RVHS H2 Mathematics Remedial Programme

Topic: Discrete Random Variables

Basic Mastery Questions

1. **CJC MYE 9758/2021//Q9 (Parts)**

A biased red die is such that the probability of any face landing upwards is proportional to the square of the number on that face. The random variable X denotes the score obtained in one throw of this die with $\mathrm{P}(X=r)=kr^2$, where $r=1$, 2, 3, 4, 5, 6, and k is a constant.

(i) Find the exact value of k. [2]

A second biased die is yellow and the random variable Y denotes the score obtained when the yellow die is thrown once. The probability distribution of Y is

y	2	4	6
$\mathrm{P}(Y=y)$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{2}{5}$

(ii) Find $\mathrm{E}(Y)$ and show that $\mathrm{Var}(Y)=\frac{56}{25}$. [3]

(iii) Given that Y_1 and Y_2 are two independent observations of Y, find $\mathrm{E}(Y_1-Y_2)$ and $\mathrm{Var}(Y_1-Y_2)$. [2]

Answer: (i) $k=\frac{1}{91}$ **(ii)** $\frac{22}{5}$ **(iii)** 0, 4.48

(i)

r	1	2	3	4	5	6
$\mathrm{P}(X=r)$	k	$4k$	$9k$	$16k$	$25k$	$36k$

$$k+4k+9k+16k+25k+36k=1$$
$$91k=1$$
$$k=\frac{1}{91}$$

(ii)

$$\mathrm{E}(Y)=2\left(\frac{1}{5}\right)+4\left(\frac{2}{5}\right)+6\left(\frac{2}{5}\right)=\frac{22}{5}$$
$$\mathrm{E}(Y^2)=2^2\left(\frac{1}{5}\right)+4^2\left(\frac{2}{5}\right)+6^2\left(\frac{2}{5}\right)=\frac{108}{5}$$

	$\text{Var}(Y) = \text{E}(Y^2) - [\text{E}(Y)]^2$ $= \frac{108}{5} - \left(\frac{22}{5}\right)^2$ $= \frac{56}{25}$ (shown)
(iii)	$\text{E}(Y_1 - Y_2) = \text{E}(Y_1) - \text{E}(Y_2) = \frac{22}{5} - \frac{22}{5} = 0$ $\text{Var}(Y_1 - Y_2) = \text{Var}(Y_1) + \text{Var}(Y_2) = \frac{56}{25} + \frac{56}{25} = \frac{112}{25}$ (or 4.48)

2. **RI MYE 9758/2021//Q10(i)**
In a game, a player tosses a fair die, whose faces are numbered from 1 to 6. If the player obtains a 6, he tosses the die a second time, and in this case, his score is the absolute difference of 6 and the second number. Otherwise, his score is the number obtained in the first toss.

Let the player's score be denoted by X.
Show that $\text{P}(X = 1) = \frac{7}{36}$ and tabulate the probability distribution of X. [3]

$\text{P}(X = 1)$ = P(first throw = 1) + P(1st throw = 6, 2nd throw = 5)

$$= \frac{1}{6} + \left(\frac{1}{6}\right)\left(\frac{1}{6}\right)$$

$$= \frac{1}{6} + \frac{1}{36}$$

$$= \frac{7}{36} \quad \text{(Shown)}$$

Probability distribution of X

x	0	1	2	3	4	5
$\text{P}(X = x)$	$\frac{1}{36}$	$\frac{7}{36}$	$\frac{7}{36}$	$\frac{7}{36}$	$\frac{7}{36}$	$\frac{7}{36}$

Standard Questions

1. **HCI MYE 9758/2020//Q8 (Parts)**

A bag contains 9 numbered balls of identical size. Four of the balls are numbered 3, three of the balls are numbered 4 and two of the balls are numbered 5. In a game, three balls are drawn from the bag at random, without replacement. The random variable *S* is the sum of the numbers on the three balls drawn.

(i) Show that $\mathrm{P}(S=12)=\dfrac{25}{84}$ and find the probability distribution of *S*. [4]

(ii) Show that the probability where the sum of the numbers on the three balls drawn is a multiple of 3 is given by $\dfrac{29}{84}$. [1]

(i)	When listing out all the outcomes, do it systematically: (a) All 3 balls have the same number 3+3+3 = 9 **4+4+4 = 12** 5+5+5 = 15 (b) Only 2 balls have the same number 3+3+(4 or 5) = 10 or 11 4+4+(3 or 5) = 11 or 13 5+5+(3 or 4) = 13 or 14 (c) All 3 balls different numbers **3+4+5 = 12** $\mathrm{P}(S=12)=\dfrac{{}^4C_1\times{}^3C_1\times{}^2C_1}{{}^9C_3}+\dfrac{{}^3C_3}{{}^9C_3}=\dfrac{25}{84}$ Or $\mathrm{P}(S=12)=\left(\dfrac{4}{9}\times\dfrac{3}{8}\times\dfrac{2}{7}\times3!\right)+\left(\dfrac{3}{9}\times\dfrac{2}{8}\times\dfrac{1}{7}\right)=\dfrac{25}{84}$ (shown) $\mathrm{P}(S=9)=\dfrac{{}^4C_3}{{}^9C_3}=\dfrac{1}{21}$ Or $\mathrm{P}(S=9)=\dfrac{4}{9}\times\dfrac{3}{8}\times\dfrac{2}{7}=\dfrac{1}{21}$ $\mathrm{P}(S=10)=\dfrac{{}^4C_2\times{}^3C_1}{{}^9C_3}=\dfrac{3}{14}$ Or $\mathrm{P}(S=10)=\dfrac{4}{9}\times\dfrac{3}{8}\times\dfrac{3}{7}\times3=\dfrac{3}{14}$

$$P(S=11)=\frac{{}^{4}C_2\times{}^{2}C_1}{{}^{9}C_3}+\frac{{}^{4}C_1\times{}^{3}C_2}{{}^{9}C_3}=\frac{2}{7}$$

Or $$P(S=11)=\left(\frac{4}{9}\times\frac{3}{8}\times\frac{2}{7}\times3\right)+\left(\frac{3}{9}\times\frac{2}{8}\times\frac{4}{7}\times3\right)=\frac{2}{7}$$

$$P(S=13)=\frac{{}^{4}C_1\times{}^{2}C_2}{{}^{9}C_3}+\frac{{}^{3}C_2\times{}^{2}C_1}{{}^{9}C_3}=\frac{5}{42}$$

Or $$P(S=13)=\left(\frac{3}{9}\times\frac{2}{8}\times\frac{2}{7}\times3\right)+\left(\frac{2}{9}\times\frac{1}{8}\times\frac{4}{7}\times3\right)=\frac{5}{42}$$

$$P(S=14)=\frac{{}^{3}C_1\times{}^{2}C_2}{{}^{9}C_3}=\frac{1}{28}$$

Or $$P(S=14)=\frac{2}{9}\times\frac{1}{8}\times\frac{3}{7}\times3=\frac{1}{28}$$

s	9	10	11	12	13	14
$P(S=s)$	$\frac{1}{21}$	$\frac{3}{14}$	$\frac{2}{7}$	$\frac{25}{84}$	$\frac{5}{42}$	$\frac{1}{28}$

(ii)

Required probability $=P(S=9)+P(S=12)$

$$=\frac{1}{21}+\frac{25}{84}=\frac{29}{84}$$

2. **RVHS MYE 9758/2020//Q8**

In a funfair game, a game-master set up two boxes with each box containing four cards, numbered 1, 2, 3, 4.

A player draws one card at random from each box and his score X, is the product of the numbers on the two cards.

(i) Find the probability distribution of X. [2]

(ii) Calculate the mean score and the variance exactly. [2]

The game-master charges \$$p$ for each game. If the player's score is odd, the player wins a \$5 cash voucher. Otherwise, the game ends.

(iii) Find the range of values of p for the game to be in favour of the game-master. [2]

Answer: (ii) 6.25, 17.1875 **(iii)** $p > \frac{5}{3}$

(i)

Probability distribution of X:

x	1	2	3	4	6	8	9	12	16
$P(X = x)$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{2}{16}$	$\frac{2}{16}$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{1}{16}$

(ii)

$$\begin{aligned} E(X) &= \frac{1}{16}(1\times1+2\times2+3\times2+4\times3+6\times2 \\ &\quad +8\times2+9\times1+12\times2+16\times1) \\ &= 6.25 \end{aligned}$$

$$\begin{aligned} E(X^2) &= \frac{1}{16}(1^2\times1+2^2\times2+3^2\times2+4^2\times3+6^2\times2 \\ &\quad +8^2\times2+9^2\times1+12^2\times2+16^2\times1) \\ &= 56.25 \end{aligned}$$

$\text{Var}(X) = E(X^2) - E(X)^2 = 17.1875$

(iii)

Let Y denote the gain of the game-master.

For the game to be in favour of game-master, $E(Y) > 0$

$$\Rightarrow p \times \mathrm{P}(X \text{ is even}) + (-5) \times \mathrm{P}(X \text{ is odd}) > 0$$

$$\Rightarrow p\left(\frac{3}{4}\right) - 5\left(\frac{1}{4}\right) > 0$$

$$\therefore p > \frac{5}{3} \quad (\text{or } p > 1.67)$$