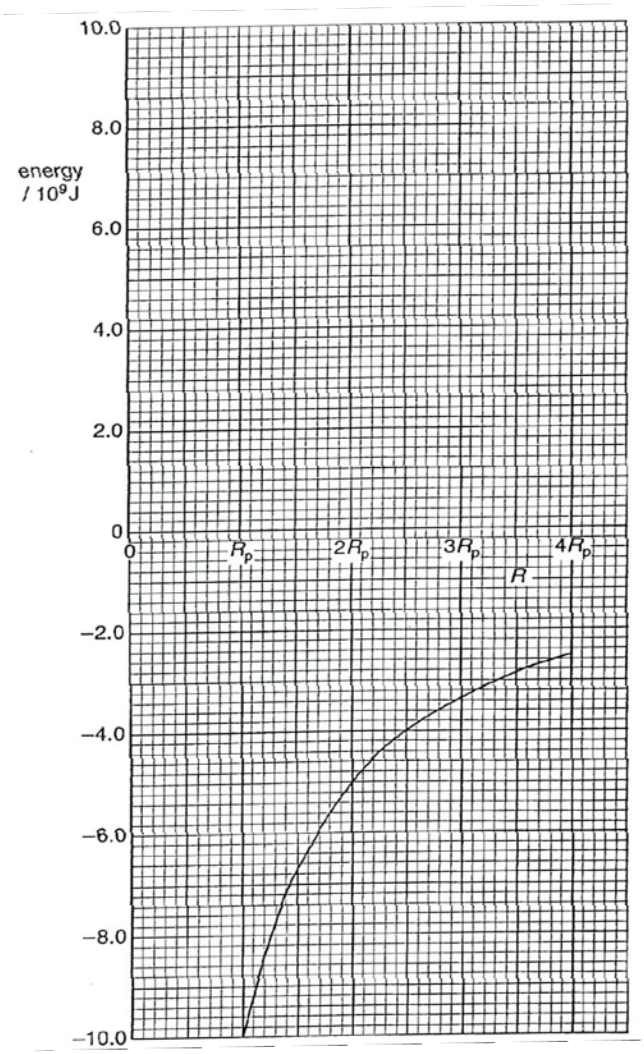
(c) The variation with orbital radius *R* of the gravitational potential energy of the satellite is shown in the figure on the right.

On the figure, draw the variation with orbital radius of

(i) the kinetic energy of the satellite,

(ii) the total energy of the satellite.

Your line should extend from $R = 1.5\ R_p$ to $R = 4\ R_p$.



(d) As the satellite orbits the planet, it gradually loses energy because of air resistance by the atmosphere of the planet.

(i) State whether the total energy E_t becomes more or less negative.

(ii) Hence, state and explain the effect of this change on

1. the radius of the orbit,

2. the speed of the satellite.

(iii) The mass *m* of the satellite is 1600 kg. The radius of the orbit of the satellite is changed from $R = 4\ R_p$ to $R = 2R_p$. Use the figure to determine the change in orbital speed of the satellite.

D7 CIE J87/II/8 (part)

A point mass *m* is at a distance *r* from the centre of the Earth.

(a) Write down an expression, in terms of *m*, *r*, the Earth's mass m_E and the gravitational constant *G*, for the gravitational potential energy *V* of the mass. (Consider only values of *r* greater than the Earth's radius. Mass of Moon = 7.4×10^{22} kg; mass of Earth = 6.0×10^{24} kg.)

Certain meteorites (tektites) found on Earth have a composition identical with that of lunar granite. It is thought that they may be debris from a volcanic eruption on the Moon. Fig. 6, which is not to scale, shows how the gravitational potential between the surface of the Moon and the surface of the Earth varies along the line connecting their centres. At the point P the gravitational potential is at a maximum.

(b) State how the resultant gravitational force on the tektite at any point between the Moon and the Earth could be deduced from Fig. 6.

(c) When a tektite is at P the gravitational forces on it due to Moon and Earth are F_M and F_E respectively. State the relation which applies between F_M and F_E. Hence find the values of *X/Y*, where *X* and *Y* are the distances of P from the centre of the Moon and the centre of the Earth respectively.

(d) If a tektite is to reach the Earth, it must be ejected from the volcano on the Moon with a certain minimum speed V_0. Making use of appropriate values from Fig. 6, find this speed. Explain your reasoning.

(e) Discuss very briefly whether a tektite will reach the Earth's surface with a speed less than, equal to or greater than the speed of projection. (Neglect atmospheric resistance.)

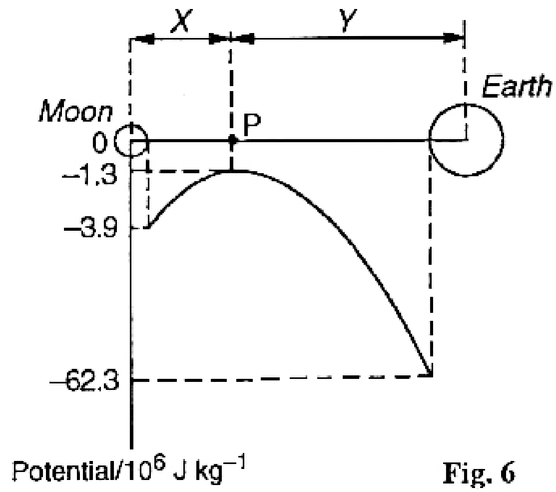
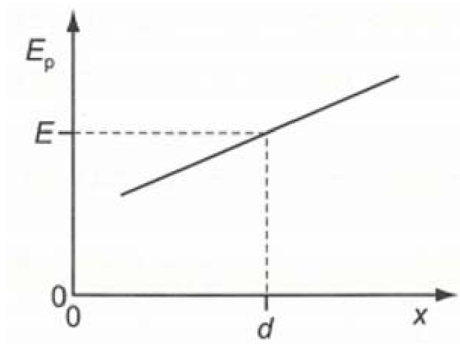


Fig. 6

D8 2011/I/12

A stone of mass *m* moves radially away from Earth.

For a small distance *x* above the Earth's surface, the variation with *x* of the stone's gravitational potential energy E_P is shown.



At point P, a distance *d* from Earth, the potential energy of the stone is *E* and the rate of change of potential energy with distance is *R*.

What is the force acting on the stone?

A E/d **B** mE/d **C** mR **D** R

D9 2021/I/10

Near the surface of a planet, the gravitational field strength is uniform. The gravitational potential difference between two points that are 10 m apart vertically is 6.0 J kg^{-1}.

How much work is done in raising a mass of 2.0 kg vertically through 2.5 m?

A 1.5 J **B** 3.0 J **C** 12 J **D** 15 J

Numerical solutions

S1) D, S2) 7.22×10^8 J, S3) (a) -3.0×10^6 J kg^{-1}, (b) 4.5×10^9 J, S4) 5.8×10^7 m s^{-1}, S5) - 0.8 MJ

D1) (a) -2.5×10^{-10} J (b) -4.3×10^{-10} J
D4) (b)(i) – 1.2×10^7 J kg^{-1}
D5) 9.9×10^{12} J
D6) (d)(iii) 518 m s^{-1}
D7) (c) X/Y = 0.11; (d) 2.3×10^3 m s^{-1}