Name:_____ **Class:**_____

JURONG JUNIOR COLLEGE

Preliminary Examination

MATHEMATICS
Higher 2

9740/02

11 September 2012

3 hours

Additional materials: Answer Paper
List of Formulae (MF15)
Cover Page

READ THESE INSTRUCTIONS FIRST

Write your name and civics class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.
Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

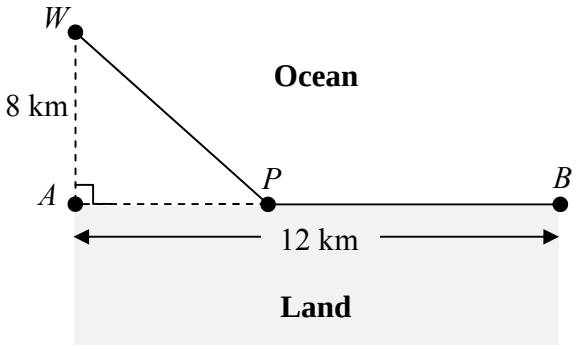
The number of marks is given in brackets [ ] at the end of each question or part question.
At the end of the examination, fasten all your work securely together, with the cover page in front.

This document consists of **6** printed pages.

[Turn over

Section A: Pure Mathematics [40 marks]

1



An offshore oil well is located in the ocean at point W, which is 8 km from point A on land as shown in the figure above. Oil is to be piped to point B, which is 12 km from point A, using a straight pipe laid underwater from W to some point P between A and B, and then on to point B via another straight pipe laid on land.

If the cost of laying a pipe is \$60 000 per km underwater and \$45 000 per km on land, how far should point P lie from point A to minimise the cost of laying the pipe? [5]

2 In the theory of learning, the amount of material memorised by a person, y units, in time t minutes satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}t} = k(10 - y),$$

where k is a positive constant.

Initially, zero units of material were memorised.

(i) Find y in terms of t, given that five units of material were memorised in two minutes. [6]

(ii) Sketch the graph of y against t and explain what will eventually happen to the amount of material memorised. [3]

(iii) The rate at which material is forgotten is known to be proportional to the amount memorised in time t. Write down a differential equation involving y when forgetfulness is taken into account. [2]

3 The complex number z satisfies the relations $|z-2\mathrm{i}|<4$ and $|z+4-6\mathrm{i}|\leq|z-2\mathrm{i}|$.

(i) Illustrate both of these relations on a single Argand diagram. [3]

(ii) Find the range of values of arg z. [3]

Another complex number w is such that $\arg(w-2-4\mathrm{i})=\arg(-1-\mathrm{i})$.

(iii) Sketch the locus of the points representing w on the same Argand diagram in part **(i)** and hence find the exact least value of $|z-w|$. [5]

4 The planes Π_1 and Π_2 have equations $\mathbf{r}\bullet(\mathbf{i}-2\mathbf{j}+\mathbf{k})=4$ and $\mathbf{r}=3\mathbf{i}-\mathbf{j}+\mathbf{k}+\lambda(-\mathbf{i}+\mathbf{j}+\mathbf{k})+\mu(4\mathbf{i}+\mathbf{j})$ respectively, and meet in a line l_1.

(i) Find the acute angle between Π_1 and Π_2. [3]

(ii) Find a vector equation of l_1. [2]

(iii) The points A and B have coordinates $(6,\ 3,\ -5)$ and $(2,\ 3,\ 1)$ respectively. Find the length of projection of $\overrightarrow{AB}$ onto the line l_1. [2]

The line l_2 passes through the point C with position vector $p\mathbf{i}+(2p+1)\mathbf{j}-3\mathbf{k}$ and is parallel to $3q\mathbf{i}-3\mathbf{j}+q\mathbf{k}$, where p and q are positive constants. Given that the perpendicular distance from C to Π_1 is $\frac{15}{\sqrt{6}}$ and that the acute angle between l_2 and Π_1 is $\sin^{-1}\left(\frac{2}{\sqrt{6}}\right)$, find the values of p and q. [6]

Section B: Statistics [60 marks]

5 A company has 1500 employees. 35% of employees are in the 21-40 age group, 50% are in the 41-60 age group, and the rest are in the 60 and above age group.

A sample of 50 employees is to be chosen to take part in a survey on staff morale. A list of the employees in alphabetical order by name is generated. A random number k from 1 to 30 is chosen. The kth employee on the list is chosen and thereafter, every 30th employee on the list will be chosen until a total of 50 employees are chosen.

Give a disadvantage of this sampling method. [1]

State the name of a method of sampling that would not have this disadvantage, and describe how such a sample could be carried out. [3]

6 Find the number of ways in which 4-letter code-words can be obtained from the word ENDANGERED if

(i) there are no repeated letters, [1]

(ii) there are three "E"s, [2]

(iii) there is at least one repeated letter. [3]

7 A tennis match is played between two players, A and B. The match consists of at most three sets. Each set is won by either A or B, and the match is won by the first player to win two sets. The probability that A wins the first set is p.

For each set after the first, the conditional probability that A wins the set, given that A won the preceding set, is 0.7.

For each set after the first, the conditional probability that B wins the set, given that B won the preceding set, is 0.6.

Construct a probability tree showing this information. [2]

(i) When a match is decided by playing all three sets, the probability of A winning is the same as that of B winning. Show that the value of p is $\frac{8}{11}$. [2]

(ii) Find the probability that B wins the first set, given that A wins the match. [3]

8 Fruits are sold by weight. Apples are sold at \$2 per kg and oranges at \$3 per kg. The masses, in kg, of apples and oranges are modelled as having independent normal distributions with means and standard deviations as shown in the table.

	Mean mass	Standard deviation
Apples	0.2	0.05
Oranges	0.3	0.08

(i) Find the probability that the difference between the weight of 3 apples and 1 orange is less than 0.2 kg. [3]

(ii) Find the probability that the total selling price of 5 apples and 5 oranges exceeds \$5. [3]

(iii) Find the least value of n such that there is a probability of less than 0.3 that the mean weight of n randomly chosen apples is greater than 0.21 kg. [3]

9 To maintain the good condition of library books, any book with more than 7 defects will be donated away. The mean number of defects per book in the reference section is 3. The number of defects in a reference book is the random variable R.

(i) State, in the context of this question, two assumptions needed to model R by a Poisson distribution. [2]

(ii) Find the probability that a reference book is donated away. [1]

(iii) There are 1000 reference books in the library. Use an appropriate approximation to find the probability that at least 10 but no more than 15 reference books will be donated away. [3]

The mean number of defects per book in the children's section is λ.

(iv) Given that $\lambda = 5$, find the probability that the total number of defects in a randomly chosen reference book and in a randomly chosen children's book is fewer than 10. [2]

(v) The probability that a children's book has at most one defect is 0.1. Find an equation for λ and hence find the value of λ. [3]

10 An insurance company declares that the mean payment made on motor insurance claims is \$10 000. The auditor examines a random sample of 25 claims and records the payment made for each claim. The payments, \$ x, are summarised by

$$\sum(x - 10\,000) = -6560 \qquad \text{and} \qquad \sum(x - 10\,000)^2 = 7\,493\,500.$$

(i) Stating any necessary assumptions, test whether the mean payment made is less than \$10 000 at the 1% significance level. [6]

(ii) Explain the meaning of "at the 1% significance level" in the context of the question. [1]

(iii) Without carrying out any further hypothesis tests, state with a reason whether the conclusion in part **(i)** would be the same if a two-tailed test is carried out at the same significance level. [1]

Suppose now that the standard deviation of the payments made on motor insurance claims is known to be \$500, and that any assumptions made in part **(i)** are still valid.

(iv) Find the largest value of the mean payment made that the insurance company should declare so that it will not be rejected as an overestimate at the 5% significance level, giving your answer correct to the nearest dollar. [3]

11 An old film roll is treated with a chemical in order to improve the contrast. The table below shows the amount of chemical applied, x ml, and the contrast index, y, which takes values between 0 (no contrast) and 100 (maximum contrast), for nine samples drawn from the film.

x	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
y	19	46	64	73	80	55	88	93	95

(i) Draw a scatter diagram for the data. [2]

It is discovered that one of the samples of film was damaged and produced an incorrect result.

(ii) Indicate the corresponding point on your scatter diagram by labelling it as P. [1]

In all subsequent calculations, data point P is omitted.

(iii) Calculate the equation of the regression line of y on x. [1]

(iv) Estimate the value of y when $x = 5.0$, and comment on the suitability of the linear model. [2]

It is suggested that the relationship between x and y can be modelled by

$$\text{A:} \quad y = a + \frac{b}{x},$$

$$\text{B:} \quad y = ax^b.$$

(v) Find the value of the product moment correlation coefficient for models A and B, and determine which is the better model. [3]

(vi) Use a regression line to give the best estimate that you can of the amount of chemical applied when the contrast index is 75. Comment on the reliability of your estimate. [3]